faces of the second order is made according to the form taken by their cones. It is interesting to compare this with the classifications of quadrics in elliptic space given by J. L. Coolidge (Non-Euclidean Geometry, page 156) and T. J. I'a. Bromwich ("The classification of quadric loci," *Transactions*, volume 6, 1905). In these articles the principles of classification are entirely different from that employed here.

In the section dealing with linear complexes, right and left complexes are distinguished, the existence of "diameter parallel nets" is proved, and the appearance of parallels in the linear complex and in the corresponding null space are investigated. Of special interest is the parallel complex, which possesses a whole net of axes and admits ∞^4 motions carrying it into itself, while the ordinary complex has only ∞^2. The article is concluded by a discussion of the properties of the general linear congruence and some of its special forms.

E. B. COWLEY.

Das Gruppenschema für züfallige Ereignisse. Von HEINRICH BRUNS. Des XXIX Bandes der *Abhandlungen der Mathematisch-Physikalischen Klasse der Königl. Sächischen Gesellschaft der Wissenschaften,* No. VIII. Leipzig, B. G. Teubner, 1906. Pp. 579–628.

THIS monograph is an extension of the brief development of the subject in the eighteenth lecture contained in the treatise by Bruns on Wahrscheinlichkeitsrechnung und Kollektivmasslehre. It is assumed in setting the simplest problem of the work that n balls are drawn, one at a time, from a bag containing balls of various colors, and that each time the ball drawn is returned to the bag before another drawing is made. The n consecutive drawings are called a draw series (Zugreihe) indicated by $Z(n)$. The draw series is written in the form

$$Z(n) = z_1 z_2 \cdots z_n, \tag{1}$$

where z_h denotes the hth drawing.

If the draw series are collected into sets of s with subscripts 1 to s, 2 to $s+1$, 3 to $s+2$, etc., the sets of s are called s-membered draw groups and the symbol $G(s)$ is used to designate such a group. In the formation of such groups, the author distinguishes between what he calls linear and cyclical groups. If, from (1), we take merely z_1 to z_s, z_2 to z_{s+1}, $\cdots$, z_{n-s+1} to z_n, the formation is called linear, and the corresponding groups linear groups. If, in addition to these groups, we proceed to take sets of s by repeating z's at the left of the draw series (1), the formation is called cyclical and the groups cyclical groups. The larger n is relative to s, the less significant is the difference of the two methods of group formation. It turns out that it is simpler to treat the cyclical groups than the linear groups, and it is of importance that the two do not differ significantly for large values of n.

Associated with each drawing Z_h, the author writes p_h, the probability of drawing that particular color. If in any group $G(s)$ we replace each z by the corresponding p, then, according as the elements p_h occur in $G(s)$, the group takes different forms. To illustrate in a simple case, for three colors $1, 2, 3$ the group $G(2)$ takes forms

$$p_1p_1,\ p_1p_2,\ p_1p_3,\ p_2p_1,\ p_2p_2,\ p_2p_3,\ p_3p_1,\ p_3p_2,\ p_3p_3.$$

It is a general analytic representation of the frequency distribution of such forms in a set of drawings that is the first concern of the present work. The simplest case is to determine the frequency with which a form of the group $G(s)$ occurs in each $Z(n)$ where a number of draw series $Z(n)$ have been obtained by drawing say n times. In this case, there is obtained an observed distribution indicated by $U(x)$, where $U(x)$ expresses the relative frequency with which the argument x occurs, and x refers to any group form. Next, the inquiry is for that theoretical distribution $U_1(x)$ of which the observed distribution may be regarded as a sample, or for the distribution that would be obtained if the draw series $Z(n)$ were taken an infinite number of times. The form of the theoretical distribution is given by means of the ϑ-operation introduced in Bruns's treatise.

The problem is next extended in various directions. Instead of confining the inquiry to the distribution of a single group form, several forms are considered simultaneously and different weights are given to the forms considered.

These considerations suggest building an argument

$$x = a_1x_1 + a_2x_2 + \cdots,$$

where a_h is the weight and x_h the frequency of the form of that weight in the totality.

It is shown that by transformations the consideration of one form for purposes of distribution may be replaced by that of one or more other forms, and that it is possible to reduce all the forms until they possess the same number of members. On account of such transformation it is sufficient to keep s fixed in $G(s)$ in treating the problem of distribution.

While the notation is rather complicated, the analytic expression for the frequency of the forms in draw groups seems to be a result fundamental in the theory of "collective quantity" (Kollektivgegenstand) in general, and for problems of statistics in particular, as drawings z_h are representative of any events back of which lies that mode of origination that belongs to problems of chance. The work appears to the reviewer to be of considerable importance for the mathematics of statistics.

H. L. RIETZ.

Elementary Treatise on the Differential Calculus. By W. W. JOHNSON. New York, John Wiley and Sons, 1908. x + 191 pp.

IF phrases current in the present political situation be allowed in reviewing a text in the calculus, the best possible way to describe the impressions made on the reviewer by the present volume would be to say that it is very plainly written from the viewpoint of the "stand-patter" who refuses to be convinced of the value for purposes of instruction in the calculus of the methods of limits and function theory as promulgated by the "progressives," or of the "insurgent," methods of modern disciples of the Perry movement. And the analogy goes further than the stand-pat attitude taken on the method of rates; for it applies throughout to the contents of the 7 chapters of the volume of 191 pages.

The new text is in great part an abridgment of the author's larger treatise on the differential calculus. The contents are very similar to the old, but seemingly compounded in a more digestible form for beginners. The attitude on rates having been taken, the author naturally makes a maximum use of the student's geometric intuition in explaining the fundamental notions of the differential calculus, a point of view sometimes lost sight of by those who, regardless, hold fast to rigor of demonstration.

The derivative, or differential coefficient, is defined as the relative rate of increase of the function as compared with the