$$A_n = \gamma^{\lambda-2},\quad B_n = \alpha\gamma^{\lambda-3},\quad C_n = \alpha^2\gamma^{\lambda-4},\quad \cdots,\quad K_n = \alpha^{\lambda-2},$$
$$(-1)^{n-1} Q_n = \alpha^\lambda - R\gamma^\lambda,\ (-1)^n P_n = \alpha^{\lambda-1}\beta - R\gamma^{\lambda-1}\delta,$$

where α/γ and β/δ are the nth and $(n-1)$th convergents in the expansion of $R^{1/\lambda}$. Other remarkable relations between the coefficients are indicated.

2. From two logical classes can be developed a system of sixteen elements which is closed under the operations of logical addition, multiplication, and negation and forms a group under a certain other operation. Professor Whitney discusses this system and shows that it has a high degree of symmetry and that its internal structure is the same as that of a regular tetrahedron.

3. In the paper of Professor Lewis there is set up a revised system of implications in the algebra of logic which will exclude such doubtful theorems as "A false proposition implies any proposition" and "A true proposition is implied by any proposition." This system indicates that definitions in mathematics are relations of reciprocal implication, and that such relations can sometimes be deduced instead of assumed.

T. M. PUTNAM,
Secretary of theSection.

THE FIFTH INTERNATIONAL CONGRESS OF MATHEMATICIANS. SECTIONS II–IV.

SECTION II. GEOMETRY.

IN geometry, four sectional meetings were held, the chairmen being H. F. Baker, F. Severi, J. Drach and F. Morley. A. L. Dixon and E. Bompiani were elected permanent secretaries for all the sessions. The following papers were presented before this Section.

(1) BROUWER, L. E. J., Amsterdam: "Sur la notion de classe de transformations d'une multiplicité."

(2) MORLEY, F., Baltimore: "On the extension of a theorem due to W. Stahl."

(3) EISENHART, L. P., Princeton: "Continuous deformation of surfaces applicable to quadrics."

(4) BOMPIANI, E., Rome: "Recent progress in projective differential geometry."

(5) NEVILLE, E. H., Cambridge, England: "On generalized moving axes."

(6) BRÜCKNER, M., Bautzen: "Ueber Raumteilung durch 6 Ebenen und die Sechsfläche."

(7) MARTIN, A., Washington: "On rational right-angled triangles."

(8) STÉPHANOS C., Athens: "Sur l'équivalent analytique du problème des principes de la géométrie."

(9) ESSON, W., Oxford: "On the characters of plane curves."

(10) DRACH, J., Toulouse: "Résumé de recherches géométriques."

(11) GROSSMANN, M., Zürich: "Die Zentralprojection in der absoluten Geometrie."

(12) SCHOUTE, P. H., Groningen: "On the characteristic numbers of the polytopes $e_1e_2 \cdots e_{n-1}S_{n+1}$ and $e_1e_2 \cdots e_{n-1}M_n$ of space S."

(13) KASNER, E., New York: "Conformal geometry."

(14) TZITZEICA, G., Bucharest: "Sur les surfaces isothermiques."

(15) SOMMERVILLE, D. M. Y., St. Andrews: "The pedal line of the triangle in non-euclidean geometry."

(16) HOSTINSKÝ, B., Prague: "Sur les Hessiennes successives d'une courbe du troisième degré."

(17) FINSTERBUSCH, J., Zwickau: "Geometrische Maxima und Minima mit Anwendung auf die Optik."

(18) HUDSON, Miss H. P., Croydon: "On binodes and double curves."

(19) STUDY, E., Bonn: "Conformal mapping of complex domains."

(20) HATZIDAKIS, N., Athens: "Sur les paires de trièdres de Frenet."

(21) KÖNIG, D., Budapest: "Zur analysis situs der Doppelmannigfaltigkeiten und der projectiven Räume."

(22) SINZOV, D., Charkow: "Sur la théorie des connexes."

(23) JANISZEWSKI, Z., Warsaw: "Ueber die Begriffe Linie und Fläche."

(24) WEITZENBÖCK, R., Bonn: "Ueber das sechs-Ebenen-Problem im R_4."

