

one of 48 pages by Painlevé on "The development of analytic functions." The latter will perhaps be to some the most interesting part of the book, as in it M. Painlevé gives for the first time a connected development of the results which he has recently published in the *Comptes Rendus* concerning the expansion of analytic functions in series that are valid in the whole complex plane with the exception only of certain half-lines, *i. e.*, in the whole "star of holomorphism." But this review is already too long; we can not go into details.

The following are the only typographical errors we have noticed; page 7, line 2, for F read P , and line 8, for M read N ; page 16, line 5 from bottom, for E_1 read E_2 , and line 4 from bottom, for E_2 read E_1 ; page 33, line 9, after "limites" insert "inférieures et"; page 58, line 8 in the formula replace y_i by $(y_i - y_{i-1})$; page 60 in the expansion for $\sqrt{x^2}$ the last factors in the numerators of the coefficients of the second and third terms should be removed; page 80, the equations defining $\phi_{p,q}(x)$ are not correct when $p = 0$; page 82, in the last sentence of the footnote for Kircherberger read Kirchberger; page 85, line 15 from bottom, for A read A' .

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SHORTER NOTICES.

Advanced Algebra. By HERBERT E. HAWKES. Ginn and Company, 1905. 285 pp.

IN writing an Advanced Algebra, in the current acceptation of the phrase, two courses are fairly open to an author. He may lay his foundations deep and build on them with unflinching rigor, bringing teacher and pupil into intimate touch with some of the epoch-making researches of the past fifty years. From the standpoint of pure science, this is, of course, admirable; but the meat which it supplies is only for strong men. Foundation-laying is for the hardest of frontiersmen; and the methodical account of its severely logical steps makes demands upon the average freshman which are far beyond his power to meet.

The second course consists in a frank and explicit assumption of the postulates, as they become necessary in the order of

thought, and a frequent appeal to intuition to bridge over awkward chasms. The words "frank and explicit" are used advisedly, in order to differentiate this second course from a vicious middle one, which has often characterized our text books, namely that of making a show of thoroughness without furnishing the genuine article. Nothing is more fatal to deep and fruitful mathematical thinking than such pseudo-rigor.

The book before us follows very conscientiously the second of the courses noted above. The assumptions, so far from being veiled, appear with refreshing boldness as captions and in italics, and the pages ring true. Incidentally it may be remarked that the clear-cut "euclidean" mode of presentation greatly enhances the value of all the books of this Yale series. Every important discussion has a definite outcome and issues in a clearly stated and italicized proposition.

The author also shows courage in his omissions. He leaves out a number of things which for years have been held to belong to the orthodox canon. We look in vain for cube root, the familiar but forbidding proofs of the generalized binomial expansion, convergency, exponential and logarithmic series and the multiplication theorem for determinants. Derivatives, multiple roots and Sturm's theorem have also disappeared. Many thoughtful teachers will miss some of these things and find the book too meager, but the author would rightly rejoin, in the spirit of his preface, that they require the calculus or other portions of mathematics for their complete comprehension. At any rate it must be admitted that he has made good use of the space saved by these omissions, in devoting it to an unusually extended and interesting treatment of graphical representation. If one is seeking for a fine illustration of the utility of graphs in picturing a highly abstract bit of elementary analysis, he need only turn to the figure on page 105, introduced to illustrate the behavior of the general quadratic when the coefficient of x^2 is infinitesimal.

The book begins with a concise account of the fundamental operations and their laws, then proceeds with a rapid review of the usual topics as far as quadratics. Next follows a chapter on mathematical induction, a method which is immediately applied to the derivation of the binomial theorem in the case of positive integral exponents. From this point on we find the customary material of a manual of the kind, with the omissions already referred to. In nearly every chapter, however, the

subject matter has been refashioned, clarified and enlivened. If, in the modesty of the author's programme, scholarship may not have its perfect work, yet intimations of its existence are everywhere apparent. Above all else he rarely loses the viewpoint of the teacher or forgets the fact that he is forging a classroom tool.

While in this he has been exceptionally successful, his work is not without its faults. There are errors, some minor and others serious and misleading. One of the characteristics of the book is the repeated use of mathematical induction, yet in the first case in which this method is called into service (page 129) it is abandoned at the end for mere inference. A second attempt to use it (page 177) fails badly because of errors in subscripts and the repeated occurrence of the words "negative roots"—an error which is certainly not typographical. The proposition that imaginary roots enter in pairs is laboriously proved, when eight or ten lines would suffice for a satisfactory demonstration. On page 167 the converse of the corollary should have been stated and proved. While the discussion of the generalized binomial theorem has been perhaps wisely omitted, attention should have been called to the fact that it holds for negative and fractional exponents, and examples furnished to illustrate its use in such cases. In the chapter on determinants (page 221) $A_1, A_2,$ etc., are not "minors," as the author has defined this term, but the old-fashioned cofactors, and this error vitiates the proof. The value of the book, in the opinion of the reviewer, would have been increased by the addition of a brief chapter on the "gradients" of curves, immediately after that on graphs. An experience of several years shows that a very few pages would suffice—and few portions of algebra interest students more. An answer book has been compiled but its value is impaired by the errors which it contains.

But such faults as have been indicated can be easily corrected in a subsequent impression and do not invalidate the conclusion that the book under consideration is admirably adapted to the field which it undertakes to cover. Alike in its theory and in its practice it furnishes fresh, rich and wholesome material for the advanced course in fitting schools and first year work in our colleges.

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