THE NINTH REGULAR MEETING OF THE SOUTHWESTERN SECTION.

THE ninth regular meeting of the Southwestern Section of the Society was held at Washington University, St. Louis, Missouri, on Saturday, November 27, 1915. Thirty-eight persons attended the sessions, including the following twenty-one members of the Society:

Professor L. D. Ames, Mr. Charles Ammerman, Professor E. W. Davis, Dr. W. W. Denton, Professor E. P. R. Duval, Dr. E. A. Engler, Professor A. B. Frizell, Professor E. R. Hedrick, Professor G. O. James, Professor O. D. Kellogg, Mr. J. C. Rayworth, Professor S. W. Reaves, Professor W. H. Roever, Dr. H. M. Sheffer, Professor C. H. Sisam, Professor H. E. Slaught, Professor E. J. Townsend, Professor J. N. Van der Vries, Professor C. A. Waldo, Dr. Eula A. Weeks, Professor W. D. A. Westfall.

The morning session opened at 10 A.M. and the afternoon session at 2 P.M. Professor Roever presided. It was decided to hold the next meeting of the Section at the University of Kansas, Lawrence, Kansas, on Saturday, December 2, 1916. The following program committee was appointed: Professor J. N. Van der Vries (chairman), Professor S. W. Reaves, Professor O. D. Kellogg (secretary). Attending members were entertained at a smoker at the Washington Hotel on Friday evening, and at luncheon at the Tower Dormitory on Saturday noon.

The following papers were presented at this meeting:

(1) Professor C. H. SISAM: "On sextic surfaces which have a nodal curve of order eight."

(2) Professor G. H. HARDY: "Weierstrass's non-differentiable function."

(3) Professors E. R. HEDRICK and LOUIS INGOLD: "Note on the continuity of the function ξ in the law of the mean."

(4) Dr. S. LEFSCHETZ: "On the n-dimensional cycles of an algebraic n-dimensional variety."

(5) Professor A. B. FRIZELL: "The postulate of time."

(6) Professor K. P. WILLIAMS: "A theorem on real functions."

(7) Professor W. H. ROEVER: "Mathematical theory of

the optical phenomenon observed in viewing a light through a screen."

(8) Dr. C. H. FORSYTH: "A method of interpolating single values among groups of values."

(9) Professor S. W. REAVES: "Metric properties of flecnodes on ruled surfaces."

(10) Professor H. C. GOSSARD: "Note on the Euler line."

(11) Mr. J. C. RAYWORTH: "On the generation of epicycloidal and hypocycloidal curves."

(12) Professor W. H. ROEVER: "Graphical construction for a function of a function and for a function parametrically given."

(13) Professor O. D. KELLOGG: "On the roots of minors in the secular equation."

(14) Professor M. B. PORTER: "On Savary's construction for the centers of curvature of a roulette."

In the absence of their authors, the papers of Professor Hardy, Dr. Lefschetz, Professor Williams, Dr. Forsyth, Professor Gossard, and Professor Porter were read by title. Abstracts of the papers follow.

1. In this paper, Professor Sisam points out some fundamental properties of sextic surfaces which have a nodal curve of order eight, of the sextic surfaces in space of five dimensions of which they are the projections, and of certain line congruences associated with them. The paper will appear in the *American Journal of Mathematics*.

2. It was proved by Weierstrass that the function

$$f(x) = \Sigma\, b^n \cos a^n \pi x,$$

where $0 < b < 1$ and a is an integer, has no differential coefficient for any value of x if $ab > 1 + \frac{3}{2}\pi$. The last condition is evidently artificial. In Professor Hardy's paper Weierstrass's function is discussed by a new method which leads to much more general and natural results. In particular it is shown that the function (and the corresponding function defined by a series of sines) has no finite differential coefficient for any value of x if $ab > 1$. The restriction that a is an integer is shown to be unnecessary. Similar results are established for other classes of functions. This paper will appear in the *Transactions*.

3. In this note, Professors Hedrick and Ingold show that the function $\xi(h)$ in the equation $[f(a+h) - f(a)]/h = f'(\xi)$ is continuous for h in the closed interval from 0 to $b - a$ provided there is, for each value of h in the interval, a single number ξ between a and b satisfying the above equation.

Incidentally, under the same hypothesis, certain other properties of the derivative and of the difference quotient are obtained.

