THE SEPTEMBER MEETING OF THE SAN FRANCISCO SECTION.

THE tenth regular meeting of the San Francisco Section of the AMERICAN MATHEMATICAL SOCIETY was held at the University of California on Saturday, September 29, 1906. The following sixteen members were present:

Professor R. E. Allardice, Professor H. F. Blichfeldt, Mr. A. J. Champreux, Professor G. C. Edwards, Professor M. W. Haskell, Professor L. M. Hoskins, Professor D. N. Lehmer, Professor A. O. Leuschner, Dr. J. H. McDonald, Dr. W. A. Manning, Professor H. C. Moreno, Professor C. A. Noble, Dr. T. M. Putnam, Professor Irving Stringham, Mr. L. C. Walker, Professor E. J. Wilczynski.

Professor R. E. Allardice occupied the chair at both the morning and afternoon sessions. The following officers were elected for the ensuing year: Professor E. J. Wilczynski, chairman; Dr. W. A. Manning, secretary; Professor M. W. Haskell, Professor H. F. Blichfeldt, and Dr. W. A. Manning, program committee. It was arranged to hold the next meeting on Saturday, February 23, at Stanford University.

The following papers were read at this meeting:

(1) PROFESSOR R. E. ALLARDICE: "Additional note on the multiple points of unicursal curves."

(2) PROFESSOR H. F. BLICHFELDT: "A theorem concerning the Sylow subgroups of simple groups."

(3) PROFESSOR M. W. HASKELL: "On the collineation group belonging to triangles quadruply in perspective."

(4) PROFESSOR L. M. HOSKINS: "The effect of viscosity on the propagation of free vibrations in an elastic solid."

(5) DR. W. A. MANNING: "On the order of primitive groups."

(6) PROFESSOR R. E. MORITZ: "On the symbolic representation of quotiential coefficients of the second order."

The paper by Professor Moritz was read by title. Abstracts of the papers follow below. The abstracts are numbered to correspond to the titles in the list above.

1. If a unicursal curve be given by the equations $x = (a_0, a_1, \cdots, a_n)(\lambda, 1)^n$, etc., the multiple points depend on the

quantities $A = (a_0, b_1, c_2)$, etc. It is known that the multiple point of a cubic curve is given by the hessian of the cubic $(A, B, C, D)(\lambda, 1)^3$. In the case of curves of a higher order the problem becomes complicated on account of the fact that the quantities (a_0, b_1, c_2), (a_0, b_1, c_3), etc., are not independent. In a previous paper Professor Allardice gave a theorem for the quartic curve analogous to the theorem for the cubic stated above; and in an additional note, read at this meeting, curves of the fifth degree in space of two, three and four dimensions were considered.

2. Frobenius has proved ("Ueber auflösbare Gruppen V," *Sitzungsberichte der Berliner Akademie*, 1901, page 1324) that in a simple group G of composite order there must be contained a subgroup H whose order is a power of a prime p, and a substitution S of order prime to p which transforms H into itself yet is not commutative with every substitution of H. Professor Blichfeldt proves that we may use for H a subgroup of G of order p^λ, this being the highest power of p which divides the order of G, provided such a subgroup is abelian, or of some other specified type.

3. Professor Haskell showed that it is possible to construct a system of four triangles, each pair quadruply in perspective and inscribed in a conic, and that the perspective reflections thus defined generate a group of 72 collineations which is equivalent to the group

$$y_1 : y_2 : y_3 = \gamma^\lambda x_p : \gamma^\mu x_q : \gamma^\nu x_r,$$

where $\gamma^6 = 1$, $\lambda + \mu + \nu \equiv 0 \pmod 6$ and p, q, r are 1, 2, 3 in any order. This group might well be regarded as trivial, apart from its relation to the above configuration. This configuration exhibits further an interesting relation to the nine points of inflexion of a cubic and their harmonic polars.

