when f, A, B and C are bounded and have their discontinuities regularly distributed in the rectangle $0 \leqq x \leqq a$, $0 \leqq y \leqq b$. This proof avoids the usual decomposition of the rectangle (a, b) into smaller rectangles. An application to partial differential equations of the hyperbolic type is given.

31. If the power series $f(x) = x + a_2x^2 + \cdots + a_nx^n + \cdots$ is such that $z = f(x)$ defines the conformal representation of the unit circle in the x-plane on a simple region in the z-plane, then, as Koebe has shown, there exist upper and lower boundaries for $|f(x)|$ and $|f'(x)|$ when $|x| = r < 1$, these boundaries being independent of the coefficients of $f(x)$.

Professor Gronwall obtains the following approximations to these boundaries:

$$\frac{2}{9}r < |f(x)| < \frac{3r}{(1-r)^2}, \quad \frac{1}{5}\cdot\frac{1-r}{(1+r)^3} < |f'(x)| < 5\cdot\frac{1+r}{(1-r)^3}.$$

These approximations are of the correct order of magnitude, as is shown by the example $f(x) = x/(1-x)^2$.

F. N. Cole,
Secretary.

THE WINTER MEETING OF THE SOCIETY AT CHICAGO.

The thirty-fourth regular meeting of the Chicago Section of the American Mathematical Society was held at the University of Chicago on Monday and Tuesday, December 28–29, 1914, it being the third regular meeting of the Society at Chicago. Eighty persons were in attendance upon the sessions, including the following sixty-four members of the Society:

Professor R. P. Baker, Professor W. H. Bates, Mr. William Betz, Professor G. A. Bliss, Professor Henry Blumberg, Professor Daniel Buchanan, Professor H. E. Buchanan, Dr. Josephine Burns, Professor W. H. Bussey, Professor W. D. Cairns, Professor R. D. Carmichael, Professor A. F. Carpenter, Dr. E. H. Clarke, Professor H. E. Cobb, Professor D. R. Curtiss, Dr. W. W. Denton, Professor L. E. Dickson, Mr. C. R. Dines, Professor L. W. Dowling, Professor Arnold Dresden, Professor Arnold Emch, Professor A. B. Frizell, Dr. M. G. Gaba, Professor E. R. Hedrick, Professor T. H. Hildebrandt, Mr. L. A. Hopkins, Professor A. M. Kenyon, Dr. H. R. Kingston, Professor Malcolm McNeill, Professor W. D. MacMillan, Professor H. W. March, Professor J. L. Markley, Dr. T. E. Mason, Professor G. A. Miller, Professor U. G. Mitchell, Professor E. H. Moore, Professor E. J. Moulton, Professor L. C. Plant, Mr. V. C. Poor, Professor S. W. Reaves, Professor W. J. Risley, Miss Ida M. Schottenfels, Mr. A. R. Schweitzer, Professor J. B. Shaw, Dr. H. M. Sheffer, Mr. T. M. Simpson, Professor C. H. Sisam, Professor E. B. Skinner, Professor H. E. Slaught, Professor C. S. Slichter, Professor R. B. Stone, Professor E. B. Stouffer, Professor A. L. Underhill, Professor E. B. Van Vleck, Dr. G. E. Wahlin, Professor Mary E. Wells, Professor Marion B. White, Professor E. J. Wilczynski, Professor K. P. Williams, Professor R. E. Wilson, Mr. C. H. Yeaton, Professor J. W. A. Young, Professor W. A. Zehring, Professor Alexander Ziwet.

Professor E. B. Van Vleck, President of the Society, presided at the session on Monday afternoon. Professor E. J. Wilczynski, chairman of the Section and Vice-President of the Society, presided at all the other sessions, except when relieved by Professor C. S. Slichter during part of the session on Monday morning.

