*Then all the solutions of* (1) *continuous in $t$ except for a finite number of points are such that the function $u(x)g(x)$ remains continuous in $t$.*

This theorem has several applications. If neither $f(x)$ nor $g(x)$ vanishes in $t$, there can be no solution becoming infinite at any point in $t$; therefore the continuous solution is the only solution of the equation continuous except at a finite number of points. If $K(a, a) \neq 0$, and if $\lim_{x=a} \phi(x)g(x) = 0$, the theorem of page 134 holds, as we have already noticed. The solutions there given are the only ones continuous except at a finite number of points, such that $d[u(x)g(x)]/dx$ remains finite in $t$; they are also the only solutions possible, continuous except at a finite number of points, provided that $K(x, \xi) - K(a, a)$ vanishes identically when $\xi = x$.

If the kernel of the integral equation (1)* is analytic in $T$, and if $\phi(x)$ is continuous in $t$, a proof similar to that of the above theorem shows that the continuous solution is the only solution of (1) continuous in $t$ except for a finite number of points.

HARVARD UNIVERSITY,
*September*, 1909.

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## DESCRIPTIVE GEOMETRY.

*Lehrbuch der darstellenden Geometrie für technische Hochschulen*, Volume I. By Professor EMIL MÜLLER, of the Imperial Technical School at Vienna. Leipzig and Berlin, Teubner, 1908. xiv + 368 pages, 273 figures, and three plates.

*Vorlesungen über darstellende Geometrie.* By GINO LORIA. Volume I: *die Darstellungsmethoden.* Authorized German translation from the Italian manuscript, by FRITZ SCHÜTTE. Teubner's Sammlung, volume $XXV_1$. Leipzig and Berlin, Teubner, 1907. xi + 218 pages and 163 figures.

*Descriptive Geometry*, a treatise from a mathematical standpoint, together with a collection of exercises and practical applications. By VICTOR T. WILSON, Professor of drawing and design in the Michigan Agricultural College. New York, John Wiley and Sons, 1909. 8vo, viii + 237 pages and 149 figures.

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* If the kernel of the integral equation with constant coefficients is analytic in $S$ and if $\phi(x)$ is continuous in $t$, the continuous solutions are the only solutions continuous except for a finite number of points.

DURING the last few years Teubner has published twenty-four treatises, several of them of two or more volumes, on descriptive geometry, and his latest circulars announce that ten more are in an advanced state of preparation. And this is only one instance; a number of other publishers announce the early appearance of one or more books on the same subject, and the periodic literature of Europe (particularly German and Italian) contains during the last ten years some five hundred memoirs, essays, notes, solutions, etc., pertaining to this science. Under these circumstances the question may be asked, why still another book in a language represented by so many? But an examination of Professor Müller's work will dispel any doubts as to the wisdom of its publication. Descriptive geometry was taught to most of us in America who had any instruction at all in the subject, not as a science, but rather as a clever device for producing certain graphical representations. Courses in it are given only in our technical schools, conducted by engineers for purely practical purposes, and largely without proofs. The students soon think of the procedure as empirical and frequently wonder why the drawings come out so well, when they do not really know just what they are doing. For this reason, many of our American graduates who have extensive responsibilities in draughting operations have had to learn most of the elaborate processes in the offices, and begin years later really to understand the geometric principles upon which the constructions were based.

The present book has a very different character; a student who masters it will not doubt that the various devices will accomplish the desired ends. The author has taught the subject for years, has had extensive experience with graphical methods, and by his achievements in other fields of mathematics has proved that he can speak with authority. While on every page emphasis is laid on the fact that the entire subject is to be regarded as an auxiliary science for engineers and architects, and hence the practical applicability is the principal aim, yet nothing is taken for granted, every step being carefully analyzed. One striking feature is the early insistence on the interpretation of a figure as a whole, rather than simply the sum of a number of individual points and lines. This is an excellent means for retaining the active interest of the student. Thus, a general parallel projection of a hexagonal pyramid is discussed as early as page 25, and shades and shadows are systematically introduced on page 48. Within the next few pages the ground, front, and end elevations of complicated frames are constructed, and their shadows found.

A concise system of abbreviations is used throughout. This greatly aids in reading demonstrations, the only objection being that it is different from that of other writers on this and related subjects.

In the first part of the book, which treats of the ordinary properties of rectilinear figures, two principles are successfully employed, both of which are unusual at this stage, but are always met with in practice. The first is the frequent use of the general profile, that is, projection on a plane perpendicular to but one of the fundamental planes; the other is the early omission of the ground line. These principles are used throughout the volume.

Another feature is the prominence of the discussion of projective properties, which always precede the metrical ones. Incidentally, the student is gradually gaining a comprehensive knowledge of projective geometry, but always in organic relation to graphical problems. Affinity is indeed given a separate consideration, the first part of which is quite independent of graphical notions.

The second part of the book is devoted to the general theory of curves and surfaces. The treatment is partly algebraic and partly differential. Here the author seems rather too ambitious; he acknowledges in the preface that the presentation may be too brief, but hopes by presupposing considerable knowledge of analytics and the calculus that the student may see his way through. In thirty pages we find such varied concepts as Plücker's numbers for algebraic plane curves, the circular points and absolute circle, the imaginary generators of ellipsoids, and a number of properties of space curves, including lines of curvature, asymptotic, and geodesic lines on a given surface. Either the student must have learned these things before, or he will not sufficiently know them even after this discussion. This is the only part of the book that does not seem to be carried out in the best way.

