Taking $f(x) = \log(1 + x)$, it is seen that the conditions of Theorem 2 are satisfied.

COROLLARY 2. *A sufficient condition that* $k(x, y)$ *defined,* $0 < y \leqq x$, *and integrable in* y *for each* x, *correspond to a regular transformation is*

$$|k(x, y)| \leqq \frac{M}{x^{1-p}y^p}, \quad 0 < y \leqq x, \quad \lim_{x=\infty} \int_0^x k(x, y)dy = 1 - \alpha,$$

where p *and* M *are constants,* $0 \leqq p < 1$, $M \geqq 0$.

Taking $f(x) = x^{1-p}$, it is seen that the conditions of Theorem 2 are satisfied.

Similar theorems hold for transformations of sequences.

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SHORTER NOTICES.

Contributions to the Founding of the Theory of Transfinite Numbers. By GEORG CANTOR. Translated, and provided with an introduction and notes, by PHILIP E. B. JOURDAIN. Chicago and London, Open Court Publishing Company, 1915. ix+211 pp.

THIS volume contains a translation of G. Cantor's two fundamental memoirs on transfinite numbers which appeared in the *Mathematische Annalen* for 1895 and 1897 under the title "Beiträge zur Begründung der transfiniten Mengenlehre." The translator has changed the title to that given above because "these memoirs are chiefly occupied with the investigation of the various transfinite cardinal and ordinal numbers." The book is put forth by the publishers as number 1 of "The Open Court Series of Classics of Science and Philosophy."

It is not too much to say that the work of Cantor on the theory of classes of points has brought about both a mathematical and a philosophical revolution, that in philosophy perhaps being even greater than that in mathematics, notwithstanding the fact that "these theories of Cantor are permeating modern mathematics."*

In the opinion of the translator K. Weierstrass, R. Dedekind, and G. Cantor are the three men who have exerted the most marked influence on modern pure mathematics and indirectly on the modern logic and philosophy which abut on it. In

* E. H. Moore, Introduction to a Form of General Analysis, p. 2.

order that the work of Cantor—and, in particular, the two memoirs here translated—may be best understood it is desirable on the one hand to compare it with the work of Dedekind which has developed along a parallel direction and on the other hand to trace its origin backwards through the earlier theory of functions and especially through the work of Weierstrass. To facilitate the latter historical study this book is provided with an excellent introduction (pages 1–82), based in large part on the translator's "Development of the theory of transfinite numbers," published in the *Archiv der Mathematik und Physik* in 1906, 1909, 1910, and 1913.

This introduction begins with a brief account of the contribution of Fourier toward the general definition of function, followed by remarks concerning the labors of Dirichlet, Cauchy, Riemann, and Hankel in the theory of functions as well as by some further discussion of the theory of Fourier series. A fuller account is then given (pages 10–23) of the contributions of Weierstrass to the theory of functions. The remainder of the introduction (pages 23–82) is devoted to an illuminating account of the development of Cantor's ideas as seen from his publications prior to 1895.

The translation of Cantor's articles covers pages 85–201. On the whole the work of the translator is done in a satisfactory manner. One regrets, however, that the term "power" is used to translate both "Potenz" and "Mächtigkeit," thus introducing an ambiguity in the English which is unnecessary and is absent from the German. Sometimes (as on pages 97, 178) the translator finds it necessary to introduce a footnote to remove the ambiguity.

Brief notes at the end (pages 202–208) contain a short but valuable account of the development of the theory of transfinite numbers since 1897.

A few misprints and slips need correction, as follows: on page 16, line 5 below for s_{n+} read $s_{n+\gamma}$; on page 31, lines 8 and 9, for P', P'', P read P'', P''', P'; on page 33, line 8 below, for *consists of* read *contains*; on page 47, end of line 13, insert *at*; on page 69, line 1, for *in* read *is*; at the middle of page 71, for last exponent 3 read 2; on page 96, line 4, after $x \geqq$ insert 0; on page 96, line 9, for $2^{\aleph}$ read $2^{\aleph_0}$; on page 109, line 12 below, for $\aleph$ read $\aleph_0$; on page 184, line 3 below, for the first ω read ω^{α_0}; on page 194, line 3, for $\alpha^{(\rho_0}$ read $\alpha^{(\rho_0)}$; on page 196, line 11 below, for ω^{γ} read ω^{γ_ν}.

In making this fundamental work of Cantor readily accessible to a wider range of English readers both the translator and the publishers have rendered a useful service in the development of science.

R. D. CARMICHAEL.

Euclid's Book on Divisions of Figures (περὶ διαιρέσεων βιβλίον), with a restoration based on Woepcke's text and on the Practica Geometriæ of Leonardo Pisano. By RAYMOND CLARE ARCHIBALD. Cambridge, Eng., University Press, 1915. ix+88 pp.

OF the nine works attributed to Euclid the "Elements" is, of course, by far the most important and the most widely known. The "Data" is known to us through the τόπος ἀναλυόμενος of Pappus, as stated in the Commandino edition of 1660, page 241; the "Porisms" was restored by Chasles, and earlier by Robert Simson; the "Optics" was known to earlier scholars through Theon, and has recently appeared in a modern edition through the labors of Heiberg; the "Phænomena" is nearly complete and was edited by Menge; the "Conics" is lost, except as part of it may have been embodied in the works of Apollonius; and the "Pseudaria" and "Surface Loci" are known only through fragments. The ninth work, entitled "On Divisions" (of figures), was for a long time known only through references by Proclus, but in 1570 it appeared in print under the editorship of John Dee and Federico (sometimes printed Federigo) Commandino in Latin translation from the Arabic. In 1851 Woepcke found an Arabic manuscript of the work at Paris, and this was published in translation in the *Journal Asiatique*.

It seems that John Dee, when he visited Commandino at Urbino in 1563, gave to the latter a Latin manuscript of the work as translated into Arabic by one Muhammed Bagdedinus, and this together with an Italian version was published seven years later. An English translation appeared in London in 1660 and again in 1661. David Gregory included the Latin text in his edition of Euclid in 1703 with the statement: "Joannes Dee Londinensis, cum Librum de Divisionibus superficierum, Machometo Bagdedino (qui floruisse creditur seculo Christi decimo) vulgo adscriptum, ex Arabico (uti credo, licet hoc expresse non dicat) in Latinum verteret." As to the conjectured date of "Machometo Bagdedino" it may be said in