At the business meeting of the Chicago Section the report of the committee of the Section appointed last April to consider the relation of the Society to the field now covered by the *American Mathematical Monthly* was received and ordered spread upon the minutes. In accordance with the recommendation of this committee, it was then voted that the Chicago Section request the Council of the Society to appoint a committee to consider and report concerning possible relations of the Society to the field now covered by the *American Mathematical Monthly*. In transmitting this request to the Council the Secretary of the Section was instructed to send with it a copy of the report of the committee of the Section.

The following officers of the Chicago Section were elected: Chairman, Professor E. J. Wilczynski; Secretary, Professor H. E. Slaught; and third member of the program committee, Professor R. D. Carmichael. Professor Wilczynski was requested by the Section to deliver an address upon the completion of his second term of office as chairman.

The usual social gathering and dinner on Monday evening was attended by sixty members, this being an unusually large number.

The following papers were presented at this meeting:

(1) Professor ARNOLD EMCH: "On some general theorems concerning analytic curves."

(2) Professor A. F. CARPENTER: "Ruled surfaces with plane flecnode curves."

(3) Professor E. J. WILCZYNSKI: "The general theory of congruences."

(4) Professor A. B. FRIZELL: "A non-enumerable well-ordered set of infinite permutations."

(5) Professor HENRY BLUMBERG: "On a generalization of certain theorems relating to discontinuous functions."

(6) Professor DANIEL BUCHANAN: "A new isosceles triangle solution of the three-body problem."

(7) Professor DANIEL BUCHANAN: "Oscillations near an isosceles triangle solution as the finite bodies become unequal."

(8) Mr. L. L. STEIMLEY: "On the general solution and so-called special solutions of linear homogeneous partial differential equations."

(9) Mr. W. L. HART: "A method of successive approximations for the determination of implicit functions."

(10) Professor ARNOLD DRESDEN: "On the second derivative of the extremal integral for a general class of problems."

(11) Professor G. A. BLISS: "A note on functions of lines."

(12) Professors G. A. BLISS and A. L. UNDERHILL: "Shortest lines between curves and surfaces."

(13) Dr. G. R. CLEMENTS: "Singular point transformations in two complex variables. Supplementary note."

(14) Professor HENRY BLUMBERG: "On the factorization of certain polynomials and certain linear homogeneous differential expressions. Second communication."

(15) Dr. G. R. CLEMENTS: "The resultant of a system of polynomials."

(16) Mr. A. R. SCHWEITZER: "On the formal properties of functional equations."

(17) Professor L. E. DICKSON: "On modular cubic surfaces."

(18) Professor G. A. MILLER: "A new proof of Sylow's theorem."

(19) Professor G. A. MILLER: "Note on the potential and the antipotential group of a given group."

(20) Professor J. B. SHAW: "On parastrophic algebras."

(21) Mr. C. H. FORSYTH: "A general formula for the valuation of bonds."

(22) Dr. H. R. KINGSTON: "Metric differential properties of nets of plane curves."

Mr. Steimley was introduced by Professor R. D. Carmichael, Mr. Hart by Professor F. R. Moulton, and Mr. Forsyth by Professor J. W. Glover. The papers of Dr. Clements and Mr. Schweitzer were read by title.

Abstracts of the papers follow, the numbers corresponding to those of the titles above.

1. In an article which will appear in the *Rendiconti del Circolo Matematico di Palermo* it is proved analytically that every closed analytic curve without cusps and multiple points has an even number of points of inflexion and admits of an even number of tangents parallel to any given direction. The proofs of these theorems are based upon the co-periodicity of the parametric representation of such curves and of their derivatives.

These results may be extended to any closed analytic curve by properly extending the definition of inflexions and tangents and by taking proper care of multiplicities. Among such closed curves may be included those that result from the former by projective transformations.

A purely intuitive proof of the theorem on inflexions was given by von Staudt in his Geometrie der Lage. In the present paper Professor Emch extends his former proofs to the general cases and discusses von Staudt's method.

