points belonging to the attracting mass are developed, and the discontinuity of the second derivatives as the point goes through a bounding surface is shown and the values of the saltus determined. Following this, like problems are taken up for surface distributions. The closing chapter of the section is devoted to proving that the properties enumerated for the potential as necessary are also sufficient, hence characteristic.

The second section considers the function for other laws than that of the inverse square of the distance. It is shown in particular that the Newtonian is the only law which gives a constant potential inside a spherical shell whose density is a function of the distance from the center. It is not however the only law for which the attraction on an external point due to the shell is equal to that of an equal mass concentrated at the center of the shell. The shape of the "solid of greatest attraction" is considered. The logarithmic potential and the potential due to a double distribution, as a Leyden jar, are each given a chapter.

The book would seem to be quite teachable. Gauss's, Green's, and Stokes's theorems are not dragged in, but show up naturally when needed to further the development. The student sees clearly all the time the drift of the development and why it proceeds as it is does. He learns how to attack such problems, but he also becomes acquainted with a class of point functions particularly useful in mathematical physics. Difficult questions of higher analysis are passed over, yet the treatment is careful and tends to inspire to further research.

JAMES BYRNIE SHAW.

Geometrie der Kräfte. By H. E. TIMERDING. Leipzig and Berlin, Teubner, 1908. 8vo. xi + 381 pages.

In this book the author has developed the geometry of forces as an independent discipline, a branch of pure mathematics. While the word force (Kraft) has been retained in preference to stroke or vector, great pains have been taken to free it from the "physiological, physical, and metaphysical" attributes which belong to it originally. A force is a matter of definition, being defined as a vector with which is associated a numerical factor. The resulting theory is then applicable to any quantity which satisfies the definition, for example to momentum quite as well as to force in the ordinary physical sense. The subject matter is not new. In different forms and from various points of view it has appeared in several places. For example the part which is applicable to the kinetics of rigid bodies has been worked out in Ball's Theory of Screws, and a parallel development for the statics of deformable bodies appears in the writings of Sir William Thomson. The authors' purpose in rewriting to a large extent the Theory of Screws is explained by the following quotation from the preface. "As a consequence of the English genius it (Ball's work) shows, even in those parts which do not have immediate practical applications, a fine appreciation for the essence and claims of science. On the other hand it seeks in no way to attain to that unity and purity which we Germans have been accustomed to regard as the goal of all scientific work. It appears more as a geometric illustration of mechanics than as a special subject independent of mechanics. The latter appears to us as the only purpose which a geometry of forces has to fulfill." It is intended that this development shall provide a bridge from geometry to mechanics without assuming a hybrid character; that it shall be a development such as the geometry of motion has had under the name of kinematics.

The first five chapters stand apart from the rest of the book and are devoted to an exposition of Grassmann's Ausdehnungslehre and Hamilton's Quaternions. Any mention of vector products carries with it the question as to whose notation is employed. After giving in a footnote the notations of Grassmann, Hamilton, Heaviside, Gibbs, and Lorentz, Professor Timerding adopts one which is not in agreement with any of the others. He denotes the inner or scalar product by $\mathbf{a} \times \mathbf{b}$ and the outer or vector product by $[\mathbf{ab}]$. In the following chapters almost no use is made of the terminology and results of vector analysis, and much of the first five chapters could have been omitted. It is remarked in the preface that the introduction of the methods of Grassmann's analysis into the first part of the book without making use of them in the sequel may be open to criticism. The justification is to be found in the purpose of a strictly systematic development of the subject without regard to subsequent application of the early steps.

For the principal part of the book the choice of material has been determined by two main considerations. On the one hand a development of the geometry of forces must be broad enough to include the two manifestations of mechanical energy, the kinetic energy of motion and the potential energy of deformation. On the other hand the author has demanded a minimum prerequisite knowledge of analysis. This requirement is limited to elementary analytic geometry and calculus, to which may be added a few of the formulas for vector products given in the introductory chapters.

The exposition of the geometry of forces begins in Chapter 6 with a consideration of instantaneous rotations. This is followed by chapters on forces and force systems, foundations of line geometry, and equilibrium. The next six chapters are devoted to the theory of screws and are followed by two chapters on deformations, the point of view throughout this portion being purely geometric. The concepts of mechanics are taken up in the last six chapters, of which two are given to deformable bodies. The four chapters on kinetics of rigid bodies deal with the equations of motion in general, free motion under no applied forces, motion with two degrees of freedom, and with three degrees of freedom. The special case of a system of forces in a plane is excluded throughout, and in the free motion of a rigid body it is assumed that the axis of rotation does not have a fixed direction in space.

The necessary complications of notation in this subject have been reduced by the systematic use of different styles of type, thus avoiding an excessive number of accents and subscripts. For the convenience of the reader the scheme of notation is exhibited in a table at the end of the book.

There are many misprints, but fortunately most of them are self-evident and will not cause confusion.

W. R. LONGLEY.

NOTES.

THE seventeenth summer meeting of the AMERICAN MATHEMATICAL SOCIETY will be held at Columbia University on Tuesday and Wednesday, September 6–7. Abstracts of papers intended for presentation at this meeting should be in the hands of the Secretary not later than August 20.

THE April number (volume 32, number 2) of the *American Journal of Mathematics* contains the following papers: "The reduction of families of bilinear forms," by H. E. HAWKES; "Basic systems of rational norm-curves," by J. R. CONNER; "Surfaces invariant under infinite discontinuous birational