

SHORTER NOTICES.

The Theory of Determinants ; in the Historical Order of Development. Second edition, revised. By THOMAS MUIR. London, Macmillan and Co., 1906. xii + 491 pp.

THE present work "is intended for the student who aims at acquiring such a knowledge as can only be got by a study of the subject in the historical order of its development, for the investigator who is specially interested in this branch of mathematics and wishes to become acquainted with the various lines of attack opened up by previous workers, and for the general working mathematician who requires guide books and books of reference concerning special domains."* It covers the period from the beginning of determinants in 1693 to the close of 1841. Jacobi's long and exhaustive memoirs published in volumes 27 and 29 of *Crelle's Journal* are also considered in connection with skew determinants, although they appeared three or four years after the close of the period under consideration.

About one-third of the space is devoted to quotations from the works of the authors under consideration. These quotations enable the reader to form an independent judgment in reference to questions of priority and are especially valuable to those who do not have access to large libraries. Each new theorem of importance is numbered in Roman figures, and if the same result is either generalized or obtained in a different way by a subsequent writer it is marked among the contributions of the latter with the same Roman figure followed by an Arabic numeral.

Two very useful tables are inserted, showing the advance of the subject from 1693 to 1812 and from 1813 to 1841 respectively. There is also an index to the numbered results, but there is no general subject index. Its value as a reference book would have been greatly enhanced by such an index, especially since the chronological order of arrangement made it frequently impossible to bring together the material relating to the same topic.

The book is divided into two parts. Part I, in ten chapters, treats of general determinants and occupies considerably more

* From the preface.

than half the space. Complete chapters are devoted to the memoirs appearing in each of the years 1812 and 1841, while only one is given to those which appeared in the eighty-seven years from 1693 to 1779. Part II is devoted to special determinants which began to be studied, though without the present notation, by Lagrange and Gauss as early as 1773 and 1801 respectively. One chapter is devoted to each of the following subjects: axisymmetric determinants, alternants, jacobians, skew determinants, orthogonants.

Chapter XVI treats the miscellaneous special forms which prior to 1841 had received but little attention. The most fertile originator of such forms was Wronski, a poor Polish enthusiast whose egotism and wearisome style tended to attract few followers. The chapter is mainly devoted to the four special forms of determinants studied by Wronski, three of which were introduced by him. The other writers considered in this chapter are Scherk, Schweins, Jacobi, and Sylvester, the first two in relation to extensions of Wronski's results, the last two in relation to special forms of determinants arising from Bezout's method of eliminating the unknown between two equations of the n th degree and from Sylvester's dialytic process.

The last chapter consists of a single paragraph, devoted to a retrospect on special forms. Two other brief chapters (V and X) also are devoted to retrospects. In the former of these the fact is emphasized that nearly all of the earlier theorems are due to French writers, while the latter notes the rapid increase of interest after 1812 — especially in Germany, where Jacobi and Schweins took the most prominent part in the development of the subject. This spread of interest was largely due to the study of determinants whose elements have special forms. The beauty of some of these results made the notation more attractive and it spread rapidly into common use. Its great advantages as a thought saver, replacing the repeated thought tensions by a single demonstration, soon won for it universal favor, and, in some instances, undue prominence.

As the author of the present volume has published the most complete bibliography of determinants* and has contributed towards its advancement during a period of more than twenty years, we naturally expect the work to have the charm of ripe

* *Quar. Jour. of Mathematics*, vols. 18, 21, 36. The bibliography includes 1,744 titles and covers the periods from the earliest writings on the subject down to the close of the nineteenth century.

scholarship. This expectation is enhanced by the fact that the author confines himself to an early period and that the present volume is a second edition of a valuable work that has been before the public for sixteen years. The typographical errors are few and insignificant. The most serious noticed occurs on page 4, where it is stated that the earlier edition appeared in 1900, instead of 1890.

In 1838 Liouville demonstrated a property of the special forms which were afterwards called wronskians by Muir (1881) and later writers. According to Anissimov* this seems to be the first important result relating to these useful forms, but the paper is not mentioned by our author although it falls within the period covered. Neither is it included in the bibliography cited above.

The present volume is almost twice as large as the first edition and includes a number of papers not there mentioned. The greatest difference between the two editions is in the facts that the papers relating to special forms are brought together in the new edition and that a table showing the advance of determinants from 1813 to 1841 has been added. The use of smaller type for quotations has greatly improved the appearance of the text. The completeness and attractiveness of the book combine to make it indispensable to the student of determinants and their history.

G. A. MILLER.

A First Course in Analytical Geometry, Plane and Solid, with Numerous Examples. By CHARLES N. SCHMALL. New York, Van Nostrand, 1905. 8vo. 7 + 318 pp.

ON the whole, this little book resembles so closely the ordinary text on the subject, both as to material and treatment, that a detailed account is scarcely necessary. Mention of the few points in which it does depart from the general type of text will be sufficient. In the chapter on loci, the usual examples from physics, economics, statistics, etc., are omitted, and attention is confined exclusively to purely geometric problems. This omission will probably be regarded as a fault or a merit according to one's views on the recently discussed relations of mathematics to physics and other subjects. The chapter on the circle precedes that on the transformation of coordinates, reversing

* Encyclopédie des Sciences Mathématiques, Tribune publique I, 1906, p. 3.