2. In the general theory of ruled surfaces as developed by Wilczynski, many important theorems are obtained by taking for directrix curves the two branches of the flecnode curve. In Professor Carpenter's paper the condition that these curves be plane leads to the theorem: If the two branches of the flecnode curve of a ruled surface are distinct plane curves, then the two branches of the complex curve are plane and the four planes of these curves form a harmonic pencil of planes in which the planes of the two branches of the flecnode curve are separated by the planes of the two branches of the complex curve.

3. Professor Wilczynski's paper is concerned with the projective differential properties of congruences of rays.

By introducing a number of new congruences covariantly related to the given one, many new theorems present themselves and most of the familiar ones appear in a new light. Incidentally a new geometric interpretation is found for the condition that the Laplace-Darboux invariants of a conjugate system be equal. There is also found the geometrical significance of the systems which Bianchi calls isothermally conjugate.

4. In a previous paper (Chicago, April, 1914) Professor Frizell has called attention to a non-denumerable well-ordered set of finite permutations. The present note applies this result to obtain a non-enumerable well-ordered set of infinite permutations of the series of natural numbers. This is done by arranging the natural numbers in series higher than ω.

5. In his first paper, Professor Blumberg shows that almost all of the known general theorems on real, discontinuous functions involving the oscillation function (which is associated with every given function, whether discontinuous or continuous) hold for the case where the range of the independent variable is an abstract aggregate $\mathfrak{S}$ subject to two simple conditions. The first demands for every element s of $\mathfrak{S}$ the existence of a sub-aggregate $\mathfrak{M}$ of $\mathfrak{S}$ having the relation R to s, R being an undefined relation. The second says that if $\mathfrak{N}_1$ and $\mathfrak{N}_2$ are two "neighborhoods" (the term "neighborhood" having been defined in terms of the undefined relation R) of an element s of $\mathfrak{S}$, there exists a common sub-aggregate $\mathfrak{N}$ of $\mathfrak{N}_1$ and of $\mathfrak{N}_2$ that is a "neighborhood" of s. In particular, for example, the Sierpinski theorem holds. Certain general theorems on discontinuous functions communicated by the author in former papers are similarly generalized.

6. The isosceles-triangle solutions of the three body problem are the periodic solutions in which two of the bodies have equal mass, and the third body moves so that its distances from the other two bodies are always equal. The third body, then, moves in a straight line which passes through the center of gravity of the system and is perpendicular to the plane of the initial motion of the equal masses. In a paper presented to the Society at Chicago, April, 1911, Professor Buchanan discussed the periodic solutions in which the two

equal masses move in ellipses and the third body is infinitesimal. Two periodic solutions were shown to exist. They are expansible as power series in a certain parameter λ, which is introduced through the factor representing the mass of the finite bodies, and the coefficients are power series in e, the eccentricity of the ellipses. The period T is a function of λ but not of e. In the present paper two periodic solutions, having period T, are shown to exist when the third body becomes finite. They are expansible as power series in a parameter representing the mass of the third body, and the coefficients are power series in λ and e similar to those appearing in the previous paper.

7. Starting with the simplest case of the isosceles-triangle solutions in which the third body is infinitesimal and the finite bodies move in circles, Professor Buchanan in his second paper deals with the oscillations near this solution as the finite masses become unequal. If the finite bodies have masses $m + \mu$ and $m - \mu$, where the unit of mass is chosen so that $m = \frac{1}{2}$, then the solutions are expansible as power series in μ. The coefficients are power series in another parameter which represents the initial projection of the infinitesimal body from the plane of motion of the finite bodies where $\mu = 0$. Two solutions are shown to exist and they are expansible as power series in $\mu^{\frac{1}{3}}$ or μ according as the purely imaginary characteristic exponents arising from the equations of variation are imaginary integers or not.

8. It is known that the customary classification of the integrals of linear partial differential equations is incomplete even in the case where the coefficients are single valued and analytic in a given domain. For this latter case Mr. Steimley gives a complete classification of all possible integrals and exhibits a solution which is general in the sense that it includes all possible solutions (even the so-called special solutions).