1. Two uniform and continuous representations, one of a closed multiplicity μ and the other μ', are said to belong to

the same class if it is possible to go from one to the other by continuous modification. Professor Brouwer proved that in the case in which both entities are spheres, all the representations of the same degree belong to the same class.

2. In the article of Stahl (*Crelle*, volume 104) referred to by Professor Morley, the following theorems appeared: Any conic on the double lines of a rational quartic determines by the other four common lines the fundamental involution. In the present paper the following theorems are added: any pencil of curves of class four on the double lines of a rational quintic contains five which touch the quintic. The five parameters of contact are in the fundamental involution.

Any set of curves of class six on the double lines of a rational sextic contains six which osculate the sextic. The six parameters of osculation are in the fundamental involution. Similarly for class n.

3. Professor Eisenhart's paper is in abstract as follows:

If S is a surface applicable to a quadric Q and S_1 is a Bianchi transform of S, the joins of corresponding points on S and S_1 form a W-congruence for which these surfaces are the focal surfaces. By the general theory of W-congruences one has accordingly an infinitesimal deformation of S and S_1. Professor Eisenhart has made use of these infinitesimal deformations to build a suite of continuous deforms of Q. Such a suite, called a *system* (Q), is determined by a set of five differential equations in two dependent and three independent variables, which can be shown to admit of analytic solutions. When a solution is known, the intrinsic functions of the surfaces are given directly. Systems (Q) occur in pairs, corresponding surfaces of the two systems being focal surfaces of a W-congruence. These systems admit transformations into systems of the same kind which are a generalization of the Bianchi transformations of a single surface. There exist systems (Q) of ruled surfaces applicable to Q. When in particular Q is an hyperboloid of revolution of one sheet, there arises incidentally a continuous deformation of Bertrand curves into curves of the same kind.

4. Dr. Bompiani summarized his report as follows:

The first results of projective differential geometry of

hyperspace appear in papers by Del Pezzo (1886) on spaces tangent to a given variety immersed in a larger one, and by Segre (1888) on systems of ∞^{n-1} lines in S_n.

In his more recent memoirs, published between 1896 and 1910, Professor Segre has treated this branch of geometry in a different way, which has opened up a new field. If we consider the curves of a variety V_m issuing from one of its points with a determinate osculating $S_\nu(\nu \geqq 0)$, the spaces S_n $(n > \nu)$ which osculate it belong to a linear space which is called the n-osculant to the given variety along the fixed S_ν (when it does not coincide with the former). The first problem (of local character) concerns the distribution of the osculating S_n among the n-osculants, the law of variation of these spaces with the S_ν, etc. A second problem (of general character) concerns the possibility of associating with V_m varieties of fewer dimensions to which the osculating spaces are related. This problem corresponds to that of the determination of a double system of conjugate lines on a given surface; it is stated in terms of one or more differential equations. An application to the study of ruled surfaces and certain curves upon them has been made by Wilczynski and others. In the theory of multiply infinite systems of lines, the analogous problem to that in S_3 of finding the developables of a given congruence is that of finding the ruled surfaces in the system and of finding the minimum indices of the various developables.

6. Dr. Brückner made a further study of the problem which he discussed at the Rome congress. He considered the depiction of the 42 complete figures composed of six planes and explained the method of construction. A number of the figures were illustrated by means of models.

9. In Professor Esson's paper the changes in the Plücker numbers were derived which take place when a curve of a pencil has a double point or cusp, and a criterion was deduced for discriminating between proper tangents and lines passing through double points or cusps. The dual cases were then considered and the corresponding formulas derived.

