Vorlesungen über projektive Geometrie. By FEDERIGO ENRIQUES. Second German edition by H. FLEISCHER. Leipzig, Teubner, 1915. xiv + 354 pp. Price (cloth), 10 Marks.

THE first German translation (with a prefatory note by Klein) of Enriques's lectures on projective geometry appeared in 1903 and was ably reviewed in the BULLETIN* by Professor Virgil Snyder. As the second edition does not contain any essential changes, not much needs to be added to that review. In view of the considerable advances that have been made in this field during the past two decades, a fuller discussion of the various aspects of the fundamental theorem of projective geometry would have been of great assistance to the student. At any rate the otherwise well written historic-critical note at the end of the treatise might have been brought up to date in this respect. In the discussion of projective coordinates, pages 332–336, which is essential for a proper understanding of the one-to-one correspondence between an analytic space (x_1, x_2, x_3, x_4) and the space based upon and abstracted from intuition (parabolic, hyperbolic, or elliptic), in which the points are defined by means of cross-ratios, a detailed proof of this correspondence would be commensurate with its importance.

From a didactic standpoint Enriques's "lectures" can still be recommended as an excellent introduction to the subject.

ARNOLD EMCH.

Descriptive Geometry. By H. W. MILLER, head of the department of general engineering drawing in the University of Illinois. New York, Wiley and Son, third edition, 1915. 149 pages, 86 figures and 8 quiz sheets.

Descriptive Geometry for Students in Engineering Science and Architecture. A carefully graded course of instruction, by HENRY C. ARMSTRONG, associate professor of descriptive geometry and drawing, McGill University. New York, Wiley and Son, 1915. vi + 125 pages and 114 figures.

Darstellende Geometrie, von MARCEL GROSSMANN, professor at the technical school of Zurich. Leipzig, Teubner, 1915. v + 137 pages and 109 figures.

THE first sentence of the preface to Professor Miller's book reads: "Believing that no one study plays a larger part than

* Vol. 10, pp. 355–58, April, 1904.

descriptive geometry in the shaping of the student's mind into the analytic thinking machine, necessary to success in any engineering profession, the author has outlined and written the text with this as its chief aim."

Minute instructions as to lettering, trimming, weighting lines, notation, methods of study precede the subject proper. The variety of type and prominence of figures leave no question of clearness, but the page has in places a striking resemblance to a bill board in consequence. The method of representing a point, line, plane are explained n great detail. It provokes a smile to read on page 16, "Axiom: The two projections of a point must be on the same perpendicular to *GL*" and similar incidents on page 24, in which one statement is followed by a proof. No exercises are given for the student, and no numerical cases are worked out at all. This defect is partly remedied by a series of eight quiz sheets with drawings, put at the end of the book.

The chapter on revolution, general profile, and problems relating to them is longer than the others. The figure representing the perspective of a circle is hardly necessary at this stage and is rather too hard to understand. Most of the theorems are well explained, but their inter-relations are not well brought out. The chapter (Chapter 6) on lines and surfaces would hardly bear mathematical analysis. The classification of lines and surfaces is a curiously arbitrary one, which would greatly confuse a bright student. Single and double curvature contact are not defined, and are crudely employed. The definitions of a ruled surface, developable, and double curved surface are such as to apply only to the few elementary illustrations employed, yet this fact is nowhere stated. The concept of a tangent plane to the ellipsoid should at least have been shown to be unique. The intersections of certain surfaces with planes is more satisfactorily discussed, as is also the development of cylinders and cones. Thus far 95 pages have been covered, which includes all of descriptive geometry that is treated. Chapter 10 is on shades and shadows—well written from an architectural or structural standpoint—it contains no new mathematics. The last chapter is on perspective. It brings in no new mathematical principles; it treats of a number of elementary properties in a rather empirical way, totally overlooking the beautiful transition from descriptive to projective geometry.

The reviewer is not competent to speak of the merits of the book from the engineer's viewpoint, but had the text been more mathematical and a more scientific aim kept in mind, the claims of the first sentence of the preface would have been much more generally fulfilled.

Professor Armstrong's book shows many contrasts with the preceding. It contains no mechanical instructions for the reader; it does not use bold-faced type, and very little explanation is given concerning different kinds of lines. The book is full of exercises, an appropriate list being furnished at intervals of a few pages. In fact, so little explanation is given that a student without a competent teacher would have to exercise considerable patience to master the text. All the steps are given, but in a very concise form. Thus, restricted positions of lines, planes, plane figures, polyhedra, involving shadows on the horizontal and vertical planes, are all treated in forty pages.