9. Let there be given the system of equations

$$f_i(x_j; \mu) = 0 \qquad (i, j = 1, 2, \cdots, n), \tag{1}$$

in the field of reals, with the solution point $(a_j; \mu_0)$ at which the Jacobian, $\partial(f_1, \cdots, f_n)/\partial(x_1, \cdots, x_n)$, is different from zero. The current methods by which the fundamental theorem of implicit function theory is proved for system (1) all possess the fault that no practical scheme is given for the computation of the values of the $x_j(\mu)$ except for μ sufficiently near to μ_0. In Mr. Hart's paper there is given a method of successive approximations which furnishes a practical scheme for computing the values of the $x_j(\mu)$ over a range in general much more extended than that obtained in the usual proofs.

10. In a paper published in volume 9 of the *Transactions* of the Society, Professor Dresden treated problems of the calculus of variations by reducing them to problems in ordinary maxima and minima by means of the "extremal integral." This necessitated securing formulas for the second derivatives of this integral; these were obtained in the paper mentioned above and they are quoted on page 312 of Bolza's Vorlesungen.

It is the purpose of the present paper to secure similar formulas for more general problems of the calculus of variations, viz., such as involve integrals whose integrand contains n unknown functions and their first derivatives, in nonparametric form as well as in parametric form. In the former case these formulas are obtained without great difficulty. In the latter case, however, it has seemed to the author that a reduction of the second variation was necessary. This is done by means of a reduction of differential forms discussed in a paper presented to the Society some time ago and published in the *Annals of Mathematics* for March, 1912.

It is intended to use these formulas for the treatment of the maxima and minima of integrals of the type referred to above when one or both end points are variable, as well as when broken extremals are admitted. Furthermore the cases are to be taken up in which the unknown functions are also to satisfy a system of differential equations.

11. One of the most important applications of the theory of functions of lines is in the calculus of variations, where the integrals considered are all either functions of this sort or else quite natural generalizations of them. Unfortunately the usual definitions of continuity and the derivative of a function of a line do not apply to such integrals. In the paper of Professor Bliss the character of the continuity of an integral of the calculus of variations is discussed and the inadequacy of the definition of the derivative is shown.

Attention is called to the modifications which make these notions applicable at once to such line functions.

12. If an arc E in space joins two surfaces S and S' and is shorter than any other arc in its neighborhood joining S with S', then E must in the first place be a straight line, and in the second place it must cut the two surfaces orthogonally. These conditions are necessary but not sufficient to insure the minimizing property. The further relations which must be satisfied involve the curvatures of the surfaces at their intersections with the line E. In the paper of Professors Underhill and Bliss these relations are deduced, and analogous ones are found for the cases where the end points of E lie upon two curves, or on a surface and a curve. The results are of interest because of the preliminary information which they give with respect to the more general problem of determining the minimizing arcs, with end points variable on curves or surfaces, for an integral of the form

$$\int f\left(x, y, z, \frac{dy}{dx}, \frac{dz}{dx}\right) dx.$$

13. In the *Annals of Mathematics* for September, 1913, Dr. Clements discussed the transformation

$$T: \qquad x = f(u, v), \qquad y = \varphi(u, v)$$

in the following cases:

(*a*) $f(u, v)$ and $\varphi(u, v)$ denote functions of the complex variables u and v, single-valued and analytic throughout a neighborhood R of $u = 0$, $v = 0$;

$$(b) \qquad f(0, 0) = 0, \qquad \varphi(0, 0) = 0;$$

$$(c) \qquad J(u, v) \equiv f_u\varphi_v - f_v\varphi_u \not\equiv 0, \qquad J(0, 0) = 0.$$

In that paper the discussion was not complete for the case in which $f(u, v)$ and $\varphi(u, v)$ have a common factor in the point $(0, 0)$ but neither is a factor of the other in this point. In the present note it is shown that for this case the inverse of T exists and is finitely multiple-valued and continuous throughout a complete neighborhood of the point $x = 0$, $y = 0$, with the exception of that point itself, which explodes into a locus in the uv-region. This establishes the existence of functions of two complex variables, finitely multiple valued and continuous except in isolated points.

