THE THEORY OF ELECTRICITY.

Theorie der Elektrizität. Von M. ABRAHAM. Erster Band: *Einführung in die Maxwellsche Theorie der Elektrizität.* Von A. FÖPPL. Zweite, vollständig umgearbeitete Auflage. Von M. ABRAHAM. Leipzig, B. G. Teubner, 1904. xviii + 443 pp.

BEFORE this, we have had occasion to praise Föppl's lectures on technical mechanics.* His treatise on electricity is no less deserving. When this first appeared in 1894 it was, at all events for Germany, a new departure. Vector analysis was consistently employed throughout, and the point of view of Maxwell, which was so long in winning its proper recognition on the continent, was unequivocally and wholly adopted, even for elementary instruction. It must be a cause of gratification to Föppl to find a new edition called for in less than a decade; and to us it should be a matter for rejoicing that the new edition is not merely an improvement on the old, but an extension of it so as to form in two volumes a really modern treatise on electricity and magnetism from the Maxwell-J. J. Thomson point of view.

The theory of electrons and the atomic nature of electricity have recently become so firmly established as a highly valuable, if not yet indispensable, interpretation of nature, that a connected treatise beginning with the equations of Maxwell and leading through the motion of charged bodies to the consideration of atomic electricity and electrical mass is justified and necessary. Undoubtedly the present work will contribute to the popularization and ultimate adoption of the points of view for which it stands. Abraham, well known for his researches along just these lines, is peculiarly fitted to undertake this revision and enlargement of Föppl's treatise. The task has not been small. It has been ably executed.

The first section, containing the theory of vectors and vector fields, has been so amplified as to give the reader a summary exposition of those theorems in dynamics and hydrodynamics which form the basis of present views of electrical action. Surfaces of discontinuity, divergence, curl, and their relation to the theory of double layers (Doppelschichten, couches doubles) have their due attention. It is pointed out that the divergence and curl of a vector function **V** are given by the formulas

* Reviewed in this BULLETIN, vol. 9, pages 25–35.

$$\operatorname{div} \mathbf{V} = \lim_{d\tau \doteq 0} \frac{\int\int_S d\mathbf{S} \cdot \mathbf{V}}{d\tau}, \quad \mathbf{n} \cdot \operatorname{curl} \mathbf{V} = \lim_{dS \doteq 0} \frac{\int_C d\mathbf{r} \cdot \mathbf{V}}{dS},$$

where $d\tau$, $d\mathbf{S}$, $d\mathbf{r}$ are respectively the elements of volume, of surface, and of arc, and where $\mathbf{n}$ is a unit vector in the direction $d\mathbf{S}$. It may be observed that a very fruitful method of introducing the divergence and curl is obtained by adopting these expressions as definitions. It is not difficult to show that for ordinary vector functions these limits exist independently of the way in which $d\tau$ and $d\mathbf{S}$ approach zero — or these facts may be taken for granted without introducing any more assumptions than are frequently made in disregarding the undesired parts of integrals arising in connection with the theorems of Gauss and Stokes.

In regard to notation Föppl essentially followed Heaviside.* Abraham follows the usual convention, and adopts a new notation of his own. In the Encyclopedia,† he used italic type for vectors and denoted the components by subscript letters — A, A_x, A_y, A_z. Föppl originally used heavy German type for vectors and the corresponding letters with subscript numbers for the components — $\boldsymbol{\mathfrak{A}}$, A_1, A_2, A_3. He used $\boldsymbol{\mathfrak{A}\mathfrak{B}}$ and $V\boldsymbol{\mathfrak{A}\mathfrak{B}}$ for the scalar and vector products. Lorentz in the Encyclopedia ‡ used ordinary German type for vectors and the same with subscript x, y, z for the components. He employs $\mathfrak{A}\mathfrak{B}$ and $[\mathfrak{A}\mathfrak{B}]$ for the scalar and vector products. Abraham now uses heavy German type for vectors and the same with subscript letters for the components — $\boldsymbol{\mathfrak{A}}$, $\boldsymbol{\mathfrak{A}}_x$, $\boldsymbol{\mathfrak{A}}_y$, $\boldsymbol{\mathfrak{A}}_z$. This prolific use of heavy type, even for the components which are scalars, produces an ugly looking page. For the scalar and vector products he uses the same notation as Lorentz. It might have been as well to retain Heaviside's notation. This would facilitate reading Heaviside; and reading Heaviside must be one of the aims of every student of electromagnetic theory. But evidently we do not approach a uniform notation as time goes on and new works appear.

In the main body of the work the present editor has rewritten and rearranged to a considerable extent, showing frequently

---

* "Electrical papers" (1892) and "Electromagnetic theory" (1893, 1899).

† Volume 4, part 3, pages 1–47 (1901).

