from any lengthy discussion. Some of the logical phases of Hilbert's axioms are instructively pointed out in a controversy between Frege and Korselt in the *Jahresbericht der Deutschen Mathematiker-Vereinigung.* *

A. R. SCHWEITZER.

Vorträge über den mathematischen Unterricht an den höheren Schulen. KLEIN-SCHIMMACK. Teil I. *Von der Organisation des mathematischen Unterrichts.* Leipzig, 1907, pp. ix + 236.

PROFESSOR KLEIN has for years been calling attention to the fact that it is the duty of the universities to furnish instruction not only in the subject matter of mathematics but also in the questions relative to the teaching of the subject, since teaching is the profession to which the great majority of the students are looking forward ; and in his own university (Göttingen) he has admirably been putting into practice what he preaches. The present volume makes accessible to the wider public the substance of a course of lectures thus delivered at Göttingen in 1904–5.

It is a work that at once commanded thoughtful attention in all quarters of the nation for which it was especially intended, but it also deserves and will receive careful study far beyond the national confines.

The work is divided into the following sections, whose titles give a general idea of its plan and scope :†

Introduction, pages 1–9 ; I. Elementary schools, 10–18 ; II. The six lower classes ‡ of the (boys') higher schools, 19–43 ; III. Girls' schools and trade schools, 44–66 ; IV. The historical development of instruction in mathematics in the German higher schools, 67–99 ; V. The three upper classes § of the higher schools according to the curricula of 1901, 100–126 ; VI. Propositions for reform in the upper classes of the higher schools, 127–159 ; VII. The universities and technical schools, 158–189 ; Conclusion, 189–190 ; Appendix (containing re-

* See vol. 12 (1903), pp. 319, 368, 402 ; vol. 15 (1906), pp. 293, 377, 423 ; vol. 17 (1908), p. 98.

† With each numbered title, except No. IV, the words "mathematics in" are of course implied.

‡ In mathematics these six classes correspond roughly to our grades 4 to 8, and the first year of the high school.

§ In mathematics these classes correspond roughly to the second and the third year of the high school and the freshman year in college.

prints of two of Klein's earlier papers and of the curriculum proposed by the Commission at Meran *), 191–236.

The form of the work is, as already stated, that of lectures to students of a German university preparing to teach mathematics in Germany. It therefore naturally contains much matter that relates exclusively to German schools and much of a more general character that is expressed in the terminology of the German school system; but there is also not a little, and that of the most important and characteristic, which is intelligible without any acquaintance with the details of the German organization.† The chapters of widest general interest are Nos. II, V, and VI.

As to general methods, Klein finds that the genetic method has been steadily gaining ground and has entirely displaced the unduly formalistic systems of earlier days. In this connection, the question is raised, pertinent also elsewhere than in Germany, whether "many a university lecture course might not take a lesson in pedagogic-psychologic preparation from the modern types of instruction in the schools" (page 24).

Quite a little space is devoted to a discussion of the function concept, which, in geometric form, is to be made the *soul* of the mathematical courses in the schools (page 34). In close connection with this lies the question of the interrelations in time and subject matter of the various branches of mathematics taught in the schools, a question of prime importance for us Americans, since in this matter we are far behind the Germans, who have long been teaching algebra (including numerical work) and geometry (with trigonometry) side by side, details being left in the main to the discretion of the teacher. Let us hear whether Klein is satisfied with this, or whether perchance he even believes that the Germans have gone too far in this direction.

"According to my feeling, instruction in mathematics is at present still conducted with too much external separation of algebra and geometry, and with this external separation a

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*See BULLETIN, vol. 12 (1906), pp. 347–352.

† Various descriptions of the German secondary schools, quite sufficient to make the present work intelligible throughout, are within reach of the American reader. For example:

Russell: German Higher Schools. New York (Longmans). 1898.

Bolton: The Secondary School System of Germany, New York (Appleton), 1900.

