with this sort of mathematics. With the rigorist the question at issue is frequently not whether the proposition which apparently is being considered is *true* or not, but whether it follows from some other proposition. That is, how much must we say to include by implication a certain other body of propositions.

It seems equally legitimate to inquire what must be the properties of systems of numbers in order that they shall express conveniently and accurately the varied phenomena that daily impinge upon us.

The prospective reader must judge for himself whether this is the sort of book he wants to read. Does he wish to study the modern development of the various algebras as based upon definite assumptions? Then this book is of no use to him. Does he wish to see an attempt to develop these algebras as corollaries of physics? Then it is probably the best book he could find.

N. J. LENNES.

Grundlagen der Geometrie. Von Dr. FRIEDRICH SCHUR. Leipzig, Teubner, 1909. vii + 192 pp.

THE Neuere Geometrie of Pasch marked the first effort to set up a complete set of fundamental statements for geometry —if we except the manifoldness development of Riemann. Following the appearance of that book there came a remarkable development of the subject that has thrown great light upon the logic of geometry. Schur wishes the present book to be considered as, in a sense, a revision of Pasch. Although the author expresses very strongly his indebtedness to Pasch, in only the most general way can the book be said to be such a revision.

In the general trend of his development the author follows Peano, in that congruence is obtained from motion or from projective geometry rather than directly from postulates, as is done by Hilbert, for example.

The general problem is formulated as follows: "To set up a simple and complete system of intuitive facts or axioms, entirely independent of one another, from which geometry can be derived by purely logical processes. To deserve the name geometry, axioms must be employed which express the results of the simplest and most elementary consideration of geometric figures, from which they are obtained by abstraction." This of course bars out the idea of space as a number manifold.

Although in a general way the effort is made to have the axioms independent, there would seem to be no effort to have each axiom contain but a single statement. In this respect the book lacks the elegance of logical form of the work of Veblen.

In § 1, the "projective postulates" are developed. The point and the segment (Strecke) are employed as undefined symbols, and the postulates concerning them in this chapter are sufficient to develop the ordinary connective properties of space and the order relations of points, and to characterize the space as being of exactly three dimensions. These axioms are equivalent to the first two groups of axioms of Hilbert. The line, plane, and space are formally defined in terms of point and segment, and the separation theorems are proved.

In § 2, after proving Desargues's theorem concerning perspective 3-edges in a bundle of lines in 3-space, it is shown that a line and a plane not containing it, or two lines in the same plane, determine a set of lines having the properties of a bundle. If the existence of the point of intersection is not certain, the bundle thus obtained is said to determine an ideal point. Similarly ideal lines are defined by means of non-intersecting planes. Desargues's theorem is then proved in the plane, and the complete duality of the system of actual and ideal points is shown. That the Desargues theorem can not be proved in the plane alone is shown by means of the non-desarguesian geometry of Moulton. Finally an ideal plane is defined as the locus of lines which join an ideal point to all points of an ideal line not containing it, and the ideal connection properties of 3-space are then complete. Since the only assumption concerning the intersection of lines in a plane is that regarding the points of a triangle, i. e., a restricted domain, the development can serve as a basis for the euclidean and non-euclidean geometries which differ regarding intersection properties of two lines in the unlimited plane. The central collineation and collinear reflections are then defined, and used for the definition of harmonic points.

In § 3, postulates are introduced to make precise the concept of motion, and on this are based definitions of absolute pole and polar, perpendicular lines and planes, folding, and rotation, and the proof of Pascal's theorem for the degenerate conic. Finally congruence of segments is defined as follows: Two segments are called congruent, if one can be transformed into the other by a motion.

In § 4, the fundamental theorem of projective geometry is proved and the algebra of segments and resulting analytic geometry are developed. The fundamental theorem is proved by making use of the postulates of motion, a procedure that appears less natural from the projective point of view than it would be to choose as a postulate either this theorem itself, or the theorem of Pascal.

The projective development is then applied for the derivation of the fundamental metric forms of non-euclidean geometry. The equations of lines are obtained, and the trigonometric functions sine and cosine are defined projectively. The theory of triangles then follows readily. The characteristic constant k occurring in these formulas depends on the character of the absolute involution obtained on each line. Congruence of triangles is developed, and construction of the triangle from given parts. The proof of the independence of the parallel axiom is made by exhibiting in the classical way the analytic number spaces for which each hypothesis concerning parallels is valid. The relation of the parallel axiom to the angle-sum of a triangle and to motion is also discussed. In § 7, plane geometry is developed without the use of 3-space, following the methods of Hjelmslev. In § 8, the role of the Archimedean axiom is treated. The proof that while the Pascal theorem does not follow from the projective postulates alone, it does follow from these with the addition of the Archimedean postulate, is especially interesting.

As a whole the book is a valuable addition to the literature of the subject. More references to original sources would add much to its value. It is not free from typographical errors, but those noted are fairly evident from the context.

F. W. OWENS.

Précis de Mécanique rationnelle. Introduction à l'Étude de la Physique et de la Mécanique appliquée. Par P. APPELL et S. DAUTHEVILLE. Paris, Gauthier-Villars, 1910. 8vo. 729 pp. 25 francs.

To condense an enormous Traité in three volumes and over 1,800 pages into a *précis*—a very significant term—of one volume and about 700 pages, demands a treatment which is more than a mere deletion of certain parts. The whole must to some extent be recast. Everyone has seen abridged treatises which were worthless and by the side of the original quite ob-