SHORTER NOTICES.

Elliptische Funktionen. Von Professor Dr. KARL BOEHM.
Zweiter Teil. Göschen (Sammlung Schubert LXI), Leipzig,
1910. vii+180 pp. M. 5.

THIS little book is complete in itself and independent of the
first volume.* It is devoted entirely to the older theory,
that is, the development of the properties of elliptic integrals
and the inversion problem.

In the first chapter we have a rather thorough discussion
of hyperelliptic integrals. There is, first, the reduction of the
general hyperelliptic integral to the three possible types and,
second, the classification of these integrals according to
deficiency (Geschlecht). The elliptic integrals are thus intro-
duced as a particular class of hyperelliptic integrals (cf.
Picard, Traité, Tome I).

The second chapter deals with the behavior of the elliptic
integral of the first kind in the complex plane and upon the
proper Riemann surface, leading to the existence and meaning
of the two period moduli.

The third chapter is devoted to the inversion of the elliptic
integral of the first kind. The point of departure is the
theorem concerning the existence of the solution of the
differential equation $dz/du = F(z)$, where $F(z)$ is an analytic
function of z.

A careful proof of the uniqueness of the inverse function is
given; the final conclusion being that this function is mero-
morphic in the entire u plane. The results of the preceding
chapter enable the author to add that the inverse function is
doubly periodic and of the second order. In this way, then,
we are led to the elliptic function of the second order. The
chapter closes with the theorem:

The most general elliptic integral can be expressed as the
integral of an elliptic function $\phi(u)$, where u is the elliptic
integral of first kind belonging to the same irrationality as
appears in the general integral.

The fourth chapter considers the integral of the first kind
and its inversion when the branch points are real. There is

* Reviewed in the BULLETIN for January, 1911, vol. 17, p. 202.

a discussion of the real values of the inverse function and the various ways of representing this function geometrically.

Chapter five is devoted to a discussion of the usual normal forms of the elliptic integral of the first kind.

The sixth and last chapter contains Abel's theorem and its consequences; in particular addition theorems for the inverse function and for related functions.

As is stated in the preface, the book contains no discussion of modular functions, the transformation theory, or the applications. The author's purpose is to present, in a concise manner, the necessary concepts and theorems which form the definition and fundamental properties of elliptic integrals and their inverses. This he has succeeded in doing without digression, although the temptation to include one or more chapters on the subjects noted above must have been great.

The beginner will better appreciate the first volume if he follows Dr. Boehm's suggestion (Vorwort, Erster Teil, page iv), viz., to read this second volume first.

<div style="text-align: right">L. WAYLAND DOWLING.</div>

Éloges académiques et Discours. Volume publié par le Comité du Jubilé scientifique de M. GASTON DARBOUX. Paris, A. Hermann et Fils, 1912. 525 pp. Fr. 5.

THE present volume was published by an international committee of mathematicians, formed for the purpose of expressing an appreciation of the scientific work of M. Gaston Darboux at the completion of his fiftieth year of public instruction. This committee addressed a circular letter to the mathematicians of all countries, inviting subscriptions for the purpose of awarding M. Darboux a medal on this occasion. The responses were so numerous and so liberal that the committee was enabled to have the medal executed by the eminent French artist M. Vernon, and also to publish the present volume and send it to all the subscribers.

The greater part of the volume consists of a collection of eulogies by M. Darboux. The subjects of these are: Joseph-Louis-François Bertrand, François Perrier, Charles Hermite, Antoine d'Abbadie, Général Meusnier, Donateurs de l'Académie. These eulogies are followed by a collection of discourses by M. Darboux and by an account of the Jubilé, including the various addresses and a list of the subscribers. The Jubilé was to be held towards the end of October, 1911,