# THE NINETEENTH SUMMER MEETING OF THE AMERICAN MATHEMATICAL SOCIETY.

THE nineteenth summer meeting of the Society was held at the University of Pennsylvania on Tuesday and Wednesday, September 10–11, 1912, extending through two sessions on Tuesday and a morning session on Wednesday. The following twenty-nine members were in attendance:

Professor Frederick Anderegg, Professor M. J. Babb, Mr. A. A. Bennett, Dr. Elizabeth R. Bennett, Professor G. G. Chambers, Dr. A. Cohen, Professor F. N. Cole, Dr. G. M. Conwell, Professor E. S. Crawley, Professor G. H. Cresse, Professor E. W. Davis, Professor W. P. Durfee, Professor H. B. Evans, Mr. Meyer Gaba, Professor O. E. Glenn, Professor G. H. Hallett, Dr. S. Lefschetz, Professor J. A. Miller, Dr. H. H. Mitchell, Mrs. Anna J. Pell, Professor M. B. Porter, Professor Arthur Ranum, Professor E. D. Roe, Jr., Dr. J. E. Rowe, Professor F. H. Safford, Professor I. J. Schwatt, Professor H. D. Thompson, Professor Oswald Veblen, Professor H. S. White.

Ex-President H. S. White occupied the chair, being relieved by Professors Crawley and Davis. The Council announced the election of the following persons to membership in the Society: Professor W. A. Bratton, Whitman College; Professor Florence P. Lewis, Goucher College; Mr. Leslie MacDill, Indiana University; Professor H. W. March, University of Wisconsin; Mr. M. R. Richardson, University of Chicago; Dr. J. I. Tracey, Johns Hopkins University; Mr. H. S. Vandiver, Philadelphia, Pa. Five applications for membership in the Society were received.

On both days of the meeting luncheon was provided by the University of Pennsylvania. On Tuesday evening twenty-six of the members gathered at the usual dinner. In the interval between the sessions the visitors were conducted through the university buildings and grounds. On Wednesday afternoon an automobile excursion about the city was arranged. At the close of the meeting a vote of thanks was tendered to the university authorities for their generous hospitality.

The following papers were read at this meeting:

(1) Professor R. D. CARMICHAEL: "On the theory of relativity: analysis of the postulates."

(2) Professor F. H. SAFFORD: "An irrational transformation of the Weierstrass $\wp$-function curves" (preliminary communication).

(3) Dr. E. L. DODD: "The least square method grounded with the aid of an orthogonal transformation."

(4) Dr. E. L. DODD: "The probability of the arithmetic mean compared with that of certain other functions of the measurements."

(5) Dr. HENRY BLUMBERG: "Algebraic properties of linear homogeneous differential expressions."

(6) Dr. J. E. ROWE: "The relation between tangents and osculant $(n - 1)$ics of rational plane curves."

(7) Dr. H. H. MITCHELL: "Determination of all primitive collineation groups in $n$ ($> 4$) variables which contain homologies."

(8) Professor ARTHUR RANUM: "Lobachevskian polygons trigonometrically equivalent to the triangle."

(9) Professor G. A. MILLER: "A few theorems relating to Sylow subgroups."

(10) Mrs. ANNA J. PELL: "Linear equations in infinitely many unknowns."

(11) Mr. L. B. ROBINSON: "Invariants of two tetrahedra."

(12) Professor F. R. SHARPE: "The Klein-Ciani quartic."

(13) Professor F. R. SHARPE: "The (2, 1) ternary correspondence with a sextic curve of branch points."

(14) Professor F. R. SHARPE and Dr. F. M. MORGAN: "A type of quartic surface invariant under a non-linear transformation of period 3."

(15) Dr. S. LEFSCHETZ: "Double curves of surfaces projected from $S_4$."

(16) Dr. HENRY BLUMBERG: "Sets of postulates for the rational, the real, and the complex numbers."

(17) Professor OSWALD VEBLEN: "Decomposition of an $n$-space by a polyhedron."

(18) Professor F. N. COLE: "The triad systems of thirteen letters."

(19) Professor H. S. WHITE: "Triple systems as transformations, and their paths among triads."

(20) Professor L. C. KARPINSKI: "Augrim stones."

(21) Dr. DUNHAM JACKSON: "On the approximate representation of an indefinite integral."

(22) Dr. T. H. GRONWALL: "Some special boundary problems in the theory of harmonic functions."