12. After having indicated the meaning of the expansion symbols $e_1, e_2, \cdots, e_{n-1}$ representing operations to be applied to the edges, faces, limiting bodies . . . of one of the three

regular polytopes (simplex, measure polytope, cross polytope) in order to get the semiregular polytopes corresponding to the semiregular polyhedra of Archimedes, and having recalled that the application of all these operations one after another to the measure polytope and the cross polytope leads to the same result, Professor Schoute explains how the characteristic numbers, i. e., the numbers of vertices, edges, faces, etc., of the two polytopes of space S_{n+1} can be deduced from those of the two polytopes of space S_n by means of a rule nearly as simple as the generally known rule of the triangle of Pascal. In the case of the simplex we have

S_2	6	6	1			
S_3	24	36	14	1		
S_4	120	240	150	30	1	
S_5	720	1800	1560	540	62	1
. . . . .						

in the case of the measure polytope we get

S_2	8	8	1			
S_3	48	72	26	1		
S_4	384	768	464	80	1	
S_5	3840	9600	8160	2640	242	1
. . . . .						

In both cases the unit always represents the thing itself. Now each number in these tables is deduced from the one that is immediately above it and the one on the same horizontal line as the latter one place to the left. This may be shown for both cases by indicating how we pass from the plane S_2 to ordinary three dimensional space S_3. For the simplex we find

number of faces $= \mathbf{2}(1 + 6)$
number of edges $= \mathbf{3}(6 + 6)$
number of vertices $= \mathbf{4}.6$

and for the measure polytope

number of faces $= \mathbf{2}.1 + \mathbf{3}.8$
number of edges $= \mathbf{4}.8 + \mathbf{5}.8$
number of vertices $= \mathbf{6}.8$

In these two cases there is only a difference as to the multipliers represented in heavy type.

After having given the geometrical proof of these rules as far as the step leading up from S_2 to S_3 is concerned, the author

indicates how the two forms obtained in S_n may be represented by the coördinates of their vertices. Thus the symbol (3, 2, 1, 0) represents the vertices of the polyhedron with the characteristic numbers (24, 36, 14) deduced from the simplex in a certain kind of barycentric coordinates, whilst $[1 + 2\sqrt{2}, 1 + \sqrt{2}, 1]$ represents in the same way in ordinary cartesian coordinates the vertices of the polyhedron with the characteristic numbers (48, 72, 26). In the case of (3, 2, 1, 0) the round brackets mean that we have to take all the permutations of the values; in the case of $[1 + 2\sqrt{2}, 1 + \sqrt{2}, 1]$ the square brackets indicate that we have to take all the permutations of the values with all the possible combinations of the signs plus and minus. By the introduction of these symbols the analytical proof with respect to the step from n to $n + 1$ is nearly as simple as that from 2 to 3.

In the end it was indicated that the proved recurrent relations lead up to the following results by means of the method of induction. If $a_{p,\,n}$ represents the number of limits $(l)_p$ of p dimensions of a polytope in S_n, we have

$$\text{for } e_1e_2 \cdots e_{n-1}\, S(n+1)$$

$$\begin{aligned} a_{n-1,\,n} &= 2(2^n - 1) \\ a_{n-2,\,n} &= 3(3^n - 2.2^n + 1) \\ a_{n-3,\,n} &= 4(4^n - 3.3^n + 3.2^n - 1) \\ &\cdots\cdots\cdots\cdots \end{aligned}$$

$$\text{for } e_1e_2 \cdots e_{n-1}\, M_n$$

$$\begin{aligned} a_{n-1,\,n} &= 3^n - 1 \\ a_{n-2,\,n} &= 5^n - 2.3^n + 1 \\ a_{n-3,\,n} &= 7^n - 3.5^n + 3.3^n - 1 \\ &\cdots\cdots\cdots\cdots \end{aligned}$$

the laws of succession of which are sufficiently transparent.

It is almost unnecessary to add that the operations of expansion were introduced by Mrs. A. Boole Stott ("General deduction of semiregular polytopes and space fillings of regular ones," *Verhandelingen* of Amsterdam, volume 11, number 1), but it is necessary to say that the geometrical proof given by the author was also suggested by her.

13. The geometry of the plane based upon the infinite group consisting of all conformal transformations has been developed only in those directions which are suggested by the

theory of functions of a complex variable. Certain simple and fundamental problems have therefore been overlooked. The simplest configuration of real interest is the curvilinear angle (two analytic arcs having a point in common). The conformal transformation is required to be regular all around the vertex. When are two angles equivalent? Equality of magnitude is of course necessary, but not always sufficient. Professor Kasner studies the differential invariants of higher order which thus arise. It makes an essential difference whether the magnitude of the angle is commensurable or incommensurable with respect to 180°. The case of the horn angle (two curves touching each other) is of special interest.

