But this selection from examples must suffice to illustrate the criticisms made above with regard to errors and careless presentation.

As to additions, I suggest two or three, at random, in connection with things geometrical.

Why not give a reference for Hamilton's "letter to De Morgan (1852)" with regard to the construction of the regular polygon of 17 sides?* And would not the reproduction of Gauss's original announcement of the discovery of the possibility of construction of such a polygon, with ruler and compass,† be worth while?

Why leave out Prior's

> "Circles to square, and cubes to double,
> Would give a man excessive trouble;"‡

And finally, might not the plan of the work permit the inclusion of the verses of the British Museum MS. which shows that Euclid was studied in England as far back as 924–940 A. D.?

> The clerk Euclyde on this wyse hit fonde
> Thys craft of gemetry yn Egypte londe
> Yn Egypte he tawghte hut ful wyde,
> Yn dyvers londe on every syde.
> Mony erys afterwarde y vnderstonde
> Gher that the craft com ynto thys londe.
> Thys craft com ynto England, as y ghow say,
> Yn tyme of good kyng Adelston's day.§

R. C. ARCHIBALD.

BROWN UNIVERSITY,
PROVIDENCE, R. I.

SHORTER NOTICES.

The Development of Arabic Numerals in Europe. By G. F. HILL. Oxford, Clarendon Press, 1915. 125 pp. Price 7 shillings 6 pence.

IT is a commonplace remark that noteworthy achievements in this world often have their inception in the most trivial incidents, and this semiparadoxical law is well illustrated in the work under review. Mr. Hill is the curator of the depart-

* Graves's Life of Sir Wm. R. Hamilton, vol. 3 (1889), pp. 433–435.

† *Intelligenzblatt der allgem. Literatur-Zeitung*, Nr. 66, 1 Junius, 1796, col. 554.

‡ In Alma, canto 3, lines 366–7, published in 1717. Or in Poetical Works of Matthew Prior in 2 volumes, London, 1779, vol. 1, p. 404.

§ J. O. Halliwell, Rara Mathematica, second ed., London, 1841, p. 56.

ment of coins and medals in the British Museum, and a few years ago the date (1481) of an Italian medal of the Sultan Mahomet II, by Costanzo of Ferrara, being called in question, he set about the study of the forms of the so-called Arabic numerals of the early Renaissance period. This naturally led him to extend his researches back to the period of probable introduction of these numerals in Europe, and forward to the time when, owing chiefly to the influence of printing, the forms became practically fixed. It is needless to say that Mr. Hill's search soon carried him beyond the field of numismatics and sigillography, and into the general domain of epigraphy, of medieval manuscripts, and of early typography, so that his essay becomes of value not merely in the field in which he is one of the great living authorities, but in other fields as well.

Such a study has long been needed to assist students in the history of mathematics; for the problem of the date of the introduction of our present numerals into Europe is by no means solved, and aids of this kind are exceedingly valuable. For example, the dates of 800 on the sarcophagus of Pegavus Petrasanta in Milan, of 1007 on a gravestone at Katharein near Troppau, and of 1084 at Castle Acre Priory, are all disconcerting to a beginner in the study of the question, and to have these dates distinctly rejected by a recognized authority on the subject is very helpful. On the positive side, to have at hand a set of tables giving the earliest authentic example of the numerals in the west, the well-known Codex Vigilanus of 976, and a score of other examples of the Boethian apices; to have the most characteristic specimens of the Arabic forms from the twelfth to the sixteenth century, not only as shown in the manuscripts, but also in the inscriptions and on the paintings of these periods; to be able at a glance to follow the lambda seven in its efforts to assume the upright form; to see the looped four giving place to the familiar form which finally replaced it; to see the beginning of the struggle of the zero for a place and for a recognized shape,—to have all this in scientific tabular form so that the changes can be traced at a glance, is to visit what may be described quite seriously as the cinematograph of our numeral system.

The tables, sixty-four in number and with hundreds of carefully drawn examples, are not without their surprises to those who have not looked into the subject. For example, the earliest known manuscript with the apices gives the upright

form of the seven instead of the lambda form which is commonly supposed to be the earlier one, and this is true, it may be said in passing, not only of the manuscript from which Mr. Hill has taken his illustration, but also the other Escorial manuscript of the same work. Likewise the upright four, which we ordinarily think of as due to the Florentines of the fifteenth century, who indeed had much to do with establishing it, is shown to have been used in the thirteenth and fourteenth centuries by English scribes and early in the fourteenth century by the Italians, probably Florentine monks, and quite commonly in the fifteenth century by writers of English manuscripts.

What strikes the reader as most gratifying is that Mr. Hill has brought to the problem a perfectly judicial mind; he has no thesis to defend; he is advocate for no party to any controversy; he is the scholar seeking absolute truth. To his researches, to his patience, to his care in weighing evidence, all who have an interest in the history of mathematics are quite as much his debtors as those whose fields of interest are in the lines of numismatics and paleography.

DAVID EUGENE SMITH.

Introduction to Infinitesimal Calculus. By G. W. CAUNT. Oxford, The Clarendon Press, 1914. xx+568 pp.

THIS book is an attempt to present the calculus in a way that will appeal to students of engineering. The author expresses a hope that he has made the book rigorous enough to satisfy the instructor in a first course in calculus for a student in pure mathematics. This seems to be rather an exception, most texts being written for the pure mathematician, or at least chiefly from his viewpoint. The subject matter is that usually found in the texts on calculus with the addition of a chapter on differential equations, and the author presents the subject from the viewpoint of the engineer. The book is written for a first course in calculus and is arranged for a minimum amount of analytic geometry to precede it. The author usually introduces a subject by means of a number of illustrative numerical examples worked out in detail, thus leading the student into a subject by means of his interest in the purpose it serves. This use of numerical examples, completely solved out, prepares the student of engineering to make use of his mathematics in his engineering courses. Too often a