(23) Dr. T. H. GRONWALL: "On analytic functions of constant modulus on a given contour."

(24) Dr. T. H. GRONWALL: "On series of spherical harmonics."

(25) Professor O. E. GLENN: "A general theorem on upper and lower limits for the order of a factor of a $p$-ary form with polynomial coefficients."

(26) Professor E. J. WILCZYNSKI: "On a certain class of self-projective surfaces."

Dr. Blumberg was introduced by the Secretary, Mr. Robinson by Dr. Cohen. In the absence of the authors the papers of Professor Carmichael, Dr. Dodd, Professor Miller, Professor Sharpe, Dr. Morgan, Professor Karpinski, Dr. Jackson, Dr. Gronwall, and Professor Wilczynski were read by title.

The papers of Professor Miller and Dr. Lefschetz appear in full in the present issue of the BULLETIN. Abstracts of the other papers follow below. The abstracts are numbered to correspond to the titles in the list above.

1. In the present paper Professor Carmichael subjects the postulates of relativity to a fresh analysis. He points out that a part of the second postulate is a consequence of the first, and calls attention to the fact that most workers in the theory of relativity have (consciously or unconsciously) made other fundamental hypotheses in addition to the two so-called postulates of relativity. These additional hypotheses should be stated as postulates; and in this paper for these additional hypotheses two very simple ones are chosen. These two together with the first and a special form of the so-called second postulate of relativity form the basis of the paper. With this as a foundation the essential general results of the theory of relativity are built up in a very elementary manner and certain extensions of previous results are obtained. Special attention is given to an analysis of those elements in the postulates which give rise to the strange conclusions of the theory. The problem of logical equivalents of the postulates is also treated.

2. In two previous papers Professor Safford has considered an irrational transformation due to Weierstrass, in which the curves obtained are of the sixteenth degree. In the case analogous to the Legendrian form of the elliptic element he

has resolved these curves into four curves of the fourth degree. But in the case corresponding to the Weierstrass form two curves are of the fourth degree, while there remains an apparently irresolvable component.

This paper is a study of the conditions under which further resolution is possible.

3. Dr. Dodd establishes the least square method, for the case in which the observation equations are linear, as a consequence of the Gaussian probability law, without recourse to infinite series, approximations, or a discontinuity factor.* Only those values for the unknowns are considered which can be obtained from linear combinations of the observation equations. It would be futile to consider all possible values for the unknowns in this connection; for under the Gaussian law, there are no "most probable values" for the unknowns.

Let $x$ be the true value of one of the unknowns and $\xi$ an approximation for $x$, obtained from a linear combination of the observation equations, with multipliers $X_i$. Theorem: The error of each measurement being subject to the Gaussian law, the probability that the error $x - \xi$ of $\xi$ will lie in any given interval $(-\alpha, \alpha)$ is greatest when the $X$'s are so chosen that $\xi$ has the value given it by the least square method.

There being $n$ observation equations, the required probability is expressed as an $n$-fold integral. This is then simplified by an orthogonal transformation—"rotation"—making the two "parallel planes" which bound the region of integration "perpendicular" to an "axis of coordinates." If the measures of precision are different, a similitude transformation is first used.

4. In his second paper Dr. Dodd investigates the consequences of the Gaussian probability law, and shows that this law is not compatible† with the principle of the arithmetic mean as the "most probable value." By the Gaussian law (G. L.) is meant the statement that

$$\frac{h}{\sqrt{\pi}}\int_{x'}^{x''} e^{-h^2x^2}dx$$

* See Czuber, Theorie der Beobachtungsfehler, p. 232.

† Bertrand, in his Calcul des Probabilités (1889), pp. 177–180, gives an example to exhibit this incompatibility.

is the probability that the error $x = a - m$ of the measurement $m$ will lie between $x'$ and $x''$; here $a$ is the true value. Under the G. L. there is, indeed, "no most probable value" for the unknown, the probability of each of the infinite number of possible values being zero.

Definition: The probability of a function $F$ of the measurements will be said to be greater than that of another function $f$ if the probability that $F$ will differ from the true value $a$ by less than $\alpha$ is greater than the probability that $f$ will differ from $a$ by less than $\alpha$, for all positive values of $\alpha$ less than some $\alpha'$.