14. After having recalled the methods of representation which he had previously developed, Professor Tzitzeica applied them to a system of partial differential equations of the third and of the fourth order. In the latter case he obtained the equation of the fourth order derived by another method by Rothe and by Calapso.

15. The loci discussed by Professor Sommerville are defined as follows: In non-euclidean geometry, if the feet of the perpendiculars X, Y, Z from P upon the sides of a triangle ABC are collinear, the locus of P (pedal locus) is a bi-partite cubic passing through A, B, C; the envelope of XYZ (pedal envelope) is a curve of the third class touching the sides of the triangle. In euclidean geometry the pedal locus degenerates into the circum-circle and the line at infinity. The only other case in which the locus degenerates into a straight line and a circle is when the triangle is equilateral with imaginary angles $\cos^{-1} 2$.

16. Let m be the parameter in the equation of a plane cubic curve reduced to the Hessian canonical form, and r the absolute invariant. Professor Hostinsky considered the successive Hessians $H_1, H_2, \cdots$ of the given curve, regarded as functions of m, and in particular the condition for periodicity, $H_n = H_0$, in which H_0 is a given cubic. If $n = 1$, the syzygetic pencil obtained by varying m contains four curves which degenerate into three straight lines, each of which is identical with its Hessian.

For $n = 2$, there are three cycles composed of harmonic curves ($r = 0$). When $n = 3$, the condition is $r^3 + 3r + 3 = 0$,

and there are eight cycles. For every value of n the equation in r is always abelian. If H_k is bipartite, H_{k+1} and H_{k-1} are unipartite.

18. In the paper of Miss Hudson two theorems were discussed:

If a point O is an isolated binode which reduces the class of a surface F by i, then $F \equiv H \cdot K + f_i$, in which f_i is small of order i near O, and H, K are surfaces each passing once only through O, and having contact of order $i - 2$ with one sheet of F.

If F is of degree n and has t triple points lying upon a general nodal curve C of degree m and rank r with d nodes, then, if there are no other singularities, the number of pinch points on F is $2\{m(n - 2) - r - 2d - t)$, and the reduction in the class of the surface is $m(7n - 12) - 4r - 8d + 3t$.

20. The paper of Professor Hatzidakis dealt with the relations existing between the curvature of a pair of generalized trihedra of Frenet. The expressions for curvature and torsion of the one (D_1) are derived in terms of those of the other (D_2), and the direction cosines of (D_1, D_2) and also the condition that D_1 is a trihedron of Frenet with regard to D_2 are found. Finally, the relations between the normal and the geodesic curvatures, and also the geodesic torsion are derived.

21. As generalization of the concept of double surface, double varieties for hyperspace may be defined. In this way spheres appear as double varieties of projective spaces having the same number of dimensions. From these hypotheses, Dr. König derived the following theorem: The n-dimensional projective space is unilateral for even values of n, and bilateral for odd values.

23. Spatial intuition, by means of which we think we know something of the concepts curve and surface, often leads us astray. An example is furnished by those curves and surfaces having properties not compatible with intuition. Dr. Janiszewski discussed the two cases: first, a curve having a strip in common with every plane of a parallel pencil, and holomorphic with a plane curve whose multiple points compose a continuum; second, a surface which cannot contain any repre-

sentation of the area of a circle—namely, a cylindrical surface whose directrix is a plane curve without a simple arc.

24. When six planes are given in R_4, then in the general case there are five lines which these planes cut. Dr. Weitzenböck explained the geometric significance of this number and derived an equation of order five whose roots determine the five lines.

SECTION III*a*. MECHANICS, PHYSICAL MATHEMATICS, ASTRONOMY.

The programme consisted of the following papers:

TURNER, H. H., Oxford: "On double lines in periodograms."

MOULTON, F. R., Chicago: "Relations of families of periodic orbits in the restricted problem of three bodies."