4. In this paper, Dr. Lefschetz extends and generalizes a mode of generation of superficial cycles of an algebraic surface obtained by Emile Picard (Picard-Simart, Traité des Fonctions algébriques de deux Variables, volume 2, page 335), largely by the use of topological methods. The method followed in this note is different in that (*a*) only an infinitesimal deformation is used, and (*b*) it is based upon the value of Picard's invariant ρ_0. The generalization, which by Picard's method is arduous at best, is thus made comparatively simple. The paper is to appear in the *Rendiconti del Circolo Matematico di Palermo.*

5. Discussions on Mengenlehre have put in evidence two opposing tendencies; one is represented by the formal reasoning of Zermelo, the other finds remarkably clear expression in the philosophy of Bergson. To Bergson reality is the experience of change. Therefore the intuition of time, or duration as he prefers to call it, is essential to reality. Zermelo in his Auswahlprincip sets up an axiom which does not need duration and can not be tested by experience. By aid of a scheme described in the November BULLETIN, page 71, Professor Frizell develops a process for producing well-ordered sets by successive steps which taken singly may be described in terms of duration but constitute a sequence that overlaps duration. It follows that for mathematics time is not an *a priori* intuition; it is only a postulate which distinguishes intuitionism from formalism.

6. The theorem given by Professor Williams is a generalization of the theorem that a continuous function actually assumes all values between any two of its values. The paper has appeared in full in the December number of the *Annals of Mathematics.*

7. When a source of light is viewed through a screen certain curves of light become visible. Professor Roever shows that these curves, of which there are three, are the loci of the brilliant points (images of light) of the wires of the screen and that, in general, they are cubics. For an infinitely distant source of light they are conics and, in any case, they are nearly straight in the neighborhood of the light, i. e., of the point in which the plane of the screen is pierced by the ray from the light to the eye, through which point they all pass. One of the curves is the locus of the brilliant points of the practically straight cross wires of the screen. The other two are the loci of the brilliant points of the lengthwise wires, which bend in and out around the cross wires. When the ray from the light to the eye is perpendicular to the plane of the screen only two curves are seen (at least in the neighborhood of the point of crossing) and these cut at right angles. Geometrical constructions are given for all of the curves.

8. Dr. Forsyth's method of interpolation can be explained best by an illustration: Given the following age group of deaths, (5–9) 4129, (10–14) 2617, (15–19) 4317, the method gives the number of deaths for any single age, all computation being based upon the differences of the age groups. Dr. Forsyth incidentally gives the leading term and differences for interpolating several (such as five) values at a time. Thus, a group, such as given above, may be broken up completely into the several single values. The paper will be offered to the *Journal of the Royal Statistical Society* (England).

9. Professor Reaves in this paper makes use of the Wilczynski projective theory of ruled surfaces to study some metric properties of such surfaces. These relate chiefly to the flecnodes, flecnode curves, osculating quadric, and the locus of the center of the osculating quadric.

10. Euler proved that orthocenter, circumcenter, and centroid of a triangle are collinear, and the line through them has received the name "Euler line." He also proved that the Euler line of a given triangle together with two of its sides forms a triangle whose Euler line is parallel with the third side of the given triangle. By the use of vector coordinates or ordinary projective coordinates, Professor Gossard

proves the following theorem: the three Euler lines of the triangles formed by the Euler line and the sides, taken by twos, of a given triangle, form a triangle triply perspective with the given triangle and having the same Euler line. The orthocenters, circumcenters and centroids of these two triangles are symmetrically placed as to the center of perspective.

11. In Mr. Rayworth's paper, the number of ways of generating any epicycloidal or hypocycloidal curve, the locus of a point on the nth rolling circle, is shown to be $(n+1)!$ Since the arcs of the circumferences rolled over are arbitrary multiples of the preceding arcs, a dependence exists between them which finds expression thus: the sum of the products of the angles in the terms of the equations by their respective coefficients vanishes identically. Similar results are found when the fixed circle is replaced by a straight line. Some particular cases are considered.

12. A simple construction, suggested by the methods of descriptive geometry, for the graphical representation of a function of a function, and of a function given by two parametric equations is given in Professor Roever's second paper. The paper will appear in the *American Mathematical Monthly.*

13. In a problem connected with integral equations, it is of interest to know about the signs of the first minors of the symmetric matrix $[a_{ij} - \delta_{ij}\rho]$ for the roots of the secular equation $|a_{ij} - \delta_{ij}\rho| = 0$, where $\delta_{ij} = 0$ for $i \neq j$ and $\delta_{ij} = 1$ for $i = j$. It is known that any set of principal minors of this matrix, each of which is a first minor of the preceding, forms a Sturm sequence. Professor Kellogg shows that if all the minors of $[a_{ij}]$ are $\geqq 0$, the functions $|a_{ij} - \delta_{ij}\rho|$, and the minors of the elements of its first row, $A_{11}(\rho)$, $A_{12}(\rho)$, $\cdots$, $A_{1n}(\rho)$, form, for positive values of ρ, a Sturm sequence. This gives information on the signs of all first minors, because of the proportionality subsisting among first minors of a vanishing determinant.

14. The purpose of Professor Porter's paper is to show how Savary's elegant construction can be derived by the most elementary considerations from projective geometry. The paper will be offered to the *American Mathematical Monthly.*

O. D. KELLOGG,
Secretary of the Section.