4. In an isotropic elastic solid without internal friction or viscosity, dilatational and rotational vibrations travel with velocities which depend upon the elastic moduli and the density but are independent of the frequency. The object of Professor Hoskins's paper is to investigate how this result is affected by viscosity. The discussion is restricted to the case in which viscosity is specified by a single modulus ν, the differential

equations for the displacements being obtained from those applying to a non-viscous solid by substituting $n + \nu d/dt$ for the rigidity-modulus n. It is found that the velocity of propagation is not independent of the frequency, and that for each kind of vibration it may have any value from 0 up to the value corresponding to no viscosity. It thus appears that viscosity furnishes a sufficient explanation of the fact that earthquake vibrations lasting but a fraction of a minute near their place of origin are recorded for several hours at distant places; the explanation being that the original complex disturbance analyzes into simple vibrations which travel with velocities varying from the greatest value (corresponding to no viscosity) down to zero.

5. Dr. Manning presented the following theorem: Let q be one of the numbers 2, 3, 4; p any prime greater than $q + 1$; the degree of a primitive group which contains a substitution of order p with q cycles (without containing the alternating group) cannot exceed $pq + q$. This is directly connected with the similar theorem stated, in part without proof, by Jordan in the *Bulletin de la Société mathématique de France*, volume 1, page 221.

6. Professor Moritz's paper deals with analogies between certain differentiation formulas and the corresponding quotientiation formulas. It is first shown that for a function of n dependent variables $y = F(u_1, u_2, \cdots, u_n)$

$$\frac{qy}{qx} = \left(\frac{qy}{qu_1}\right)\frac{qu_1}{qx} + \left(\frac{qy}{qu_2}\right)\frac{qu_2}{qx} + \cdots + \left(\frac{qy}{qu_n}\right)\frac{qu_n}{qx},$$

where the parentheses are used in the eulerian sense to denote partial variations of the variables. The second quotiential coefficient is given by

$$\frac{q^2y}{qx^2}\;\frac{qy}{qx} = \sum_{i=1}^{n}\left(\frac{qy}{qu_i}\right)\left(\frac{q^2y}{qu_i^2}\right)\frac{\overline{qu_i}^2}{qx} + 2\sum_{\substack{k=2\\ i<k}}^{k=n}\left(\frac{qy}{qu_k}\right)\left(\frac{q^2y}{qu_iqu_k}\right)\frac{qu_i}{qx}\;\frac{qu_k}{qx}$$
$$+ \sum_{i=1}^{n}\left(\frac{qy}{qu_i}\right)\frac{qu_i}{qx}\;\frac{q^2u_i}{qx^2}.$$

It is pointed out that this expression may be written symbolically thus

$$\frac{q^2y}{qx^2}\frac{qy}{qx}=\left[\sum_{i=1}^{n}\left(\frac{qy}{qu_i}\right)\frac{qu_i}{qx}\right]^{(2)}+\sum_{i=1}^{n}\left(\frac{qy}{qu_i}\right)\frac{qu_i}{qx}\frac{q^2u_i}{qx^2},$$

where the exponent in parenthesis signifies that the expression to which it is attached is to be squared, and after squaring

$$\left(\frac{qy}{qu}\right)^2, \quad \left(\frac{qy}{qu}\right)\left(\frac{qy}{qv}\right), \quad \left(\frac{qy}{qv}\right)^2$$

are to be replaced by

$$\left(\frac{qy}{qu}\right)\left(\frac{q^2y}{qu^2}\right), \quad \left(\frac{qy}{qv}\right)\left(\frac{q^2y}{quqv}\right), \quad \left(\frac{qy}{qv}\right)\left(\frac{q^2y}{qv^2}\right),$$

respectively.

W. A. MANNING,
Secretary of the Section.

PROJECTIVE DIFFERENTIAL GEOMETRY.

AN ABSTRACT OF FOUR LECTURES DELIVERED AT THE NEW HAVEN COLLOQUIUM, SEPTEMBER 5–8, 1906.

BY PROFESSOR E. J. WILCZYNSKI.

THESE four lectures were devoted to an exposition of the principal results belonging to the subject of projective differential geometry. The place of this subject in a systematic treatment of geometry is indicated by the following discussion.

A first important basis for the classification of the various geometries is furnished by the group concept. There is metric geometry, projective geometry, the geometry of the birational transformations, to mention only the most important. Together with this classification by means of the characteristic groups, there is the distinction between differential and integral geometry. The differential properties of a geometric configuration merely depend upon the fact that in a certain, perhaps very small, region, certain conditions of continuity are satisfied, that derivatives of a certain order exist, etc. These differential properties are studied by means of the differential calcu-