THE SEPTEMBER MEETING OF THE SAN FRANCISCO SECTION.

THE fourteenth regular meeting of the San Francisco Section of the AMERICAN MATHEMATICAL SOCIETY was held at the University of California, Saturday, September 26. The following eleven members of the Society were present: Professor R. E. Allardice, Professor H. F. Blichfeldt, Professor M. W. Haskell, Professor L. M. Hoskins, Professor D. N. Lehmer, Professor J. H. McDonald, Professor H. C. Moreno, Professor C. A. Noble, Professor T. M. Putnam, Mr. H. W. Stager, Mr. J. D. Suter.

Professor D. N. Lehmer was elected to succeed Professor Hoskins as chairman of the Section. The present Secretary was reëlected, and Professors Blichfeldt and McDonald were named to serve with the secretary on the program committee, Dr. Blichfeldt being designated as chairman of the committee.

The two meetings in 1909 will be held on February 27, at Stanford University, and on September 25, at the University of California.

The following papers were read at this meeting:

(1) Professor H. F. BLICHFELDT: "A theorem on simple groups."

(2) Professor J. H. MCDONALD: "On Fourier series and expansions in spherical harmonics."

(3) Professor H. C. MORENO: "On a class of ruled loci."

(4) Dr. ARTHUR RANUM: "The general term of a recurring series."

(5) Professor C. A. NOBLE: "Necessary conditions that three or more partial differential equations of the second order shall have common solutions."

Abstracts of these papers are given below in the order of their presentation.

1. Professor Blichfeldt's paper is in substance as follows: Let F be a group of order $p^\lambda n$, p being prime to n, containing a Sylow subgroup G of order p^λ. This has a subgroup G_1 of order $p^{\lambda-1}$. Let S be one of the substitutions of lowest order contained in G but not in G_1. Then, either F will contain an invariant subgroup of index p, or there is in G a subgroup K

containing at least every substitution A of G for which $SAS^{-1}A^{-1}$ is an invariant substitution of G_1, — this group K being transformed into itself by a substitution T of H, though T does not transform S into itself. The order of T is prime to p. Further properties of K were stated; e. g., $K = G$ when the latter contains no substitutions of order p^2. The types of certain Sylow subgroups G contained in a simple group H were shown to be restricted by this theorem. Cf. Frobenius, "Über auflösbare Gruppen, V," *Berliner Sitzungsberichte*, 1901, pages 1324–1329.

2. Professor McDonald's paper discusses the application of Poisson's integral to the summation of a Fourier series and investigates Laplace's method of expansion in spherical harmonics in a similar way.

3. The general problem of finding, in a space of n dimensions, the locus of all the lines meeting n $(n - 2)$-flats was presented by Professor Moreno at a previous meeting of this section. In the present paper, the same problem for $n = 4$ is treated more in detail. By a special choice of coordinates the equation of the 3-spread was shown to be $\Sigma xyz + n^2\Sigma x = 0$. This is Segre's cubic spread. In this form of the equation, many properties of the spread are readily deduced.

4. Let $u_0 + u_1 + u_2 + \cdots$ be a recurring series of the nth order, in which every term after the nth is expressed as a linear function of the n preceding terms by means of the formula $u_m = a_1u_{m-1} + \cdots + a_nu_{m-n}$. By successive applications of this formula the general term is expressible as a linear function of the first n terms, and a rational integral function of the n constants of the scale, say $u_m = F_m(u_0, \cdots, u_{n-1}; a_1, \cdots, a_n)$. Nevertheless, the classical method of finding it involves the irrational process of decomposing the generating function of the series into partial fractions. In this paper Dr. Ranum gives a rational method and determines the explicit form of the function F_m. As an application, the mth power of a linear substitution in n variables is expressed in terms of its first $n - 1$ powers.

5. Following a suggestion made by Hilbert in his lectures in 1900, and which was carried out in some detail by Yoshiye

in volume 57 of the *Annalen*, Professor Noble extends the results of Yoshiye by finding the necessary conditions that three or more equations of the form

$$F\left(x, y, z, \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, \frac{\partial^2 z}{\partial x^2}, \frac{\partial^2 z}{\partial x \partial y}, \frac{\partial^2 z}{\partial y^2}\right) = 0$$

shall have solutions in common.

W. A. MANNING,
Secretary of the Section.

NOTE ON STATISTICAL MECHANICS.

BY PROFESSOR EDWIN BIDWELL WILSON.

(Read before the American Mathematical Society, September 11, 1908.)

IN developing the elements of statistical mechanics it is customary though by no means essential to remark the analogy between that subject and hydromechanics.* The analogy arises primarily through the fact that the equation of continuity of hydrodynamics,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \text{or} \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0,$$

where u, v or u, v, w are the velocities in the fluid in two or in three dimensions, exists in the form

$$(1) \qquad \frac{\partial \dot{p}_i}{\partial p_i} + \frac{\partial \dot{q}_i}{\partial q_i} = 0 \quad \text{or} \quad \sum_{i=1}^{i=n} \frac{\partial \dot{p}_i}{\partial p_i} + \frac{\partial \dot{q}_i}{\partial q_i} = 0$$

for dynamical systems regulated by the hamiltonian canonical equations

$$(2) \qquad \dot{q}_i = \frac{\partial H}{\partial p_i}, \quad \dot{p}_i = -\frac{\partial H}{\partial q_i} \quad (i = 1, 2, \cdots, n);$$

and it is further exemplified by connection of the Boltzmann-Larmor hydrodynamical interpretation of Jacobi's last multiplier with the principle of conservation of extension in phase.† The object of this note is to comment upon the analogy in question.

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* Jeans, The Dynamical Theory of Gases, p. 62. Gibbs, Elementary Principles in Statistical Mechanics, p. 11.

† Compare Whittaker, Analytical Dynamics, p. 272, and Gibbs, loc. cit., p. 29.