Young: The Teaching of Mathematics in Prussia. New York (Longmans), 1900.

deeper one often goes hand in hand. The principle of purity of methods may be interesting in the higher special courses of the university, but unfortunately it is also sometimes proclaimed in the schools. 'Geometria geometrice'; 'No numbers in geometry'; 'Happily, algebra does not need geometric crutches'; 'By all means, no diagrams in arithmetic'; such are the mottoes for the sacrifice of thorough treatment to the higher honor of purity of method. It is needless to discuss the unpedagogic character of this position. The secondary schools, whose ideal aim is general culture, should evidently present mathematics as an organism whose various parts stand in active and vital interrelations.

"If, gentlemen, any mathematical question presents itself to you in later life you will have no leisure to ask: 'Shall I proceed purely geometrically, perhaps even projectively, avoiding all metric concepts, or shall I follow the method of the pure theory of numbers?' No; the pressing thing will be to be able to help yourself out on the basis of the totality of your mathematical acquisitions. And this will in general be easier the closer together the different forms of mathematical thought have been brought in your instruction. Just on that account it will be well if the arithmetic, algebra, and geometry of the lowest classes coalesce in 'Obertertia' and 'Untersecunda' (ages 13–15) and culminate in a grasp of the function concept. Of course, geometry is not to be sacrificed to algebra or vice versa; by mutual assistance, both forms of mathematical thinking will not lose but gain."

The fifth chapter is devoted to a discussion of the course in mathematics in the three upper classes, the last of which takes up approximately our freshman work in college. The list of subjects prescribed by the curricula for these years is taken up in detail (Algebra, pages 101–109, Geometry, pages 121–126), and the considerations are interesting and suggestive to Americans also. A large portion of the chapter (pages 109–121) is devoted to an "Excursus on the question of the infinitesimal calculus in the schools." So far as the *last* school year (our freshman year) is concerned, the reasons so cogently stated would find ready acceptance by some in America. The writer has for over a decade given an introductory course in calculus for freshmen, and no doubt the ideas of the calculus have been more or less extensively introduced into freshmen work in numerous instances. Mention should be made especially of the work in mathematics of the first year in the Massachusetts Institute of Technology, as set forth in the excellent and suggestive work of Woods and Bailey,* which aims to organize college algebra, plane analytic geometry, and the elements of the calculus into a whole, whose unifying note shall be *mathematics* rather than any one of the subjects named. But to carry the calculus back into precollegiate work, even in the most elementary way, would not at present be practicable with us. Still Klein's proposal to introduce the ideas of derivative and integral in what corresponds roughly to our third high school year may well be feasible in Germany (where the pupils remain in the same school, and probably with the same teacher during the years in question), and has there met with strong, though by no means unanimous, support.

Mere mention must suffice for the interesting discussion of proposed reforms in Chapter VI, and the sketch (Chapter VII) of mathematics at the universities, especially Göttingen, with its graphic representation in colors of the galaxy of famous men that in unbroken sequence have justly constituted Göttingen's pride.

The work is written in an informal, clear, and interesting style, and should find a considerable circle of readers on this side of the Atlantic.

Upon the heels of the work discussed above there has followed a lithographed volume of a course of lectures by Klein (in 1907–8 †) treating topics related to elementary mathematics from the higher standpoint, along the lines of the synoptic course recommended by the German Commission in 1907.‡ This is a further valuable and stimulating fruit of Klein's attitude mentioned at the outset, and merits attention and study by a wide circle of readers. A second volume dealing with geometric topics is to follow quite shortly, and a full review of both will appear in the BULLETIN in due course.

J. W. A. YOUNG.

* A Course in Mathematics, vol. I., Boston (Ginn), 1907.

† Elementar-Mathematik vom höheren Standpunkte aus. Teil I: Arithmetik, Algebra, Analysis. Leipzig (Teubner), 1908, pp. 590.

‡ See BULLETIN, vol. 15 (1909), p. 262.