such points is two parabolas, one on each side of *RS*." A similar statement follows the definition of the hyperbola. Is it not unfortunate to speak of a locus which is not a complete locus?

The definition of equivalence of sets of equations (page 3) is somewhat vague; and it hardly seems wise to say that the equation $(x + y)(x - 2y) = 0$ is equivalent to the two equations $x + y = 0$ and $x - 2y = 0$, even though the sense in which this is meant is immediately explained.

The statement (page 15) that "a set of homogeneous equations can often be solved for the ratios of the variables when there are not enough equations to determine the exact values" might seem to imply that the "exact" values could be determined if there were enough equations.

In chapter five there is a paragraph on "infinite values" which reminds one of the school algebras of the last generation. It seemed to the present writer to be a really serious defect in what is in many respects an excellent book.

The mechanical features of the book are attractive, the figures (with a few exceptions) are accurate, and the typographical work is free from errors.

WALTER B. CARVER.

A Budget of Paradoxes. By AUGUSTUS DE MORGAN. Reprinted, with the author's additions, from the *Athenæum*. Second edition, edited by DAVID EUGENE SMITH. Two volumes, I, viii+402 pp.; II, 387 pp. Chicago, The Open Court Publishing Co., 1915. Price, $3.50 per volume.

THE first edition of this interesting work by Augustus De Morgan (1806–1871) appeared in 1872, after the author's death, under the editorship of his widow, Sophia De Morgan. Some ten years later Mrs. De Morgan wrote a "Memoir of Augustus De Morgan," which is worthy of mention in connection with the "Budget of Paradoxes." De Morgan's articles which constitute the present work appeared from time to time, in the years from 1863 (Oct. 10) to 1866 (Dec. 1), in the London *Athenæum*. From other facts which we have concerning the life of De Morgan it appears that some of the popular writing which he did, for encyclopedias and for journals, was stimulated by financial pressure; at this distance we can properly rejoice at the conditions which fostered the growth of the present work.

The wide range of interests of De Morgan is nowhere so well shown as in the somewhat random discussions of the "Budget of Paradoxes." In particular his interest in the history of mathematics, and in the history of science, is revealed again and again in these pages. De Morgan holds that a reasonable familiarity with the development of any field of scientific research is an indispensable, necessary condition for a contribution to the field. "All the men who are now called discoverers, in every matter ruled by thought, have been men versed in the minds of their predecessors, and learned in what had been before them. There is not one exception. I do not say that every man has made direct acquaintance with the whole of his mental ancestry; many have, as I may say, only known their grandfathers by the report of their fathers. But even on this point it is remarkable how many of the greatest names in all departments of knowledge have been real antiquaries in their several subjects.

"I may cite, among those who have wrought strongly upon opinion or practise in science, Aristotle, Plato, Ptolemy, Euclid, Archimedes, Roger Bacon, Copernicus, Francis Bacon, Ramus, Tycho Brahé, Galileo, Napier, Descartes, Leibnitz, Newton, Locke. I take none but names known out of their fields of work; and all were learned as well as sagacious. I have chosen my instances: if any one will undertake to show a person of little or no knowledge who has established himself in a great matter of pure thought, let him bring forward his man, and we shall see."

Taking into account the special interests of De Morgan and his great activity in the popularizing of mathematics the selection of David Eugene Smith as editor of this edition was almost inevitable. Between the time of the English mathematician and the present time no one could have been found more admirably fitted by nature and by training to edit the "Budget" than Professor Smith. The similarity of his literary and public activity to De Morgan's is striking; both men have been widely known as the authors of elementary textbooks of unusual excellence, both have acted as editors of the mathematical department of encyclopedias and dictionaries, both have been energetic collectors of old mathematical books and other mathematical material, both have made notable contributions to the history and bibliography of mathematics, and both have been distinguished by a wide and human

interest in mathematics. Smith's "Rara Arithmetica," the illuminating, descriptive catalogue of Mr. G. A. Plimpton's unparalleled collection of arithmetical books and manuscripts, is a continuation and extension of De Morgan's bibliographical work, "Arithmetical Works from the Invention of Printing to the Present Time" (London, 1847).