Then, under the G. L., the measurements having the same "measure of precision" $h$, the probability of the root-mean-square, $\sqrt{\frac{1}{2}(m_1^2 + m_2^2)}$, of two measurements $m_1$, $m_2$ is greater than that of the arithmetic mean $M = \frac{1}{2}(m_1 + m_2)$ if $ha > 2$. But in the case of three measurements the probability of the arithmetic mean is the greater.

Let $M$ be the arithmetic mean of any number of measurements, with the same $h$. There exist values of the constant $b$, a little less than unity, such that the probability of $bM$ is greater than that of $M$. But if $b > 1$, the probability of $bM$ is less than that of $M$.

Under the G. L., the probability of the geometric mean, and of the median and of $M + c$,—this $c$ being a constant, not zero,—is less than the probability of $M$.

If the measures of precision are $h_1, h_2, \cdots, h_n$, respectively, the probability of the weighted mean $W = p_1m_1 + p_2m_2 + \cdots + p_nm_n$ (where $p_1 + p_2 + \cdots + p_n = 1$) is greatest when the weights are given their usual values $p_i = h_i^2/\Sigma h^2$; but values of $b$ exist making the probability of $bW$ greater than that of $W$.

5. In his first paper Dr. Blumberg sketches the main results in his dissertation, entitled "Ueber algebraische Eigenschaften von linearen homogenen Differentialausdrücken" (Göttingen, 1912). The dissertation deals with algebraic properties (such as those connected with reducibility, irreducibility, etc.) of expressions of the form

$$a_0(x)\frac{d^ny}{dx^n} + a_1(x)\frac{d^{n-1}y}{dx^{n-1}} + \cdots + a_{n-1}(x)\frac{dy}{dx} + a_n(x)y,$$

where the functions $a_0(x)$, $a_1(x)$, ... belong to a fixed domain of rationality, in the sense of the Picard-Vessiot group theory of linear homogeneous differential equations. The chief point to be emphasized is that the proofs make no use of integrals, but are given with the aid of a very small number of properties of linear homogeneous expressions, so that the results are of a much more general character than those hitherto obtained.

6. A rational plane curve of order $n$, an $R^n$, possesses $2n - 2$ tangents through every point of the plane; also, there are $2n - 2$ osculant $(n - 1)$-ics of the $R^n$ through every point of the plane. Dr. Rowe's paper consists in proving that any projective property imposed upon the parameters of the $2n - 2$ osculant $(n - 1)$-ics of the $R^n$ through a point holds for the tangents through the point as a pencil of lines.

7. In a recent paper (*Proceedings of the London Mathematical Society,* November, 1911) Burnside made a determination of all finite collineation groups in $n$ variables with rational coefficients which contain the symmetric group on those variables. In addition to the already known groups, those of order $(n + 1)!$ (for any $n$) and $2^7 \cdot 3^4 \cdot 5$ (for $n = 6$), he found two additional primitive groups, of order $2^9 \cdot 3^4 \cdot 5 \cdot 7$ (for $n = 7$) and $2^{13} \cdot 3^5 \cdot 5^2 \cdot 7$ (for $n = 8$).

Dr. Mitchell solves a more general problem, i. e., the determination of all primitive groups in $n (> 4)$ variables which contain homologies, the results for $n \leqq 4$ being already known. A transformation in the symmetric group which interchanges two of the variables and leaves the rest unaltered is evidently an homology of period 2. In addition to the groups mentioned, there are two others, of order $2^6 \cdot 3^4 \cdot 5$ (for $n = 5$) and $2^8 \cdot 3^6 \cdot 5 \cdot 7$ (for $n = 6$). Neither of these can be represented with rational coefficients. The former is isomorphic with the known simple group of that order, and the latter contains a self-conjugate subgroup of index 2, which is (1, 1) isomorphic with a known simple group.

8. Professor Ranum shows that in the Lobachevskian plane there are nine polygons whose trigonometry is equivalent to that of the triangle and three special polygons whose trigonometry is equivalent to that of the right triangle. Among the former is the rectangular hexagon (whose angles are all

right angles). Among the latter is the rectangular pentagon, which bears somewhat the same relation to the right triangle in the Lobachevskian plane that Gauss's "Pentagramma Mirificum" bears to the right triangle in the Riemannian plane. The hyperbolic cosine of any side of the rectangular pentagon is equal to the product of the hyperbolic sines of the two opposite sides and to the product of the hyperbolic cotangents of the two adjacent sides.