FÖPPL, L., Göttingen: "Stabile Anordnungen von Elektronen im Atom."

SMOLUCHOWSKI, M. S., Lemberg: "On the practical applicability of Stokes's law of resistance and the modifications of it required in certain cases."

LOVE, A. E. H., Oxford: "The application of the method of W. Ritz to the theory of the tides."

LEUSCHNER, A. V., Berkeley: "The Laplacian orbit methods."

BENNETT, G. T., Cambridge: "The balancing of the four-crank engine."

KÁRMÁN, TH. VON, Göttingen: "Luftwiderstand und Hydrodynamik."

BROMWICH, T. J. I'A., Cambridge: "Some theorems relating to the resistance of compound conductors."

EWALD, P. P., Göttingen: "Dispersion and double refraction of electrons in rectangular grouping (crystals)."

MILLER, D. C., Cleveland: "The graphical recording of sound waves; effect of free periods of the recording apparatus."

TERRADAS, E., Barcelona: "On the motion of a chain."

ABRAHAM, M., Milan: "The gravitational field."

MCLAREN, S. B., Birmingham: "Aether, matter, and gravity."

SILBERSTEIN, L., Rome: "Self-contained electromagnetic vibrations of a sphere as a possible model of the atomic store of latent energy."

SOMIGLIANA, C., Torino: "Sopra un criterio di classificazione dei massimi e dei minimi delle funzioni di piu variabili."

ESSON, W., Oxford: "On a law of connection between two phenomena which influence one another."

BLASCHKE, W., Eldena: "Reziproke Kräftepläne zu den Spannungen in einer biegsamen Haut."

BLUMENTHAL, O., Aachen: "Ueber asymptotische Integration von Differentialgleichungen mit Anwendung auf die Berechnung von Spannungen in Kugelschalen."

BOULAD, F., Cairo: "Extension de la notion des valeurs critiques aux équations à 4 variables d'ordre nomographique supérieure."

BRODETSKY, S., Cambridge: "The solution of dynamical problems."

DENIZOT, A., Lemberg: "Theoretisches über den freien Fall eines Körpers bei rotierenden Erde."

DOUGALL, J., Kippen: "The method of transitory and permanent nodes in the theory of elasticity."

HAGEN, J. G., Rome: "How the Atwood machine proves the rotation of the earth, even quantitatively."

LAMB, H., Manchester: "On wave-trains due to a single impulse."

SAMPSON, R. A., Edinburgh: "Some points in the theory of errors."

SECTION III*b*. ECONOMICS, ACTUARIAL SCIENCE, STATISTICS.

The following papers were read:

LEHFELDT, R. A.: "Equilibrium and disturbance in the distribution of wealth."

AMOROSO, L.: "I caratteri matematici della scienza economica."

SHEPPARD, W. F., Surrey: "Reduction of errors by means of negligible differences."

BRODIE, R. R., Edinburgh: "Curves of certain functions relating to mortality and compound interest."

PEEK, J. H.: "Application of the calculus of probabilities in calculating the amount of securities, etc., in the Dutch State Insurance Office."

QUIQUET, A., Paris: "Sur une méthode d'interpolation exposé par Henri Poincaré et sur une application possible aux fonctions de survie d'ordre n."

STEFFENSEN, J. F. A. F., Hellerup: "On the fitting of Makeham's curve to mortality statistics."

EDGEWORTH, F. Y., Oxford: "A method of representing frequency groups by analytic geometry."

HERON, D.: "Fallacious methods of measuring association."

SHEPPARD, W. F., Surrey: "The calculation of moments of an abrupt frequency distribution."

ARANY, D., Budapest: "Ein Beitrag zur Laplace'schen Theorie der erzeugenden Funktion."

GÉRARDIN, A., Nancy: "Statistique des vingt séries parues du Répertoire Bibliographique des Sciences Mathématiques."

SECTION IV*a*. PHILOSOPHY, HISTORY.

The list of titles of papers is as follows:

BURALI FORTI, E.: "Sur les lois générales pour l'algorithme des symboles de fonction et d'opération."

BLUMBERG, H.: "Ueber ein Axiomen-System für die Arithmetik."