‡ Volume 5, part 2, pages 63–280 (1904).

more the influence of J. J. Thomson* and Lorentz, where the original author inclined rather to Heaviside. Thus Abraham writes

$$\mathbf{I} = \frac{\mu - 1}{4\pi}\mathbf{H},$$

where Föppl wrote

$$\mathbf{I} = \frac{\mu - \mu_0}{4\pi\mu_0}\mathbf{H};$$

and $\mu = 1 + 4\pi\kappa$ where the original had $\mu = \mu_0(1 + 4\pi\kappa)$. Heaviside and Gray† would probably think this a step backward — and so indeed it is when viewed from the theory of dimensions. For κ is a pure number and not of the doubtful dimensions of μ. To be sure, the matter is carefully explained in the text, where it is pointed out that the dimensions of ϵ, μ, and c are chosen differently in different systems of units; and that in the system which is here adopted $[\epsilon] = 0$, $[\mu] = 0$, and $[c] = LT^{-1}$. Nevertheless in a work which is for the most part theoretical and where computation in any particular system is generally foregone it would seem wiser not to specialize the dimensions in this manner. The world over, seconds are used as units of time. Time might therefore be assumed as a pure scalar devoid of dimensions — but mechanics would thereby lose. Although we might not agree with Heaviside that one system of units is more "rational" than another (it may be more symmetrical, but irrationality is bound to come in if one has squares and circles to measure), we are inclined to agree with him in the matter of dimensions and to regret the alteration of Föppl equations to a form in which they are good only for one particular system of units.‡

To one acquainted with recent English and American works on electricity it appears very strange to read through a considable volume without finding much if anything concerning Faraday tubes. The reason for the omission is not far to seek. If one is familiar with the underlying conceptions of dynamics, and

---

*We may note J. J. Thomson's "Elements of Electricity and Magnetism" just out in a third edition. It covers largely the same ground as the book under review.

† Treatise on Magnetism and Electricity," Volume 1 (1898).

‡ The whole matter of systems of units for electromagnetic theory seems unfortunately becoming more and more complicated and diversified, as is the case with vector methods. More than half a dozen systems are set forth by Lorentz, *loc. cit.*

especially of hydrodynamics as set forth in the earlier chapters of the book, this Faraday tube is not a crying necessity. It offers a good visual image of a number of things going on in the medium — tension along the lines of force and pressure perpendicular to them. It may even serve to give a good idea of what takes place in the medium when a condenser discharges. And we should have been glad to see some exposition of the Faraday tube by Abraham and Föppl. But it is better to leave this mechanism or model or visualization — whatever one wishes to call it — untouched than to push it so far as to leave the reader with too crudely exaggerated a notion of its real existence if it is but a visualization, or with a conception which the reader cannot push further without bewilderment in case the tube is a reality.

The statement is often seen that from each negative electric atom there issues one unit Faraday tube terminating upon some surface of positive charge. We have been asked by physicists of considerable eminence whether the electron dragged its tube after it, or pushed it in front, or how the thing *did* look in motion or at rest. And small wonder, when the chief authority writes: * "The Faraday tubes stretching through the ether cannot be regarded as entirely filling it. They are rather to be looked upon as discrete threads embedded in a continuous ether, giving to the latter a fibrous structure; but if this is the case, then, on the view we have taken of a wave of light, the wave itself must have structure, and the front of the wave instead of being, as it were, uniformly illuminated, will be represented by a series of bright specks on a dark ground, the bright specks corresponding to the places where the Faraday tubes cut the wave front."

We are not prepared to dispute Thomson. His piercing imagination and scientific method have rendered too great services to physics. We are simply unable to follow him. An ether quasilabile or rigid, and atoms of electricity positive and negative, and Faraday tubes of the old-fashioned hydrodynamic sort we readily admit; but that the harmony of the spheres should be due to the transverse vibrations of myriads of tightly stretched strings (how long, and stretched between what?) embedded in an ether is a little more than we are yet ready to comprehend. One is tempted to feel that the harmony would

* J. J. Thomson, "Electricity and matter," page 63 (1904).

become a terrible and discordant jangle after a few reflections, interferences, rotations, and the like. Certainly the introduction of this additional stringiness adds a considerable complication to our conception of the ether, which for the most part has always been regarded as homogeneous except as to points (matter) rather than lines (Faraday tubes). There seem to be, however, phenomena † which this picture explains. Perhaps the universe does indeed look like a tremendous boiling pot of spaghetti. On this point Föppl and Abraham are silent.

After thus developing at some length a few points which the Theorie der Elektrizität has suggested, we shall not take away the zest of the reader by entering into details of the work under review. It remains to state in closing that in the revision the modern point of view is so fully adopted and the connections with the older theories so little discussed that one who had been brought up on the old would have some difficulty in immediately comprehending the new, notwithstanding its logical excellence and clear exposition. But the utility of modulations has largely vanished during the past decade, and Abraham is undoubtedly right in omitting them. This book is written by the young school for the young school — and we wish it an increase of its deserved success. The second volume will be awaited with impatience.

EDWIN BIDWELL WILSON.

YALE UNIVERSITY,
February, 1904.

NOTES.

THE seventeenth regular meeting of the Chicago Section of the AMERICAN MATHEMATICAL SOCIETY will be held at the University of Chicago on Saturday, April 22. Titles and abstracts of papers to be presented at this meeting should be in the hands of the Secretary of the Section, Professor THOMAS F. HOLGATE, 617 Hamline Street, Evanston, Ill., not later than April 8.

THE Macmillan Company is about to issue for the AMERICAN MATHEMATICAL SOCIETY a volume of 187 pages, of BULLETIN size, containing the lectures delivered before the

* Thomson, *loc. cit.*