10. A matrix $(k_{pq})$ is unlimited if the transformation $\Sigma_q k_{pq} x_q$ does not transform every system of variables $\{x_p\}$ of finite norm into a system of finite norm. Linear differential and differential-integral equations, and other types of linear equations, correspond directly to linear equations in infinitely many unknowns for which the matrix of the coefficients is unlimited. In Mrs. Pell's paper conditions are given for the existence of solutions of systems of homogeneous and non-homogeneous linear equations with an unlimited matrix of coefficients.

11. In this paper Dr. Robinson obtains a complete set of invariants of two tetrahedra, the one taken in points, the other in planes. The tetrahedra are written $(a\xi)(b\xi)(c\xi)(d\xi) = 0$ and $(\alpha x)(\beta x)(\gamma x)(\delta x) = 0$. An invariant is defined as a rational integral function of the coefficients homogeneous in the Greek and Roman letters and unaltered by any permutation of the letters and the subscripts. The group of permutations is of order 576. A rather full discussion is given of this group and the results obtained are compared with those obtained by Feder in a paper in the *Mathematische Annalen*, volume 47.

12. The Klein quartic (*Mathematische Annalen*, 1879)

$$x^3z + y^3x + z^3y = 0 \tag{1}$$

and the Ciani quartic (*Palermo Rendiconti*, 1899)

$$\Sigma x^4 - 3\lambda\Sigma y^2z^2 = 0 \tag{2}$$

(for a special value of $\lambda$) are both invariant under 21 harmonic homologies. Ciani states that (2) must therefore be transformable into (1) by some opportune transformation. In Professor Sharpe's paper the requisite transformation is determined and is applied to prove the following theorems. There are 63 conics that pass through one vertex of each of the 8 inflectional triangles, 21 of which pass through each vertex. There are 168 conics that pass through 2 vertices of each of 4 of the 8 inflectional triangles, 56 of which pass through each vertex. There are 28 conics that pass through the 6 vertices of each pair of inflectional triangles and through the points of contact of an associated bitangent.

13. Noether (*Mathematische Annalen*, 1889), reduced the double planes that can be rationally mapped on simple planes to three types according as the curve of branch points is (1) a $C_{2n}$ with a $(2n - 2)$-fold point, (2) a non-singular quartic, (3) a sextic having 3 branches touching the same line at a common point. A complete analytical treatment of the second type was given by De Paoli (*Atti Lincei*, 1878), and of the first type by Boyd (*American Journal*, 1912). In Professor Sharpe's second paper precise analytical expressions are determined for the remaining type of (2, 1) correspondence by means of the Grassmann and Wiman (*Mathematische Annalen*, 1895) depictions of a cubic surface on a simple and double plane.

14. Professor Sharpe and Dr. Morgan consider a quartic surface whose section by any plane through a line $AB$ is the line and a cubic through $A$ and $B$. If the line joining $A$ to any point $P$ on the surface meets the surface in $Q$ and if $BQ$ meets the surface in $R$, the transformation used converts $P$ into $R$. In the most general case the conditions for periodicity give 17 relations amongst the 26 coefficients of the equation of the surface. When certain terms are absent from the equation the conditions for periodicity are simple.

16. In his second paper Dr. Blumberg deals with sets of postulates for rational, real, and complex numbers. These sets grew out of conversations with Professor Zermelo and it is the intention of the latter and Dr. Blumberg to publish their results in detail in the near future. Starting with a set ($F$) of postulates defining the most general field (Körper), where the commutative law of multiplication need not hold,—such a field

will be henceforth simply called a "non-commutative field"—we obtain by the addition of a postulate $R$ a set of postulates defining the most general non-commutative field that contains as a sub-field the field of rational numbers. By the further addition of one postulate $C$, we obtain a set of postulates defining the most general non-commutative field that contains as a sub-field the field of real numbers. Finally, the set of postulates $(F)$, $R$, $C$ and a new postulate $I$ define the most general non-commutative field containing as a sub-field the field of complex numbers. To obtain sets of postulates defining the rational, real, and complex numbers a postulate $P$ is added respectively to the second, third, and fourth set of postulates mentioned above, as shown below:

For the rational numbers: $(F)$ $R$, $P$.
For the real numbers: $(F)$, $R$, $C$, $P$.
For the complex numbers: $(F)$, $R$, $C$, $I$, $P$.

There are no postulates of order. In the proofs the concept of finite number is not presupposed.

