

*Gaston Darboux. Biographie, Bibliographie analytique des Ecrits.* Par ERNEST LEBON. Paris, Gauthier-Villars, 1910. viii + 72 pp.

THIS is the second volume of the series *Savants du Jour*, which M. Lebon is bringing out. It is as regards content and arrangement similar to its predecessor, which was devoted to the life and work of Poincaré.\* It begins with a short biography of Darboux (7 pages). A few extracts may be of interest to the readers of the *BULLETIN*. Darboux was born at Nîmes, August 13, 1842, the elder of two sons of a grocer. His father died in 1849. But his mother, noting the intellectual gifts of her two sons, provided for their education. In 1861 the youthful Gaston was awarded the rare distinction of being admitted as the first among all candidates both to the *Ecole Polytechnique* and to the *Ecole Normale*. He chose the latter. He received his doctor's degree from the University of Paris in 1866. After several years spent in secondary teaching, during which period he published several important papers, he became in 1872 "maitre de conférences" in the *Ecole Normale*. In 1875, at the age of 32 years, he was awarded the *Prix Poncelet* by the French Academy for the ensemble of his mathematical work. Since 1880 he has occupied the chair of geometry at the Sorbonne, being the successor in this chair of Chasles. He was elected a member of the Academy of Sciences in 1884, and made a permanent secretary of this body in 1900. He is a member of 21 academies and of 11 scientific societies, and is an honorary doctor of the Universities of Cambridge, Christiania, and Heidelberg.

The remainder of the little volume is devoted to an appreciation of the scientific work of Darboux taken from various sources, and a complete list of his writings. The latter numbers 419 titles, many of which are accompanied by brief abstracts. The amount and quality of such productivity is at once the inspiration and the despair of his would-be disciples. The volume also contains a fine portrait of Darboux.

J. W. YOUNG.

*Handbuch des mathematischen Unterrichts.* Von W. KILLING und H. HOVESTADT. Band I. Teubner, Leipzig und Berlin, 1910. viii + 442 pp.

NOTWITHSTANDING the great number of handbooks on mathematical instruction the authors justify the present work by the

\* Cf. review in the present volume of the *BULLETIN*, p. 42.

claim that such books in general are too abstract or are too much concerned with methods of teaching and do not embody a sufficiently thorough and painstaking investigation of those things which we try to teach. Hence, assuming that the average person who proposes to teach elementary mathematics has neither the leisure nor means to investigate the advance of the science from original sources, the authors devote at least half of the book to presenting the facts in available form for the novice. The aim is obviously to rid the mind of the teacher of the idea that Euclid is sacred, and to fill him with the modern spirit.

It is worth noting that there are practically no drawings in this volume, devoted to geometry, but the required figures are very carefully described and relations of position fully defined. The reason is obvious, for a single figure is of necessity special, whereas if the reader draws a figure from the description, he will find the latter fits any case.

The book naturally divides itself into two parts. The first half is devoted to a careful and well balanced discussion of the foundations of geometry. The authors define a complete system of axioms and proceed to investigate those of Euclid and Hilbert, the latter being assumed to be the best of modern systems. There is a most interesting general discussion of euclidean and non-euclidean geometries, showing how the latter were forced upon certain investigators against their will. The bibliography here as throughout the book is excellent and is one of its most valuable features. The distinctive characteristics of the various non-euclidean geometries are clearly presented.

In order to show the basal ideas in geometry there is an attractive discussion of natural and non-natural geometries. The futility of *defining* a line as "the locus generated by a moving point," and of similar proceedings is pointed out with some degree of humor. The ideas of right and left on the line and plane, of congruence and symmetry, of the equality of area or content in plane figures, and corresponding ideas in space are treated thoroughly. The chapter on non-real configurations of the plane contains some excellent hints for the teacher presenting this subject. The authors express the opinion that most elementary teachers are unable to comprehend and explain the exact nature of the so-called infinitely distant point. Such teachers would do well to read this vigorous treatment. The chapter on plane construction problems is no less inter-

esting. The problem of determining whether a construction is possible by means of a straight edge and compass is naturally placed at the head of the chapter. Numerous constructions are proposed, including a list of apparently simple ones which are however impossible. Constructions possible by means of other instruments are discussed also, and their relation to the first problem explained.