Any transformation T can always be replaced either by a transformation of the form $x = u$, $y = v^n$ (n a positive integer), a finite number of the form $x = u$, $y = uv$, and of those one-to-one and analytic both ways; or by the preceding combined with a transformation T for which $f(u, v)$ and $\varphi(u, v)$ have no common factor.

14. In his second paper, Professor Blumberg gives criteria for the resolvability and, in particular, the irreducibility of certain polynomials, linear homogeneous differential expressions, and other expressions. The methods used are the simplest heretofore employed in similar investigations; the results are the most general heretofore obtained and, largely on account of the simplicity of the methods, generalization is made easy. The most general theorem communicated by the author at the November, 1914, meeting of the Southwestern Section is a special case of a special case of one of the theorems in the present paper. To indicate the character of the results one of the theorems obtained and several special cases of it are given below. Let

$$P = a_0(x)\frac{d^py}{dx^p} + a_1(x)\frac{d^{p-1}y}{dx^{p-1}} + \cdots + a_p(x)y$$

be a linear homogeneous differential expression belonging to (i. e., whose coefficients belong to) the domain $\mathfrak{R}$ of rational functions; let d_ν ($\nu = 0, 1, \cdots, p$) represent the degree of a_ν; and let P be decomposable into the symbolic product RS, where R and S are linear homogeneous differential expressions belonging to $\mathfrak{R}$ and of the rth and sth orders respectively. The following theorem holds: If for a fixed κ (> 0)

$$\frac{d_\kappa - d_0}{\kappa} \geqq \frac{d_\nu - d_0}{\nu} \qquad (\nu = 1, 2, \cdots, p)$$

and

$$d_\kappa - d_0 \geqq 0,$$

at least one of the following $n - \kappa + 1$ congruences holds:

$$d_\kappa - d_0 \equiv 0 \left(\bmod \frac{\kappa}{[\kappa, \rho]}\right),$$

where ρ takes the successive values $n - r, n - r + 1, \cdots, \kappa - r$ (or $n - s, n - s + 1, \cdots, k - s$), and $[\kappa, \rho] =$ G.C.D. of κ and ρ. A special case of this theorem is the following: If

$$\frac{d_\kappa - d_0}{\kappa} \geqq \frac{d_\nu - d_0}{\nu} \quad (\nu = 1, 2, \cdots, p),$$

$d_\kappa - d_0 \geqq 0$ and $[d_\kappa - d_0, \kappa] = 1$ (i. e., $d_\kappa - d_0$ and κ are relatively prime), either $r \geqq \kappa$ or $s \geqq \kappa$. From this special case follows the theorem: If

$$\frac{d_p - d_0}{p} \geqq \frac{d_\nu - d_0}{\nu} \quad (\nu = 1, 2, \cdots, p),$$

$d_p - d_0 \geqq 0$ and $[d_p - d_0, p] = 1$, P is irreducible.

The above theorems and nearly all others obtained hold for polynomials whose coefficients are rational functions of one or more variables, for linear difference equations, and also for certain cases where the domain of rationality is more general than $\mathfrak{R}$, the only changes required in the formulation of the theorems being the substitution, for example, of "difference" for "differential" in passing to "difference equations," etc.

One of the theorems generalizes the most general criterion for the irreducibility of polynomials heretofore known, a criterion due to Perron.

15. It is well known that in any *numerical* case there exist $N - 1$ rational integral functions of the coefficients of a system of polynomials

$$f_n(x) \equiv A_{n0}x^{m_n} + \cdots + A_{nm_n} \quad (n = 1, \cdots, N)$$

whose simultaneous vanishing is the necessary and sufficient condition that these N polynomials shall have a common factor of the first or higher degree in x. Further, if $N = 2$, this resultant can be written formally in terms of the coefficients. Dr. Clements points out that when $N > 2$, if the coefficients are real, there exist $N - 1$ rational integral functions, expressible as formulas in the coefficients and hence in their definition independent of the numerical value of these coefficients, whose simultaneous vanishing is the necessary and sufficient condition that $f_1, \cdots, f_N$ shall have a common factor containing x. Further, infinitely many such sets of resultants can be found. A very simple set of resultants is exhibited for the case $N = 3$, $m_3 = 2$.