ENESTRÖM, G.: "Resolution relating to the publication of G. Valentin's general Bibliography of mathematics."

GÉRARDIN, A.: "Note historique sur la théorie des nombres."

HARDING, P. J.: "The geometry of Thales."

HUNTINGTON, E. V.: "A set of postulates for abstract geometry expressed in terms of the simple relation of inclusion."

ITELSON, G.: "Bemerkungen über das Wesen der Mathematik."

ITELSON, G.: "Thomas Solly von Cambridge als Logistiker."

JOURDAIN, P. E. B.: "Isoid relations and the modern theory of irrational numbers."

JOURDAIN, P. E. B.: "Fourier's influence on pure mathematics."

JOURDAIN, P. E. B.: "The ideas of the 'fonctions analytiques' in Lagrange's early work."

LORIA, G.: "Intorno ai metodi usati dagli antichi greci per estrarre le radici quadrate."

MUIRHEAD, R. F.: "Superposition as a basis for geometry; its logic and its relation to the doctrine of continuous quantity."

PADOA, A.: "Comparaison entre la logique de l'extension et la logique de la compréhension."

PADOA, A.: "Une démonstration du principe d'induction complète."

RUDIO, F.: "Mitteilungen über die Eulerausgabe."

VACCA, G.: "Sul valore della ideografia nella espressione del pensiero; differenze caratteristiche tra ideografia e linguaggio ordinario."

VACCA, G.: "On some points in the history of the infinitesimal calculus; relations between English and Italian mathematicians."

ZERMELO, E.: "Ueber die Grundlagen der Mengenlehre."

ZERMELO, E.: "Ueber eine Anwendung der Mengenlehre auf die Theorie des Schachspiels."

ZERMELO, E.: "Ueber axiomatische und genetische Methoden bei der Grundlegung mathematischer Disciplinen."

SECTION IV*b*. DIDACTICS.

Section IV*b* held six meetings (two of them jointly with Section IV*a*), under the chairmanship of the following gentlemen in order: Hon. B. A. W. Russell, C. Godfrey, David Eugene Smith, A. Gutzmer, E. Czuber, C. Bourlet, J. W. A. Young, Sir J. J. Thomson, R. Fujisawa, C. Godfrey.

Three of these sessions were held jointly with the International Commission for the Teaching of Mathematics. The first of these was opened by an address of welcome by the chairman, C. Godfrey. Thereupon, the following address on the work of the Commission was delivered by David Eugene Smith, who had been in recent conference with the president of the Commission, Professor Klein, and the central committee:

"As has already been mentioned, Professor Klein, to whose great energy and wisdom the success of the International Commission on the Teaching of Mathematics is largely due, is unable to be present, on account of illness. It was my privilege to propose to the delegates at our meeting on Wednesday the sending of a telegram to Professor Klein, and I now propose the same message to Section IV, as follows: 'The International Commission on the Teaching of Mathematics, and Section IV, at their first Cambridge meeting, express regret at your absence and best wishes for your recovery.' *

"The Commission was organized for the purpose of reporting upon the present status of the teaching of mathematics in the various countries of the world. Special sub-committees have also been appointed from time to time, to consider questions of international rather than merely national interest.

* By unanimous vote the telegram was duly sent to Professor Klein.

About one hundred and fifty reports on the work done in the various countries have been prepared, and at least fifty more are in contemplation. A world-wide interest in the improvement of mathematical teaching has been awakened, and the influence of the movement is certain to be very far reaching. Ten countries have completed the task set for themselves. In chronological order of completion these countries are Sweden, Holland, France, Switzerland, Austria, Japan, the United States of America, the British Isles, Hungary, and Denmark. In process of publication are the monumental work of Germany, with twenty-seven out of thirty-six reports already printed, and the reports of Italy, Roumania, Spain, and Russia. In contemplation are the reports of Greece, Norway, Australia, Portugal, Servia, and doubtless of several other countries.