The two chapters on the logic of geometry and on literary form in mathematical instruction are interesting. By a complete system of axioms, we mean one that contains all specifically geometrical ideas, and we fail to notice the axioms of logic for combining them, which we nevertheless tacitly assume. The authors attempt to build up a system of axioms of this logic. The teacher has further a right to demand good literary style as well as logical precision, but the authors deride hair splitting niceties and cranky insistence on logical and verbal completeness, which only serve to kill the student's interest without clarifying his ideas. Clinch the argument by conversation about the theorem, is their advice. The final chapter of the first half is devoted to a rational discussion of the teaching of elementary geometry. The absolute folly of having pupils commit refined definitions which have no real meaning to them is dwelt upon. In trying to aid the teacher both in methods of instruction and arrangement of material, this book assumes two objects in teaching — to give, first, an insight into the peculiar character of mathematics as a unique domain of human knowledge; second, an understanding of the important rôle it plays because of its numerous practical applications.

The purpose of the second half of the book is to orient the reader as to the essentials, both as to material and as to its arrangement and treatment. No *ideal* course is prescribed, as a course must have personal coloring to be stimulating. But great care is taken everywhere to show the true importance of individual theorems, and give the reader the proper perspective to enable him to choose wisely. Beginning with the triangle, some of the discussion may seem trivial, because we have prejudiced minds as to the proper order and methods. But the succeeding chapter on the theory of parallels is extremely clear, and reveals the thoroughness of the writers. Various methods of approaching the study of the circle are criticised, also that of the equality of plane figures. The teacher must finally answer the double question: What shall I

teach and in what order? A very useful but rather out of place chapter on the theory of irrational numbers is interjected at this point. The theory and application of the Dedekind cut as well as Cantor's theory of irrational numbers are outlined quite fully. Strenuous objection is made to the custom of American texts of applying directly the laws of arithmetical proportions to geometrical quantities. Rational advice is also given against the tendency to number theorems and operations one, two, three, etc., and to compel students to commit this enumeration.

The last seven chapters treat topics rarely found in an elementary American text. The discussion of similarity of plane figures, with some suggested applications, is valuable. The extension of the idea of similarity to circles is very fruitful. Various methods of "measuring" circles are treated, including Huygens' method, although this is pointed out as being of doubtful value in the lower schools. Anharmonic ratios of points on a line and of lines through a point are treated with elegance, and considerable space is devoted to pole and polar with respect to a circle. It seems unfortunate that American texts ignore so largely these fruitful and interesting fields. Pencils and nets of circles, with their properties, are discussed in one chapter and inversion in the plane in another. The last chapter is a long discussion of the Apollonius tac-problem. The methods of Apollonius are first discussed, then the modern solutions of Gergonne, Massfeller, and C. Adams are criticised and tested by Study's criteria. The authors' discussion of Study's criteria for the best solution of a construction problem is not without interest. This last half of the book is replete with brief scientific discussions and helpful remarks which the reviewer must refrain from pointing out in more detail. The authors are not entirely free from some of the faults they inveigh against, but the volume doubtless deserves a place among other books of its kind, for it is fearless in its criticisms of others, is thorough in research, and has the merit of bringing the history and bibliography up to date.

D. D. LEIB.

*Complementi di Analisi algebrica elementare.* By FEDERICO AMODEO. Luigi Pierro, Naples, 1909. 284 pp.

THIS little volume is one of a series of elementary text books by the same author and is intended for use in the Istituto Technico of Italy. The author says in the preface that a stu-