16. In the first part of Mr. Schweitzer's paper a set of postulates is given for a *field* in which the explicit assumption of existential properties is avoided. These postulates are stated in terms of four indefinables, $f(x, y)$, $\phi(x, y)$, $\phi_1(x, y)$, $f_1(x, y)$, and are obtained essentially by giving abstract interpretations to functional relations discussed in a previous paper.*

In the second part of the paper the following theorems are proved:

I. If $f\{\beta f(x, z), f(x, y)\} = \alpha\beta f(y, z)$, then there exists $\chi(x)$ such that $\chi f(x, y) = c_0\chi(x) - c_0\chi(y)$, $\chi\alpha(x) = c_0\chi(x)$, and $\chi\beta(x) = \chi(x) + c_1$, where c_0 and c_1 are constants.

II. If
$$f\{f(x, z), f(y, z)\} = \alpha f(x, y)$$
and
$$f_1\{\alpha f_1(x, z), f_1(y, z)\} = \alpha f_1(x, y),$$
then there exists $\chi(x)$ such that $\chi f(x, y) = c_0\chi(x) - c_0\chi(y)$, $\chi\alpha(x) = c_0\chi(x)$ and $\chi f_1(x, y) = c_1\chi(x)/\chi(y)$, that is, $\alpha^{-1}f(x, y)$ and $f_1(x, y)$ are quasi-transitive.

III. If $f\{f_1(x, z), f_1(y, z)\} = f_1(x, y)$ and $f_1\{f(x, z), f(y, z)\} = f(x, y)$, then $f(x, y)$ and $f_1(x, y)$ are identical.

17. The method employed in the paper by Professor Dickson for the investigation of surfaces of any order in modular space is based upon the projective classification of all sets of real points (i. e., points with integral coordinates). For the case of modulus 2, there are exactly $n - 1$ projectively distinct types of sets of n real points when $1 < n < 8$. Since there are just 15 real points, the number of types of sets of n points is the same as the number of types of sets of $15 - n$ points. Hence a case $n \geqq 8$ reduces to a case $n < 8$. To investigate the projectively distinct types of surfaces of a given order with integral coefficients taken modulo 2, we treat as a separate case those surfaces the only real points on which are those of a given canonical set of n real points ($n = 0, 1, \cdots, 15$), so that we have 15 linear relations between the coefficients of the

* Cf. BULLETIN, October, 1914, p. 25, (6); p. 26, (11); p. 28, the relation following (22).

equation of the surface. The transformations available for the further specialization of the coefficients are the automorphs of the set of n points.

For a cubic surface modulo 2, it is shown that the number of real points n on the surface is always odd. For $n = 15$, the surface is equivalent to $xy(x + y) = 0$ or to $xy(x + y) = zw(z + w)$, the latter having no singular point and having 27 straight lines, of which 15 are real. For $n = 1$, there is a single type of surface; it has no singular point and no real line. For $n = 13$, one of the two types has a singular point, while the other is equivalent to

$$xy(x + y) = w(w + z)(y + z),$$

9 of whose 27 lines are real. On one of the five types for $n = 3$, just one of the 27 lines is real. For $n = 11$, there is a type 5 of whose 27 lines are real. Thus there are types of cubic surfaces modulo 2 having exactly 0, 1, 5, 9 or 15 real lines. In the algebraic theory, the only possible numbers for non-singular cubic surfaces are 3, 7, 15, 27. The paper will appear in the *Annals of Mathematics.*

18. Professor Miller bases his new proof of Sylow's theorem on the following theorem: The number of the substitutions of degree p^β and of order p in the symmetric group of degree n is prime to p whenever p^β is the highest power of p which does not exceed n. By means of this elementary theorem Sylow's theorem may be proved as follows: Suppose that the order g of any group G is divisible by p^α but not by $p^{\alpha+1}$. Now represent G as a regular substitution group, and construct all the substitutions on the g letters of G such that these substitutions are of order p and of degree p^β, p^β being the highest power of p which is less than n.