"As to the future work of the Commission, the Central Committee earnestly desires that it be authorized to see to the completion of the reports. It is therefore very desirable that it be continued in power, both for this purpose and for the consideration of certain questions of great international significance. Such topics as the proper training of engineers, of calculus in the secondary schools, of the general value of intuition in the teaching of mathematics, of the training of teachers, and of the educational (cultural, disciplinary, non-technical) value of mathematics, may properly occupy the attention of the Commission in the next four years. Special conferences having already been held, at Bruxelles and Milano; it is proposed, if the committee is continued in power, to hold others between now and the time of the meeting of the Congress of 1916, if that shall be the date. Possibly such conferences may be held in France in 1914, in Germany in 1915, and in Stockholm in 1916.

"It is also hoped that each country will prepare a summary of the large features of the reports of other countries, to the end that the work that has been accomplished may have its full effect. It is further hoped that the various countries will continue the financial support that has been given to the central committee in the past.

"A word should be said at this time in memory of those distinguished teachers who have been connected with the movement, but who have been called from their labors to solve the great problem. Soon after the last Congress adjourned,

Professor Vailati, of Rome, a distinguished writer and an accomplished scholar, passed away. Scarcely in his full prime of life, his loss is felt not by Italy alone, but by all who appreciate scholarship and high educational standards. Professor Bovey, president of the Imperial Technical College at South Kensington, who was charged with the labor of reporting for Canada, has also been called from us; in his death the world lost a scholar and an administrator of prominence. And as he was planning to attend this Congress, four weeks ago to-day, Geheimrath Professor P. Treutlein of Carlsruhe, passed suddenly away. In his death Germany lost one of her foremost educators, and the International Commission one of its best supporters.

"We shall now proceed to the election of the officers for the next session, and then to the reception of the reports. The central committee has consulted with the committee on organization and it has been decided that the first set of reports shall be presented to the library of the University of Cambridge, a second set to our official hosts, the Cambridge Philosophical Society, and a third set to that great world-library, the library of the British Museum."

The general secretary of the Commission next made a statement as to the work of the central committee, and submitted its collected publications.

Thereupon the reports of the various countries were formally submitted to the congress. The countries were called in alphabetical order in the French language, and the following members of the Commission presented the reports, with a brief oral description, and a longer written statement, which will be published in the Proceedings of the Congress, and in the official organ of the Commission, *L'Enseignement Mathématique*. The statement accompanying the American report will be published in *School Sciences and Mathematics*, the official American organ.

Germany, Professor A. Gutzmer (Halle); Austria, Professor E. Czuber (Vienna); Belgium, Principal E. Clevers (Ghent); Denmark, Professor H. Fehr; Spain, Professor Toledo (Madrid); United States, Professor J. W. A. Young (Chicago); France, Professor C. Bourlet (Paris); Greece, Professor H. Fehr; Holland, Professor J. Cardinaal (Delft); Hungary, Professor E. Beke (Buda-Pesth); British Isles, Professor C. S. Jackson (Woolwich); Italy, Professor G. Castelnuovo (Rome); Japan, Pro-

fessor R. Fujisawa (Tokio); Norway, Professor M. Alfsen (Christiania); Portugal, Professor F. J. Teixeira (Oporto); Roumania, Professor G. Tzitzeica (Bucharest); Russia, Professor H. Fehr; Sweden, Professor H. Fehr; Switzerland, Professor H. Fehr (Geneva).

Also the following associated countries: Brazil, Professor E. de B. R. Gabaglia (Rio de Janeiro); Servia, Professor M. Petrovitch (Belgrade).

At the second joint session of Section IV*b* and the International Commission, the report of sub-commission *B*, on "The mathematical education of the physicist in the university," was presented by Professor C. Runge, and followed by a lively discussion.

At the last joint session of Section IV*b* and the International Commission, C. Goldziher presented a report on the work done by David Eugene Smith and himself towards preparing a bibliography of works on the teaching of mathematics, published since 1900. (This bibliography is about to be published by the United States Bureau of Education and can be obtained from the Bureau on request.) Upon motion of Professor Smith the following resolution was passed:

Resolved: that Section IV*b* of the International Congress of Mathematicians, assembled at Cambridge, expresses its thanks to the Honorable the United States Commissioner of Education for his great interest in publishing, for free distribution, the recent bibliography on the teaching of mathematics (1900–1912), and the hope that it may, through his good offices, be brought to completion to the year 1915, with such additions to the present list as may seem desirable.

