THE FEBRUARY MEETING OF THE SAN FRANCISCO SECTION.

THE fifteenth regular meeting of the San Francisco Section of the AMERICAN MATHEMATICAL SOCIETY was held at Stanford University, Saturday, February 27, 1909. The following fourteen members of the society were present:

Professor R. E. Allardice, Professor H. F. Blichfeldt, Professor R. L. Green, Professor M. W. Haskell, Professor L. M. Hoskins, Professor D. N. Lehmer, Professor W. A. Manning, Professor H. C. Moreno, Professor C. A. Noble, Mr. H. W. Stager, Professor Irving Stringham, Mr. J. D. Suter, Professor S. D. Townley, Professor A. W. Whitney.

Two sessions were held, a morning session opening at 10:30 A. M. and the afternoon session at 3:00 P. M., after a luncheon at which all those attending the meeting were present.

The following papers were read at this meeting:

(1) Professor A. W. WHITNEY. "An algebra of probable inference" (preliminary communication).

(2) Professor R. E. ALLARDICE. "Note on the reduction of a circulant."

(3) Professor R. E. ALLARDICE. "A theorem in the partition of numbers."

(4) Professor W. A. MANNING. "On the order of primitive groups. Second memoir."

(5) Mr. G. F. MCEWEN. "Forced vibrations of a pendulum in a viscous fluid."

(6) Mr. H. W. STAGER. "Some investigations in the theory of numbers."

Mr. McEwen was introduced by Professor H. C. Moreno. Abstracts of the papers follow below in the order of their presentation.

1. The paper by Professor Whitney points out the unsatisfactory character of the fundamental conceptions of the theory of probability. The resolution of the difficulty is believed to lie in the recognition of the theory of probability as a system of relations which have their origin in the fundamental laws of thought. The nature of the particular judgments in this field

have the subsidiary importance that similar particular judgments hold in the logic of certain inference, where the dissociability of the formal problem from the objective problem is recognized; in the theory of probability this distinction has been obscure.

This body of relations, which properly forms the theory of probability, is obtained by generalizing the algebra of the logic of classes so as to include the implicatory relations of classes; this is done by the introduction of a symbol, AB/A, the implicatory or subsumptive relation of the class A to the class AB, and certain defining operations.

Ordinary formal logic, or the algebra of certain inference, is included as a special case, and it is believed that this generalization serves to give expression as well to the algebra of propositions.

The algorithm gives, as an example, the following expression for a simple case of inverse probability, BA/B, that is the probability, if the event B has happened, of the hypothesis

$$A\,.\,BA/B\cdot\frac{A\cdot AB/B}{A\cdot AB/A+\bar{A}\cdot\bar{A}B/A}.$$

2. In his note on the reduction of a circulant, Professor Allardice showed that the circulant $C(x, y, z, 0, 0, \cdots)$, of prime order $2p+1$, may be expanded in the form

$$\sum x^{2p+1}+\sum_{r=1}^{r=p}\left[(-)^r\frac{2p+1}{2p-2r+1}\underset{2p-r\;r}{C}\;y^{2p-2r+1}(xz)^r\right].$$

3. Professor Allardice's theorem in the partition of numbers is as follows: If $2p+1$ is a prime, the number of r-partitions of all numbers congruent to $k \pmod{2p+1}$ by means of numbers different from one another and not greater than $2p+1$ is

$$\frac{1}{2p+1}\cdot\underset{2p+1\;r}{C};$$

and the number of double partitions of the same type is

$$\frac{p}{(2p+1)^2}\cdot\left[\frac{1}{2p}\underset{2p+1\;2r}{C}\times\underset{2r\;r}{C}+(-)^r\frac{2p+1}{2p-2r+1}\underset{2p-r\;r}{C}\right].$$

4. Professor Manning's paper is a continuation of an article in the April number of the *Transactions* under the same title. In that paper a theorem announced without proof by Camille Jordan is proved for certain cases and somewhat extended. The proof is now completed by showing that a non-alternating primitive group of degree $5p + k$, where p is a prime number greater than 5, and k is an integer greater than 6, cannot contain a permutation of order p on five cycles. An easy deduction from these results is the theorem :

If the order of a primitive group of degree n exceeds

$$n!/p_1 \cdots p_2^2 \cdots p_3^3 \cdots p_4^4 \cdots p_5^5 \cdots 3^5 \cdot 2^5,$$

the group is either alternating, symmetric, or one of a small number of groups of low class.

The numbers $p_1, \cdots, p_2, \cdots$ are the primes less than $n - 2$ written in descending order. The conditions

$$p_2 < \frac{n}{2} - 1,\ p_3 < \frac{n}{3} - 1,\ p_4 < \frac{n}{4} - 1,\ p_5 < \frac{n-1}{5} - 1,$$

must be satisfied by primes which enter the denominator to a higher power than the first.

5. Mr. McEwen gave an account of a mathematical investigation, with accompanying physical experiments, of the motion in a viscous fluid of a sphere attached to a compound gravity pendulum. Upon the support of the pendulum is impressed the horizontal motion

$$x_0 = ce^{-at} \sin at.$$

When the effect on the pendulum of both the air and the fluid is considered, the differential equation of motion is

$$P\frac{d^2x}{dt^2} + P_1\frac{dx}{dt} + P_2x + N = P_0''ce^{-at} \sin at + P_0'ce^{-at} \cos at.$$

The coefficients P, P_1, $\cdots$ involve μ, the coefficient of viscosity of the fluid, and known constants only. The particular integral that expresses the forced vibration is

$$x = ce^{-at}[A \sin (at - \theta_1) + B \cos (at - \theta_1)] - \frac{N_2}{P},$$

where

$$\tan \theta_1 = \frac{P_1 a - 2a\alpha P}{P_2 + P(\alpha^2 - a^2) - \alpha P_1} = \tan\left(a t_1 - \tan^{-1} \frac{P'_0}{P''_0} \right),$$

and in which t_1 is the phase difference in seconds between the motion of the sphere and that of the support.

6. The particular class of numbers which are discussed in Mr. Stager's paper suggested themselves in the construction of a table for use in applying Sylow's theorem. This table — now constructed for the first 10,000 numbers and eventually to be extended to 15,000 or 20,000 — exhibits the value of k for all prime factors of each number which give factors of the form $p(kp + 1)$, where p is any prime except 2 and k is any positive integer. Those numbers, called P's, which contain no factor of the form $p(kp + 1)$ form a very interesting class. The present paper deals with their fundamental properties; shows that the numbers consist of four general types; obtains several formulas for their enumeration; and suggests a connection between the number of ordinary primes and the number of P's within a given limit.

W. A. MANNING,
Secretary of the Section.

THE CONSTRUCTION OF A SPACE FIELD OF EXTREMALS.

BY DR. E. GORDON BILL.

(Read before the American Mathematical Society, December 30, 1908.)

IT is a well-known theorem of the calculus of variations,* that if all the members of a one parameter family of plane curves pass through a fixed point O, then any arc of these extremals which does not contain O nor its conjugate point, may be imbedded in a field.

Moreover, in 1879 Weierstrass stated that a field including O could be constructed and Professor Bliss † has proved this to be true.

---

* Osgood, *Annals of Math.*, ser. 2, vol. 2, p. 112.
† Bliss, BULLETIN, vol. 13, p. 321.