These substitutions are transformed under G into complete sets of conjugates, each set involving more than one. At least one of these sets must involve a number m of substitutions such that m is prime to p, since the total number of these substitutions is prime to p. Hence G contains a subgroup whose order is divisible by p^α. If the order of this subgroup exceeds p^α the given operations may be repeated, and hence we can always reach a subgroup of order p^α, by the given process, provided the order of G is divisible by p^α.

19. Professor Miller's second paper was published in full in the February BULLETIN.

20. The parastrophic algebra is defined with regard to a given algebra by the relation between the multiplication constants

$$\gamma'_{ijk} = \gamma_{ikj} \qquad (i, j, k = 1, \cdots, r),$$

where r is the order of the algebra. The parastrophic algebra is usually not associative when the original algebra is associative. In the case of algebras defined by finite groups the parastrophic algebra takes a very simple form. Professor Shaw's paper discusses the relations between these algebras.

21. In Mr. Forsyth's paper a general formula is derived for the valuation of bonds where the principal of S dollars is to be repaid in $r_1, r_2, \cdots, r_n$ installments, of which the first r_1, the next r_2, etc., are equal, there being n *different* installments. All other similar formulas apply only to cases involving equal installments.

The first installment is to be paid at the end of f_1 years, the next $r_1 - 1$ at intervals of t_1 years, etc., and in general if we designate as "major intervals" those intervals wherein all the installments are equal, there will be r_n installments at intervals of t_n years in the nth major interval, the first installment to be paid at the end of f_n years.

The rate of interest offered is g and the rate to be realized i, both to be paid m times a year.

The premium or discount becomes

$$(1) \qquad K = \left[1 - \frac{\sum_1^n a:n}{s}\right]\left(\frac{g-i}{i}\right),$$

where

$$a:n = A_n\{a_m(f_n + r_n t_n) - a_m f_n\}, \qquad A_n = I_n/a_{mt_n}$$

and I_n represents the value of each of the installments in the nth major interval. The a's with subscripts refer to the ordinary annuity certain.

If all the minor intervals are equal ($= t$), formula (1) reduces to

$$(2) \qquad K = \left[1 - \frac{\sum_0^n (I_n - I_{n+1})a(n)}{s a_{mt}}\right]\left(\frac{g-i}{i}\right),$$

where

$$a(n) = a_m\left(f_1 + t\sum_1^n r_n\right)$$

and

$$I_0 = I_{n+1} = 0.$$

22. In a memoir entitled "One-parameter families of nets of plane curves" Professor Wilczynski has studied the projective differential properties of nets of plane curves, by means of a completely integrable system of three linear partial differential equations of the second order. In Dr. Kingston's paper some of the metric differential properties of these nets are discussed. If the two linearly independent, non-constant solutions of the partial differential equations

$$y_{uu} = ay_u + by_v, \quad y_{uv} = a'y_u + b'y_v, \quad y_{vv} = a''y_u + b''y_v$$

be taken as the cartesian coordinates of the moving point, it is shown that for a given set of coefficients $a, b, \cdots, b''$, which are functions of the independent variables u and v and are subject to certain integrability conditions, these equations determine a net uniquely except for an affine transformation. If, moreover, we consider the fundamental quantities E, F, and G, which are connected with the length of arc by the relation $ds^2 = Edu^2 + 2Fdudv + Gdv^2$, and satisfy certain conditions, and if we add the condition that these shall have given values, then our original equations along with the given E, F, and G, determine a net uniquely except for a motion and a reflection. The application to orthogonal and isothermal nets gives rise to interesting results. In this paper is laid also the foundation for a detailed study of the metric properties of those nets which are composed of two families of conics.

H. E. SLAUGHT,
Secretary of the Chicago Section.