17. Professor Veblen's paper gave a proof of the theorem that an $(n-1)$-dimensional polyhedron decomposes an $n$-dimensional space into two regions. The proof is of an essentially combinatorial character and involves few geometrical ideas beyond the principle that an $n$-dimensional convex region is decomposed into two $n$-dimensional convex regions by an $(n-1)$-space containing one of its points.

18. Triad systems in thirteen letters were given by Kirkman, Reiss, Netto, and De Vries. But the complete determination of these systems, of which there are only two distinct types, was first effected by De Pasquale* and Brunel,† both of whom based their examination on the configurations of S. Kantor. Professor Cole determines the possible systems by direct construction without the use of any special apparatus.

In the discussion of the paper, Professor Veblen pointed out that the two systems might be obtained as an application of finite geometry.

19. In a triple system every pair of elements (or dyad)

* "Sui sistemi ternari di 13 elementi," *Rendiconti R. Istituto Lombardo*, ser. 2, vol. 32 (1899).

† "Sur les deux systèmes de triades de treize éléments," *Journal de Math.*, ser. 5, vol. 7 (1901).

occurs in some triad once only, and the system may therefore be regarded as relating every dyad to some one element, the one which occurs with it in a triad. By this means every triad, as containing three dyads, may be converted into another triad or set of three elements. Iteration produces trains of triads, terminating in recurrent cycles. For the two triple systems on 13 elements these trains are studied. They are found to be characteristic for the species, and incidentally to facilitate the discovery of the group which transforms the triple system into itself.

20. Professor Karpinski explains the system of calculation in use from the time of Gerbert (c. 1000 A.D.) through several centuries, employing Hindu numerals upon an abacus. The numerals were placed upon markers (or stones or apices) and used instead of the corresponding set of stones. This accounts for the self-contradictory expression *augrim* (or algorism) stones, as found in Chaucer's Canterbury Tales. The word algorism was widely used to indicate the Hindu art of reckoning, employing the zero, whereas the term stones implies the use of an abacus. This article is to appear in *Modern Language Notes.*

21. From a theorem which he recently reported to the Society, concerning the approximate representation of a periodic function satisfying a Lipschitz condition, Dr. Jackson deduces the following general result: If $f(x)$ is a function of period $2\pi$ which can be approximately represented for all values of $x$ by a trigonometric sum of the $n$th order with an error not exceeding $\epsilon$, and if $f(x)$ is such that its indefinite integral also is periodic, then this indefinite integral can be represented by a trigonometric sum of the $n$th order or lower with an error not exceeding $6\,\epsilon/n$. In a case of particular interest, the factor 6 may be replaced by 3. Of the consequences of this theorem, the following may be mentioned: If $f(x)$ has the period $2\pi$ and possesses a $(k-1)$th derivative which satisfies a Lipschitz condition with coefficient $\lambda$, then $f(x)$ can be approximately represented by a trigonometric sum of the $n$th order or lower with a maximum error not greater than $3^k\lambda/n^k$. This is an improvement over the result previously established in this connection. Corresponding but not precisely parallel theorems may be stated concerning approximation by means of polynomials.

22. Dr. Gronwall shows how the theory of the Cesàro summability of the Fourier and Laplace series, coupled with the generalizations of Abel's continuity theorem for power series due to Frobenius and Hölder, gives a simple and effective method for solving boundary problems in mathematical physics in the case of a circular ring or a hollow sphere. The method is illustrated, first by Dirichlet's problem for a circular ring (previously solved by Villat) and a hollow sphere, in which cases the proofs are considerably shortened, and second, by the determination of a harmonic function in a circle, where the derivative along the normal is given on part of the circumference, the function itself being given on the remaining part; in this case (also treated by Villat) not only the proof is shorter, but the final formula is considerably simplified.

23. In this paper, Dr. Gronwall considers a uniform analytic function $f(x)$ having a finite number of essential singularities inside or on a given circle, and being of constant modulus on a set of points on the circumference with at least one limit point which is not an essential singularity. After showing that, in consequence of the last condition, the modulus is constant in every regular point on the circumference, the singularities and zeros outside the circle are determined by Schwarz' method of analytic continuation, and finally the general analytical expression of the function is obtained.

