12. If $K_1(x, y)$ and $K_2(x, y)$ are functions of the form (1), then it is easily shown by means of (5) that the integral combination

$$\int_a^b K_1(x, \xi)K_2(\xi, y)d\xi \tag{16}$$

is of the same form, provided that the hypothesis of periodicity (the period being $b - a$) holds for the parts Ψ_1, Θ_1 and Ψ_2, Θ_2 of K_1 and K_2. This is the integral combination which has been so extensively studied by Volterra.

Now it is known that if we are given two functions K_1 and K_2 of x and y, and their respective resolvent kernels k_1 and k_2, there may be built up out of them by means of the combination (16) a new kernel and its resolvent; in fact, the function

$$k_1(x, y) + k_2(x, y) - \int_a^b k_1(x, \xi)k_2(\xi, y)d\xi \tag{17}$$

is resolvent for the function

$$K_2(x, y) + K_1(x, y) - \int_a^b K_2(x, \xi)K_1(\xi, y)d\xi.* \tag{18}$$

We have then the theorem:

If we have $K_1(x, y) = \Psi_1(x + y) + \Theta_1(x - y)$ *and* $K_2(x, y) = \Psi_2(x + y) + \Theta_2(x - y)$, *where* Ψ_1, Θ_1 *and* Ψ_2, Θ_2, *as functions of a single argument, are periodic with period* $b - a$, *and if we denote by* $k_1(x, y)$ *the function resolvent to* $K_1(x, y)$, *and by* $k_2(x, y)$ *the function resolvent to* $K_2(x, y)$, *then the functions given by* (17) *and* (18) *are of the same form, have the same properties of periodicity, and are themselves mutually resolvent kernels.*

RICE INSTITUTE,
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OPERATORS IN VECTOR ANALYSIS.

BY DR. VINCENT C. POOR.

IN a note in the April BULLETIN on "Changing surface to volume integrals," Professor E. B. Wilson asks why my paper in the January BULLETIN was not made shorter by using the Gibbs-Wilson notation. While the brevity and suggestiveness

* See the footnote to Section 10. For purposes of symmetry and convenience of statemen we have taken $\lambda = 1$ and assumed it not to be a root of K_1 or K_2.

of the Gibbs-Wilson notation is admitted, the note is misleading. For had brevity been the chief aim of my paper the notation of Burali-Forti and Marcolongo could have been made to compare very favorably with Professor Wilson's compact reproduction of the formulas in the Gibbs notation. (Compare the analytic statement of the theorems in the two notations.) However, as one of the purposes of my paper was to exhibit the operational feature of the system of Burali-Forti and Marcolongo, obviously the Gibbs-Wilson notation did not lend itself to this end. Moreover, since the notation of Burali-Forti and Marcolongo is not so well known in this country, some explanation seemed to be necessary.

Professor Wilson says in closing: "The use of words like grad, div, rot is hampering: we no longer write Cubus $\bar{m}$ Census $\bar{p}$ 16 rebus aequatur 40 for $x^3 - 8x^2 - 16 = 40$." Everybody admits the last part of this statement. But we still use for particular kinds of functions or operators such symbols as log, sin, cos, etc., arcsin, etc., sinh, cosh, etc. That the use of such "words" as grad, div, rot is hampering, seems to be a matter of opinion, since they may be used interchangeably with other symbols in both notations.

It is unfortunate that Professor Wilson introduced cartesian coordinates into his proof, since a coordinate system has no proper place in vector analysis. But this seems to be characteristic of the Gibbs-Wilson system. In fact Burali-Forti and Marcolongo have pointed out how the dyadics of Gibbs constantly depend on cartesian coordinates,* a nonlinear system.

UNIVERSITY OF MICHIGAN.

SHORTER NOTICES.

Grundlehren der Mathematik. Der zweite Band des ersten Teils: *Algebra.* By EUGEN NETTO. Leipzig, Teubner, 1915. xii+232 pp.

THE Grundlehren der Mathematik, für Studierende und Lehrer is a series of four volumes on the elements of mathematics appearing from the press of B. G. Teubner under joint authorship as follows: Part I (two volumes), Die Grundlehren

* Burali-Forti et Marcolongo, Transformations linéaires, 1912, p. 147.