method, so delicate that an expert must handle it with the greatest care, and nowadays chiefly of historical interest in view of the simpler and more general theory of Dedekind, Hensel contented himself in his address with a luminous elementary account of the essential features of Kummer's method as developed for a special set of integers. It should be stated that, while it is preferable to take the newer standpoint as regards the foundation of the theory of ideals, this change of viewpoint has not altered the validity of the rich array of fundamental results obtained by Kummer. The brief sketch on page 30 of Kummer's method of proving the impossibility of Fermat's equation, when λ is a regular prime, applies directly only to the first of the two cases into which Kummer divided the discussion. However, the same use of the class number is employed in the second case and this point is the one being emphasized by Hensel. All admirers of Kummer will take keen pleasure in reading this masterly Gedächtnisrede.

The 56 pages of letters from Kummer to his favorite and most gifted pupil Kronecker are of historic value. They give a first-hand view of the progress step by step made by Kummer in his construction of his own imperishable monument.

L. E. DICKSON.

Interpolationsrechnung. Von T. N. THIELE, Em. Professor der Astronomie an der Kopenhagener Universität, Präsident des Vereins Dänischer Aktuare. Leipzig, Teubner, 1909. xii + 175 pp.

IN this book the theory of interpolation is developed so as to emphasize its role in pure mathematics as well as its place in the calculations of applied mathematics. The book is divided into four chapters. The elementary treatment in the first chapter is based on the general interpolation formula of Newton, written

$$\begin{aligned} X = A + (x-a)[\delta'(a, b) + (x-b)\{\delta''(a, \cdots, c) \\ + (x-c)\{\delta'''(a, \cdots, d) + (x-d)\delta''''(x, a, \cdots, d)\}\}], \end{aligned} \tag{1}$$

where $a, b, c, d, \cdots$ are distinct values of the argument, $A, B, C, D, \cdots$ are corresponding tabulated values, and $\delta'(a, b), \delta''(a, \cdots, c), \cdots$ are "divided differences" defined by

$$\delta'(a, b) = \frac{A-B}{a-b}, \quad \delta''(a, \cdots, c) = \frac{\delta'(a, b) - \delta'(b, c)}{a-c}, \cdots.$$

A number of theorems are proved concerning divided differences, and it is shown, by numerical applications, that formula (1) is easily usable if the arguments be chosen in certain ways. After expressing the derivative of the nth order d^nX/dx^n in terms of divided differences with repeated arguments, the first application is to develop Taylor's series in terms of divided differences. The next application is made to the solution of numerical equations. The coefficients are expressed as divided differences and it is maintained that the interpolation method is one of the best methods of finding the real roots of numerical equations.

Interpolation by means of infinite series is treated to provide for cases where divided differences of no finite order have the value zero. The possibility that a Newton's interpolation formula, even if convergent, does not represent the function in question, is considered. The author maintains, however, that the functions of the most importance in applied mathematics, fortunately, belong to a class that can be interpolated, at least within certain limitations on the argument. He states the theorem that, for this class of functions, the interpolated values coincide with the function in question if for an infinite number of values of the argument, in a finite interval, there is exact coincidence. It seems that a characterization of functions as belonging to applied mathematics is not definite, in the sense that it is a basis for proving a theorem, and it should be emphasized that the method by which the theorem is established involves questions of convergence.

The first chapter ends with a careful investigation into the representation of a^x by a method of interpolation.

The second chapter is devoted to symbolic representation; the rather burdensome notation is made readable by the presentation of numerous numerical examples.

The third chapter treats the question of interpolation when the function has singularities, and presents graphic interpolation as an auxiliary method. Near the end of the third chapter, interpolation by reciprocal differences is presented. By means of these reciprocal differences, there arises a general method of interpolation that involves continued fractions and that is entirely analogous to the method of Newton. The general formula for interpolation by reciprocal differences is given. It is indicated that the precise applicability of reciprocal differences to interpolation is much more extensive, in a certain sense, than that of divided differences. This arises from the fact that where the method by divided differences is limited to integral functions, the method of reciprocal differences is limited to the most general form of fractional functions. To be sure, the application of reciprocal differences is also much more difficult than that of divided differences.

In the fourth chapter interpolation for functions of two or more variables is presented in an interesting and useful manner.

The book, as a whole, is a scholarly and readable presentation of the elements of the calculus of finite differences, and should be found of value not only to those interested in the arithmetical application of interpolation, but also to those interested in a theoretical treatment of the subject.

H. L. RIETZ.

Plane Geometry, with Problems and Applications. By H. E. SLAUGHT and N. J. LENNES. Allyn and Bacon, Boston, 1910. vi + 280 pp.

THIS book has several features that distinguish it from the conventional high school text. Among these the most noticeable are the gradual introduction of the severely logical forms, and the introduction, for the purpose of making the subject more attractive, of a large number of applications to geometric forms more or less commonly met with in life.

The book is divided into seven chapters, the first five of which correspond in a general way to Books I to V of Euclid, except that certain of the more difficult theorems, and the subject of incommensurable ratios are deferred to the last two chapters. Chapter I begins as usual with an introduction containing the common definitions. Numerical equality and geometric equality, or congruence, are sharply distinguished. The first propositions and problems are then introduced (pages 14–25) in an informal way. Then a few axioms are stated formally, and a number of theorems are given as "preliminary theorems," some of which are easy consequences, and some of which, for the purposes of the text, are assumed. A general discussion on the nature of a demonstration follows, after which proofs are given in the usual form. Aside from the introduction of the applications and the deferring of the matter indicated above, the content of the first five chapters is about that of the usual book. The algebraic form of the treatment is a decided improvement. One particularly pleasing feature is the willing-