Part II, which treats of planes, lines, and points in unrestricted position, is less concise. The discussion is clear, is usually mathematically correct, and the frequent exercises allow the reader to test his grasp of every point. What seems an objection is that too many figures are prepared, the drawings given are completed, before the student can understand how it was done. Of course this difficulty is at once obviated by a good teacher. It is unusual to meet with axometric projection as early in the development (page 69) as in the present book. It is also presented in an unusual way, namely, as the ordinary horizontal and vertical projections of the corner and the edges of a cube, the idea of scale being developed later. The explanations are clear, but the whole treatment of this section (8 pages) is too brief to be of much use. But the last chapter of Part II, devoted to sections of simple solids and traces of cylinders and cones is excellently well done. The figures alone completely show the whole process, and they are supplemented by a brief description and followed as usual by exercises for the student.

Part III begins with problems involving tangent planes to cones and cylinders and their applications to sections of solids; tangent planes of a sphere, common to two or three spheres. The text is almost always correct mathematically—only such problems are considered in which the tangent line to a conic

can be constructed geometrically. Then follow simple intersections and their developments; here the treatment is the usual one. One modest case of a screw-thread is worked out. Finally, a short chapter on perspective is given; it is very clear as far as it goes, and includes some excellent examples for the student, but it stops just as the student's interest is aroused. Perhaps this is first class pedagogy, but somewhat dangerous without a good teacher.

Although the title of Professor Grossmann's text could hardly convey less information than it does, this book s not for beginners—indeed it presupposes a fairly comprehensive course in orthographic descriptive geometry and considerable familiarity with the technique of mechanical drawing. Its purpose is more specifically mathematical, to explain the meaning of the processes employed, and to compare their merits. The first discussion is to show that the ground line is not needed, and that any line in a plane of projection may be used as new ground line. Then follows a carefully written discussion of axonometry. If it could have been supplemented with a generous list of appropriate exercises, what a fine presentation it would make! It includes a good demonstration of Polke's theorem, that any three segments on three arbitrary concurrent lines can be taken as the projections of three concurrent edges of a cube. The chapter on perspective is less satisfactory; the details are clear enough but the purpose of it all is not as clearly presented as it might be, metrical details coming in unusually early in the discussion. Later the problem develops in a more interesting way. An unusual theme is a full discussion of stereographic projection. A dozen pages are devoted to the interesting topic of photogrammetry. Both inner and outer position are treated, and reconstruction from two vertical photographs, or from two oblique ones.

The second part, curves and surfaces, begins with a considerable digression on the analytic theory of plane and space curves, including parametric representation and differential properties. As is too liable to be the case, this attempt has but little purpose. It is too brief really to teach one unfamiliar with the ideas concerned, and unnecessary to one already acquainted with it. In the corresponding treatment of curved surfaces, a number of theorems are stated without

references or attempts at proofs. After a very brief discussion of the simple helix, the first surfaces discussed are a topographical map and forms of embankments and excavations. Granted this is important to one learning uses of graphical methods, a less felicitous application of the earlier theory could hardly be devised. Cones and cylinders fare rather better, as they connect directly with the theory. Plane perspective is developed from the standpoint of geometric correspondence; use is made of cross-ratio, and a fairly full discussion of conics from the Steiner construction is given, including the theorems of Pascal and Brianchon, and a few applications.

The treatment of intersections of cones and cylinders is rather brief; space quartics (of the first kind) and space cubics are considered and a few examples given. Plane sections of surfaces of revolution, and illumination are next discussed. From the three-page description the average reader can expect but a very vague and indefinite idea of a ruled surface. In one line the half-dual property is disposed of. Nearly five pages are given to the helicoid, six to the ruled quadrics, and three to non-ruled quadrics. At the end of the volume is a list of a dozen other texts for references; all of them have been reviewed in the BULLETIN.

While it would certainly be desirable to have students of geometry in the technical schools and colleges familiar with the topics here cited, I cannot believe that the best way to accomplish that purpose is to attempt to acquire the necessary knowledge in such a condensed way.

VIRGIL SNYDER.

NOTES.

THE opening (January) number of volume 17 of the *Transactions of the American Mathematical Society* contains the following papers: "On functions of several complex variables," by W. F. OSGOOD; "A study of certain functional equations for the ϑ-functions," by E. B. VAN VLECK and F. H'DOUBLER; "A set of four independent postulates for Boolean algebras," by B. A. BERNSTEIN; "Transformations of surfaces Ω (second memoir)," by L. P. EISENHART; "On figures of equilibrium of a rotating compressible fluid mass; certain negative results," by E. J. MOULTON.