NOTE ON THE ORIENTATION OF A SECANT.

BY PROFESSOR L. D. AMES.

(Read at the Preliminary Meeting of the Southwestern Section of the American Mathematical Society, December 1, 1906.)

IN a previous paper * the orientation of curves and their tangents is briefly discussed. For certain problems in the theory of the potential function it is desirable to be able to define uniquely the positive sense along a secant through two variable points of an open smooth curve in such a way that the angle α between the positive direction of the secant and a fixed line shall be a continuous function of the two variable points. Such a definition will include the tangent as a special case. A natural method would be to define the positive sense uniquely and then prove that α is continuous. Another method would be to define the positive sense of the secant for one position and then define the positive sense in every other position so that the angle α shall be continuous. Either method involves somewhat troublesome considerations if the angle is defined by means of any one of the inverse trigonometric functions. The method of defining an angle used in the paper above mentioned proves useful in this case.

Given the open curve

$$x = x(t), \quad y = y(t), \qquad t_0 \leqq t \leqq T,$$

where x, y, dx/dt, dy/dt are single-valued and continuous in the interval (t_0, T), and where

$$x(t_1) \neq x(t_2), \quad y(t_1) \neq y(t_2) \quad \text{when } t_1 \neq t_2.$$

Define a secant through the points t_1, t_2 as follows:

$$X = \epsilon\lambda\rho + x(t_1), \quad Y = \epsilon\mu\rho + y(t_1),$$

where ϵ is arbitrarily chosen either $+1$ or -1, ρ is the variable parameter, and

* "An arithmetic treatment of some problems in analysis situs," *Amer. Jour. of Mathematics*, Oct., 1905, p. 349.

$$\lambda = \frac{x(t_2) - x(t_1)}{t_2 - t_1}, \qquad \mu = \frac{y(t_2) - y(t_1)}{t_2 - t_1} \quad \text{when} \quad t_1 \neq t_2$$

and

$$\lambda = \frac{dx_1}{dt}, \qquad \mu = \frac{dy_1}{dt}, \qquad \text{when} \quad t_1 = t_2.$$

Define the *positive sense* along this line to be the direction of increasing ρ. This is uniquely defined for any pair of points in the interval (t_0, T) since λ and μ are unchanged by interchanging t_1 and t_2.

Define an angle α as follows:

$$\sin\alpha = \epsilon k\mu, \quad \cos\alpha = \epsilon k\lambda, \quad k = (\lambda^2 + \mu^2)^{-\frac{1}{2}}.$$

Then α is an infinitely many-valued function of t_1 and t_2, its values for any given pair of values of t_1 and t_2 differing by multiples of 2π. If one of these values α' be assigned to a particular pair t_1', t_2', then from the possible values of α one and only one single valued continuous function can be chosen which takes the value α' at t_1', t_2'.*

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ON EULER'S ϕ-FUNCTION.

BY PROFESSOR R. D. CARMICHAEL.

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THE object of the present note is the demonstration of certain very elementary propositions concerning Euler's ϕ-function of a number.

I. *The relation $\phi(m) = n$, a given number, is never uniquely satisfied for any given value of n.* That is, there is always more than one value of m for every possible value of n.

If any solution is $m =$ an odd number, then the given relation is satisfied by $2m$ also. Likewise, if m is twice an odd number, we may show that $m/2$ will also satisfy the relation.

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* Cf., e. g., Stolz, Differential-Rechnung, vol. 2, p. 15–20.