The intention of De Morgan in publishing the "Budget" was definitely stated to be "to enable those who have been puzzled by one or two discoverers to see how they look in a lump." By "discoverers," here, is meant paradoxical discoverers. For De Morgan the paradox "is something which is apart from general opinion, either in subject-matter, method, or conclusion." That no disparagement is implied necessarily in the designation "paradoxer" is indicated by the fact that De Morgan refers to Copernicus and Galileo as paradoxers, and includes the discovery of the planet Neptune by Le Verrier as a paradox. A distinction is drawn between two types of paradoxer, as follows: "The manner in which a paradoxer will show himself, as to sense or nonsense, will not depend upon what he maintains, but upon whether he has or has not made a sufficient knowledge of what has been done by others, *especially as to the mode of doing it*, a preliminary to inventing knowledge for himself." However it must be stated that the particular interest of De Morgan in this work is in the nonsense type of paradox and paradoxer.

In mathematics the old problems of the squaring of the circle, the duplication of the cube, the trisection of the angle, and numerical juggling with 666, the number of the beast, contribute most largely to the occupation of neophytes who "in a moment by a lucky thought" wish to enter their names upon the limited roll of great mathematicians. Others, and their kind is not yet extinct, have the notion that either at home or abroad there is a great reward offered for the squaring of the circle; some even have deluded themselves into thinking that this financial reward is involved in a university professorship. What could be more illuminating as to the ignorance of these paradoxers! Astronomy contributes its fair quota to the "Budget"; religion, philosophy, and medicine, too, have their pseudo-scientists who without knowing "what has been done by others" wish to revolutionize the established order. Would that all this were, indeed, a closed book. In mathematics trisectors are fairly common, and have persuaded journals,

even with scientific titles, to publish nonsense; on Fermat's theorem several hundred articles have been printed and others are now being written by paradoxers whose tense interest in the subject is occasioned rather by a certain familiarity with the power of the $25,000, an actual prize, than by any familiarity with the powers of integers; what nonsense emanates even from a distinguished seat of learning—absolutely, let us note, without official sanction—concerning a prodigy lecturing upon the fourth dimension; what a priori philosophical nonsense, based upon ignorance of "what has been done by others," has been published concerning the nature of the number idea. A modern De Morgan would, in two volumes like these, have room only for titles of published nonsense.

The present edition of the "Budget" will prove of considerable value to public libraries as a work of reference; the value would be greatly increased by an analytical table of contents, presenting the titles of the articles comprising the work, and giving, possibly, a summary of the articles arranged with reference to subject-matter. Mathematicians and astronomers have a particular interest in the work since so much of the material is germane to their fields. To that wide circle of readers who have enjoyed the artistic ramblings of William De Morgan this work by the famous novelist's father will prove entertaining, for the peculiar literary charm of the son seems to be a direct transmission from the father.

Typographically and otherwise the book is up to the high standard which has been set by the publications of the Open Court Company. The reader who takes the volumes in hand has real pleasure in store: we commend the work to all of those who take a kindly interest in the frailty, as well as in the greatness, of their fellows.

The errors of one sort and another are so difficult to find that it seems desirable to mention those which have caught the eye of the reviewer. In the Preface, I, page iv, the reference in the last line should be to page 280, not 281, of the second volume. The one good "i" is dropped out of an "acquaintance" on page 5, volume I. From Smith's "Rara Arithmetica" three or four errors should be corrected in the title of Robert Recorde's "The Whetstone of Witte" (II, page 328): The whetstone of witte . . . containyng . . . Cossike . . . Nombers. The date of death of Sacrobosco, given as 1256, (I., page 360) is not known. LOUIS C. KARPINSKI.