24. In the first part of this paper, Dr. Gronwall considers the $(n+1)$th partial sum in the formal development of a function $f(\theta, \varphi)$ in spherical harmonics according to Laplace's formula. When $f(\theta, \varphi)$ is limited to the class of continuous functions not exceeding unity in absolute value at any point of the sphere, this sum has an upper limit $\rho_n$, and it is shown that

$$\lim_{n=\infty} \frac{1}{\sqrt{n}} \cdot \rho_n = 2\sqrt{\frac{2}{\pi}},$$

which completes the result previously found by Haar

$$\liminf_{n=\infty} \frac{1}{\log n} \cdot \rho_n > 0.$$

As applications, simple examples of everywhere continuous functions are formed, the Laplace developments of which are divergent at a given point, or non-uniformly convergent.

The second part of the paper is concerned with the summability of Laplace's series by the Cesàro (or Hölder) means. Fejér has established the summability of the second order for any absolutely integrable function in every point where it is continuous; Haar, considering the particular case of Legendre's series for any function $f(x)$ (where $x = \cos\theta$), continuous for $-1 \leqq x \leqq 1$, proved the summability of the first order except at the end points $-1$ and $+1$, which are inaccessible to his method and were treated by Chapman under additional restrictions regarding $f(x)$.

Dr. Gronwall defines a set of constants $\rho_n'$ bearing the same relation to the arithmetic mean of the $n+1$ first partial sums of Laplace's series as $\rho_n$ to the $(n+1)$th sum itself, and shows that these constants $\rho_n'$ are bounded for all values of $n$. From this result, the following general theorem is derived: The Laplace series of any absolutely integrable function $f(\theta, \varphi)$ is summable of the first order at every point where the function is continuous, and this summability is uniform over any range interior to a continuity range.

Finally, it is shown that when $f(\theta, \varphi)$ is integrable, without being absolutely integrable, the corresponding Laplace series is summable of the second order at every point of continuity.

25. Professor Glenn takes a homogeneous $p$-ary form written in the ordinary manner, without multinomial multipliers with its coefficients, and considers its coefficients to be polynomials in one letter $x$, after the method of Königsberger* in his generalizations in the binary case of Eisenstein's theorem. A general range of functions is then considered, consisting of $k$ of these polynomials arranged in a certain order. Derived ranges are defined. A formula for the number of functions on a general range is given. Certain assumptions are then made with reference to the existence of common factors of the functions on a general range, and of the functions on its derived ranges and it is proved that the order $n$ of any factor of the $p$-ary form must, under the given hypothesis lie between limits, which are determined. The theorem is applied in proving certain general families of forms irreducible.

* *Journal für Math.*, vol. 115.

26. Several years ago Professor Wilczynski showed that, by introducing a properly chosen system of projective coordinates, the equation of a non-ruled surface, in the vicinity of an ordinary point, may be replaced by a development of the form

$$z = xy + \tfrac{1}{6}(x^3 + y^3) + \tfrac{1}{24}(Ix^4 + Jy^4) + \dots,$$

where $I$, $J$ and all higher coefficients of this expansion are absolute differential invariants of the surface. The present paper is devoted to an investigation of those special surfaces for which $I$ and $J$ are everywhere equal to zero, completely determines these surfaces in certain elementary cases, and obtains a large number of properties for them.

F. N. Cole,
*Secretary.*

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# A FEW THEOREMS RELATING TO SYLOW SUBGROUPS.

BY PROFESSOR G. A. MILLER.

(Read before the American Mathematical Society, September 10, 1912.)

Suppose that a group $G$ involves more than one Sylow subgroup of order $p^m$, and that a subgroup $H$ of $G$ involves more than one Sylow subgroup of order $p^\beta$, $0 < \beta < m$. The number of the subgroups of order $p^\beta$ in $H$ cannot exceed the number of those of order $p^m$ in $G$, since any two Sylow subgroups of any group generate a group whose order is divisible by at least two distinct prime numbers, and hence each Sylow subgroup of order $p^\beta$ in $H$ occurs in one and in only one Sylow subgroup of order $p^m$ in $G$.

When $H$ is an invariant subgroup of $G$ it is easy to prove that the number of the Sylow subgroups of $G$ is a multiple of the number of the corresponding Sylow subgroups of $H$. In fact, all the operators of $G$ which transform a subgroup of order $p^m$ into itself must also transform into itself all the operators of the subgroup of order $p^\beta$ in $H$ which is contained in the particular subgroup of order $p^m$ under consideration. Hence it results that all the operators of $G$ which transform into itself a subgroup of order $p^\beta$ contained in $H$ must constitute a group involving $k$ subgroups of order $p^m$ and containing