chapter deals with Fermat's equation $x^n + y^n = z^n$ from the standpoint of the elementary theory of numbers, giving Fermat's proof for $n = 4$, the Euler-Legendre proof for $n = 3$, and remarks on the Dirichlet-Legendre proof for $n = 5$, and the Lamé-Lebesgue proof for $n = 7$. Kummer's method by ideals is beyond the scope of the work; the comment on regular primes (page 461) is corrected at the end of the book. The formulas obtained independently by Abel and Legendre are established. The developments by Sophie Germain, E. Wendt, and L. E. Dickson are then cited. In one instance (page 475), the initials of the last name are given incorrectly; while, in a quotation from Sylvester on page 104, permeating is spelled wrong. However, the book is especially free of errata and the typography is excellent. In the present text Bachmann has fully maintained his reputation as to clearness, thoroughness, and exhaustiveness.

L. E. DICKSON.

Elēments de Calcul vectoriel avec de nombreuses Applications à la Gēomētrie, à la Mēcanique et à la Physique mathēmatique. Par C. BURALI-FORTI et R. MARCOLONGO. Traduit de l'italien par S. LATTÈS. Paris, A. Hermann et Fils, 1910. vii + 229 pp.

So lengthy a review * was recently accorded to two new books on vector analysis by Burali-Forti and Marcolongo that nothing more than the mere mention of the French edition of the first of the two would be needed, were it not for the fact that in the French the authors have added a long and excellent appendix on Grassmann's geometric forms and on Hamilton's quaternions. The object of the appendix is to show the power of the authors' vector analysis by using it to set up the Grassmannian and Hamiltonian systems. There is apparently the further object to set forth these two mathematical disciplines in such a way that mathematicians in general, and in particular those mathematicians who think they know something about the systems, shall be led to conceive or reconceive, as the case may be, these systems as they should be conceived. We have no exceptions to take to the authors' presentation of the subject; it is compelling.

There is one remark, found on page 201, which deserves

* Under the title "The unification of vectorial notations," BULLETIN, vol. 16, pp. 415–436 (May, 1910).

particular attention. The authors have defined quaternions in their way and have given Hamilton's definition and then remark that they will prove that the quaternions given by their definition are the quaternions of Hamilton and not any other kind of quaternions. The importance of this remark is the implication that there are other quaternions than Hamilton's. Of course the authors are not interested in any others any more than they seem to be interested in Gibbs's algebraic point of view; but we are glad to have them imply that there are others — for in the strictest sense it is precisely other quaternions than Hamilton's original ones that most quaternionists now use. Gibbs * pointed out one defection from the strict Hamiltonian point of view; the authors have frequently pointed out others for the purposes of berating the quaternionists who use them.

It may interest some, who had not the opportunity to hear Gibbs, to see (as well as I can remember) the way in which he presented his, not Hamilton's, quaternions. He considered ordinary vectors $\alpha, \beta, \cdots$ between which he had already defined his scalar product $\alpha \cdot \beta$, his vector product $\alpha \times \beta$, and his dyad product $\alpha\beta$; and he proceeded to define his quaternion product as †

$$\alpha \circ \beta = - \alpha \cdot \beta + \alpha \times \beta.$$

Thus his quaternion was the sum of a vector and a scalar — which Hamilton's quaternion primarily was not. In particular, scalars and vectors were merely especially simple quaternions. The defining equation gave at once

$$i \circ i = j \circ j = k \circ k = i \circ j \circ k = -1$$

and all the formal laws of quaternion algebra. Here i, j, k were vectors and the product was quaternionic. In Hamilton's analysis the i, j, k would be not vectors but right quaternions, and for him the right quaternion was not primarily a vector. So far as we can judge, the authors are entirely right about Hamilton, and the followers of Hamilton are a good deal wrong about him. They do not any longer use the original quaternions; they use their own, which are practically identical with Gibbs's; it is a pity the authors have not analyzed these that are used instead of the original ones which Hamilton analyzed with such detail.

E. B. WILSON.

* The Scientific Papers of J. Willard Gibbs, vol. 2, p. 172.

† He did not use the sign o or any other special sign for the quaternion product.