On the other hand, the applications of these principles to the curves and surfaces of the second order, which is found in the succeeding chapters, is exceptionally well done. A large number of theorems usually found in books on analytics are established, including a direct graphical determination of the center of curvature of an arbitrary point on an ellipse, by means of three orthogonal projections. Cones, cylinders, and developables are considered before the general quadric is taken up. Much of this matter is far more extensive than is usual in books on descriptive geometry; it compares roughly with the corresponding chapters of Reye's Geometrie der Lage.

The excellent chapters on surfaces of revolution and helicoids contain a rich fund of information, which is of value for purposes other than graphical representation. The theory of illumination is developed, curves of equal illumination determined, and the results compared with true and apparent contour.

Among the commendable details of the book we may mention the extensive index, the excellent figures, the free-hand lettering, and the frequent instructions and admonitions as to procedure, including the preparation of washes, tints, inks, etc. — making it a serviceable hand-book as well as a thorough theoretical treatise.

The above will indicate why the publisher should undertake the production of another book on descriptive geometry. A second volume is to consider central perspective, axonometry, and related subjects.

The present first volume of Professor Loria's treatise is concerned entirely with methods of graphically representing configurations composed of points, straight lines, and planes. It presupposes some knowledge of projective geometry of the plane, including self-corresponding and double elements, Pascal's theorem, Desargues's theorem, and poles and polars. The work is divided into five books; the first (88 pages) discusses the ordinary problems of descriptive geometry, but the proofs are unusually systematic, and all particular and exceptional cases are treated. The constructions are all reduced to depend upon four fundamental ones. Results are expressed in bold-faced type and a number of unsolved exercises follow each article.

The profile and certain oblique planes are frequently employed, and frequent use is made of rotation. Among the themes treated in this short space are the determination of the two transversals of four skew lines and the construction of the regulus defined by three lines.

The second book considers most of the same problems from the standpoint of central perspective. Given a center $C$ and a plane $\pi$. Let a line pierce $\pi$ in $T$. Through $C$ draw a line parallel to the given one, cutting $\pi$ in $I$. The projection of the line is then defined by $TI$. Similarly, a plane is expressed in terms of its trace $t$ and the parallel line $i$. The third book has for subject the projective equivalent of the preceding one, the point $C$ now being at infinity in the direction normal to $\pi$. The other defining element is a plane parallel to $\pi$ at a known distance from it. The chapter closes with an instructive comparison of central and double orthogonal projection.

The discussion of axonometry in the fourth book (45 pages) is an excellent theoretic presentation, but rather too brief to be of greatest use. While all the necessary steps are given, they are so brief, and treated each alone, that the guidance of a teacher is necessary to the average reader. In the preceding parts one could easily read the text and solve the assigned problems without such assistance. The chapter on orthogonal axonometry is followed by one on oblique parallel, and one on central, but these are hardly more than outlines. Finally, there is added as a fifth book a brief treatment of photogrammetry, *i. e.*, the science of constructing a third perspective of a space figure when two are completely given, or of finding either a central or orthogonal projection when several are partially given, e. g., the drawings given, but the position of the corresponding centers not specified. The discussion of these problems requires many more theorems from projective geometry, the proofs of which are supplied. The results are very interesting; while considerable knowledge and higher maturity are required on the part of the student, this discussion cannot help being a valuable incentive to the more advanced student.

The book treats the problems it proposes with thoroughness and skill, but a reader will after all probably not get a proper idea of the beauty or of the usefulness of descriptive geometry by reading it, because it treats them in an isolated matter, and does not develop the most useful power of being able to interpret a complicated drawing as a whole. The figures of a few building fronts or plans treated entirely in straight lines would add greatly to the usefulness of this useful book. Probably such matters will be adequately treated in the second volume.

In the preface of Professor Wilson's book we find the following statement: "Descriptive geometry is essentially a mathematical subject. The applications of its principles to the making of working drawings, however, and the modifications which are made to meet the contingencies of practice have had a tendency to obscure this fact and like other theoretical subjects it has suffered mutilation in the interest of short cuts to immediate practical uses. But does not technical education after all consist chiefly in an equipment of sound theory?" In the treatment of the configurations consisting of straight lines and planes the author has well justified this statement, and has made a real improvement on the presentation found in most American texts. The presentation is didactic, the explanations being full, and theorems are followed by a number of numerical exercises. The idea of rotation of one or both axes is brought in on page 17 and extensively employed in subsequent problems.

The discussion of curves and surfaces is much less satisfactory, and only partly fulfils the promise in the second part of the title. Thus, in defining parallel lines (page 2) we meet the statement: "Hence parallel lines are said to have two points in common at infinity"; on page 89 we find: "Double curved surfaces are generated only by the motion of curves and have no straight line elements." The words consecutive and coincident on page 93 are confusing; the rectification of a curve, defined on page 95, is obtained by rolling the curve out on its tangent. The word touch, as applied to space curves, is everywhere incorrectly used. According to the definition of a warped surface on page 139 no ruled surface can have a double curve. A number of theorems are worded much too broadly, as they apply only to particular cases; thus, on page 149, after discussing a method for drawing a tangent plane at a point on a general quadric, we are assured that it is applicable to any surface having two systems of rectilinear generators. The theorem on page 151 refers only to rectangular hyperboloids. Unfortunately, the book is full of similar statements; many of them are found in most of the American text-books on the subject, but a few are new contributions. However, the author should not be too severely criticized; he has at least felt the need of more light, and has made decided improvements in the early part of the book.

In view of the sharp contrast between this, one of the best American text-books on descriptive geometry, and those mentioned above we must conclude that there is still room for a good treatise on the subject in English.

VIRGIL SNYDER.