David Eugene Smith then presented the report of Subcommission *A* on: "Intuition and experiment in mathematical teaching in secondary schools." The presentation of the report was followed by an extended discussion. This report will be published in the various official organs named above.

In the other sessions of Section IV*b* the following papers were presented:

WHITEHEAD, A. N.: "The principles of mathematics in relation to elementary teaching."

SUPPANTSCHITCH, R.: "Le raisonnement logique dans l'enseignement mathématique universitaire et secondaire."

HILL, M. J. M.: "The teaching of the theory of proportion."

HATZIDAKIS, N.: "Systematische Rekreationsmathematik in den mittleren Schulen."

GÉRARDIN, A.: "Sur quelques nouvelles machines algébriques."

CARSON, G. ST. L.: "The place of deduction in elementary mechanics."

NUNN, T. P. "The proper scope and method of instruction in the calculus in schools."

It was not possible to secure brief abstracts of the above papers for incorporation in this report. The papers will be published in full in the Proceedings of the Congress, and elsewhere.

Sir G. Greenhill made the following statement in regard to the work of the International Commission on the teaching of mathematics.

"The statement I have to make, Sir, to the Congress, is given in the formal words following:

1. The International Commission on the Teaching of Mathematics was appointed at the Rome Congress, on the recommendation of the members of Section IV.

2. The several countries, in one way or another, have recognized officially the work, and have contributed financial support.

3. About 150 reports have been published, and about 50 more will appear later.

4. The Commission will report in certain sessions of Section IV.

5. The Commission hopes to be continued in power, in order that the work now in progress may be brought to completion.

A resolution to this effect will be offered at the final meeting of the Congress."

The third general session closed the Congress. At this session, upon motion of C. Godfrey, seconded by W. von Dyck, the following resolution was unanimously passed. Its adoption had previously, upon the motion of Sir George Greenhill, been unanimously recommended to the Congress by the International Commission on the Teaching of Mathematics, in its separate session, and by Section IV*b* in joint session with the Commission.

Resolved: that the Congress expresses its appreciation of the support given to its Commission on the Teaching of Mathematics by various governments, institutions, and individuals; that the Central Committee composed of F. Klein

(Göttingen), Sir G. Greenhill (London) and H. Fehr (Geneva) be continued in power and that, at its request, David Eugene Smith, of New York, be added to its number; that the delegates be requested to continue their good offices in securing the cooperation of their respective governments, and in carrying on the work; and that the Commission be requested to make such further report at the Sixth International Congress, and to hold such conferences in the meantime as the circumstances warrant.

VIRGIL SNYDER.

THE MÜNSTER MEETING OF THE DEUTSCHE MATHEMATIKER-VEREINIGUNG.

THE annual meeting of the Deutsche Mathematiker-Vereinigung was held in affiliation with the eighty-second convention of the association of German naturalists and physicians at Münster in Westphalia, September 15 to 19, under the presidency of Professor W. v. Dyck.

As usual, ample provision was made for the entertainment of the guests, a number of excursions being arranged for the afternoons, and a reception or concert each evening. Perhaps the most interesting excursion was that to the neighboring city of Essen, to inspect the works of the Krupp manufacturing company. The guests were shown the manifold processes of casting and hardening steel, the preparation of armor, the boring of cannon, etc., and were also furnished an opportunity of seeing the domestic and social problems and usages connected with this gigantic enterprise.

The session of Tuesday afternoon was devoted to the administrative affairs of the society. Reports of the status of the Encyclopedia and of the International commission on mathematical instruction were read, as well as a statement concerning the publication of various other works in which the society is interested, primarily those of Schroeder and of Euler. At this session Professor W. KILLING read a paper "On the preparation of the gymnasium teacher," which was followed by a general discussion.

The session of Wednesday morning was held jointly with the section of physics, upon the invitation of the latter, to listen to the following more general reports in mathematical physics: