

polynomial in s of order $n - 1$, which vanishes for all values of s ; the n coefficients of powers of s severally vanish. But each coefficient is a polynomial in t of order $n - 1$, every power of t being multiplied by a linear function of x, y, z . These powers of t may be eliminated by a determinant of n rows, which is the equation of the curve.

The linear function of (x, y, z) which in the r th row and s th column is the coefficient of $s^{n-r}t^{r-s}$ in the expanded form of $\Delta/(s - t)$, and is readily written down if need be. Just as Salmon's method is an adaptation of Euler's (or Sylvester's) method of eliminating a variable from two polynomials, so the foregoing is an adaptation of Cayley's statement of Bezout's method.

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SCIENTIFIC METHOD IN PHILOSOPHY.

Our Knowledge of the External World as a Field for Scientific Method in Philosophy. By BERTRAND RUSSELL, M.A., F.R.S., Late Fellow of Trinity College, Cambridge. Chicago, London, The Open Court Publishing Company, 1914. vii + 245 pp.

THE philosophic method in science is a thing familiar enough, in sound if not in fact. But what, pray, can be meant by "scientific method in philosophy"? That is what Mr. Russell essays to make clear, in outline at all events, in this book, and he does it partly by a general critique on the methodologies of philosophy and science and partly, indeed mainly, by handling certain specific problems of philosophy in the spirit and the manner of science. The chief ones among these illustrative examples belong to the general problem of the relation between the data of sense and the time, space, and matter of mathematical physics. The discussion is guided by a highly important and exceedingly difficult aim. The aim is to sketch a method available in all departments of philosophy and "adequate to yield whatever objective scientific knowledge it is possible to obtain." Traditional philosophy, however worthy of studious attention, has been too impatient; it has been too ambitious; its pretensions and claims have been grossly extravagant; it has not

been sufficiently disinterested; it has been too subjective, too personal, colored and often vitiated by individual tastes and temperaments. Such defects, which keep the great philosophic systems of the past from being scientific, are to be cured or avoided by means of a right philosophic, or scientific, method. What is this method? It is the method of logical analysis—a method first completely exemplified by Frege, a method copiously illustrated in the first three volumes of Whitehead and Russell's *Principia Mathematica* and, so far as the great conception of the world of physics as a construction rather than an inference is concerned, to be elaborately and precisely presented by Whitehead in the fourth volume of that work.

Mr. Russell's book, which is composed of the eight Lowell Lectures delivered by the author in Boston in 1914, is a rough, general, semi-popular sketch of the method in question. The presentation does not pretend to be accurate in all its details but it is confident of being sound in general and it is animated by the conviction that the method outlined "has great rewards to offer—triumphs as noteworthy as those of Newton and Darwin—and as important in the long run for the moulding of our mental habits."

The initial lecture, which is entitled "Current tendencies," is devoted to a clearing of the ground. The current tendencies dealt with are three: the classical tradition, evolutionism, and what Mr. Russell, for want of a better name, calls logical atomism. These are in conflict. The third is born of the modern critical movement in mathematics and is the type of philosophy that the author advocates. The first and the second types contend with one another and they are both of them antagonists of the third. The classical tradition seeks "to adapt to present needs the methods and results of the great constructive philosophers from Plato downwards," thus owning such names as Kant and Hegel and represented in our own day by the *Appearance and Reality* of Mr. Bradley. It is doomed by its too great reliance on pure reason and the meagreness of its imagination in conceiving the universe. The universe as revealed by modern science it does not know. It is dying at the hands of discovered natural facts. Mr. Russell would not, I suppose, deny that this philosophy has performed an immense service in testing the resources of reason. The experiment had to be made, for there was no

way of knowing a priori that reason, which had succeeded in building an immortal geometry, was not competent to construct a true philosophy of the entire world.

Evolutionism, the regnant philosophy of our time, is a reaction against the classical tradition. It springs from and allies itself with natural knowledge, as in the works of Herbert Spencer and Bergson. But it "is not a truly scientific philosophy, either in its methods or in the problems which it considers." It is not sufficiently disinterested; it is too easy-going; it does not know the severity of logical discipline. To understand the world it is necessary to understand change and continuity. But biology, physics, chemistry, and the like can not enlighten us here. The problem is foreign to natural science. Again, evolutionism is primarily interested in human destiny, but scientific philosophy is not. It is more detached. It is not Baconian. It has no essential or primary concern with questions that the other sciences may answer. It is not tethered to mundane interests. Incidentally it may help, as it may be helped by, the other sciences, but it appeals to none except such as desire, beyond all else, "to escape from intellectual bewilderment." Towards mysticism its attitude is this: It does not deny the genuineness of the mystic's alleged insights and intuitions of truth beyond the world of sense, but when the mystic denies truth to the deliverances of sense and normal perception it submits a challenge, it asks for evidence. And unlike Bergsonianism, it regards the logical analysis practiced by a Cantor or a Frege as superior to the instinct of a wasp or of a duck that will lead a brood of chicklets to a pond.

The second lecture bears the somewhat challenging title, "Logic as the essence of philosophy." In what sense is the implied thesis to be understood or to be justified? Certainly not at all if logic is to be identified with Aristotelian syllogistic. Mr. Russell does not pause to give due credit to the great logical work of Aristotle, possibly because he thinks the world has already done more than justice in that matter. But a great outburst of genius often blinds centuries of men and certainly Aristotle fettered the mediaeval mind. In a measure the fetters were broken by Bacon and Galileo. Yet Bacon's conception of the nature and proper function of induction was superficial and mistaken. Induction can not replace deduction, but can widen its scope. Deduction is not

all of it syllogistic. But how deduce by induction? What principle justifies inference, for example, from past to future sunrises? If, with Mill, we say the law of causation, then we must ask what justifies belief in that law. That the law is known a priori is rendered improbable by the formulation of the law, which runs somewhat as follows: "A causal law is any general proposition in virtue of which it is possible to infer the existence of one thing or event from the existence of another or of a number of others." If we suppose causal law to be an assumption, then evidently it can not justify inference in any fundamental or ultimate way. Is it an empirical generalization? Then, how justify such generalization? Not by enumeration, for this can at best yield probability—whatever that is—, nor by any other empirical means, for generalization transcends experience. Inference from the observed to the unobserved, if valid, must rest, not on any law of causality, but upon the principle of induction. What is this principle? It is hard to state it quite satisfactorily but it is much like this: "If, in a great number of instances, a thing of a certain kind is associated in a certain way with a thing of a certain other kind, it is probable that a thing of the one kind is always similarly associated with a thing of the other kind; and as the number of instances increases, the probability approaches indefinitely near to certainty." The proposition may be false; it admits of neither proof nor disproof. Yet it is the only justification that the kind of inference in question admits of.

But it is not to be admitted, because general propositions can not be empirically established, that we can not know any general propositions to be true. We have such knowledge and it belongs to logic. Logic consists of two parts, and we are now to see wherein and why it is the essence of philosophy. "The first part investigates what propositions are and what forms they may have." It "enumerates the different kinds of atomic propositions," that is, propositions which, when asserted, assert that a certain thing has a certain quality or that certain things have a certain relation; it enumerates the different kinds of "molecular" propositions, or propositions containing "conjunctions—*if, or, and, unless*, etc."; and it enumerates the various kinds of general propositions, such as, all equilateral triangles are equiangular; and so on. For logic the content of propositions is of no importance, but form is

all-important. The second part of logic "consists of certain supremely general propositions which assert the truth of all propositions of certain forms." An example of such supreme propositions is: "If anything has a certain property, and whatever has this property has a certain other property, then the thing in question has the other property." Such a proposition is true, and is known to be true, but can not be established empirically. This second part of logic is the first part of mathematics; it is the first part of logic that makes logic the essence of scientific philosophy. One is reminded of the saying of Leibnitz: "Ma métaphysique est toute mathématique."

It is in the third lecture, "On our knowledge of the external world," that Mr. Russell comes to close grips with his task and best indicates the method of "logical analysis" or the philosophy of "logical atomism" by operating it on certain fundamental problems. From the earliest times philosophy has from various motives found it necessary to discredit sense-data and to suppose something more real behind the veil. And modern science and especially physics (taken in a comprehensive sense) have not escaped the like necessity. Atoms, molecules, electrons, points, instants, time, space (the time and space of mathematical physics)—none of these things and their kind is known or knowable to sense or perception. Neither can they be logically inferred from the data of sense. They can not be proved to exist nor can their non-existence be shown logically. Why assume them? It is not necessary to do so, and, if not necessary, such assumption is forbidden by the principle of Occam's Razor. What, then, is proposed instead? Instead of illegitimately pretending to *infer* such things as the points, instants, matter, time, and space of mathematical physics and instead of *assuming* the existence of such things, it is proposed and shown to be possible to *construct* them from the data furnished by sense. That done, we shall know them to exist in whatever sense we may suppose constructs to exist, because we have made them, and physics will cease to be an illegitimate inference or a structure based on pure assumption and will be instead a structure built of and upon sense data, the only facts immediately known or knowable respecting the external world. How is the task of construction to be performed? A hint, by means of one example, must here suffice.

Let us confine ourselves to the sense of sight and see how we may construct a corresponding space. The sketch, for the want of room, must needs be rough. Each of many minds, looking out from its own view-point, sees at each moment a three-dimensional world. No two minds behold precisely the same thing. Each has its own "perspective," its own "private world." A mind at an intermediate point of view will have an intermediate private world. Such perspectives or worlds may be correlated by the principle of similarity, for whilst no two are identical, they may resemble each other in various degrees. Two that are much alike may be said to be "near" one another in space but this space is entirely different from "the spaces inside the two perspectives." It is a relation, a relation between the two private worlds or spaces. Between these we can in thought intercallate others more and more similar and hence nearer and nearer together. In this way public space, which consists of relations between private spaces, can be made continuous. Thus space is a constructed affair: it is a system, a system of perceived or perceivable private worlds. In ways quite analogous, we can construct a "thing," a "point," "here," "instant," "time" and so on and on. The delineation of the process with critical remarks as to its philosophic significance occupies the fourth lecture, "The world of physics and the world of sense."

The fifth lecture is devoted to "The theory of continuity." The author ranges himself on the side of those who regard sensation as a continuous function of stimulus though he does not think the doctrine capable of being established empirically. As this doctrine involves the necessity of supposing that two sensations may be different despite the fact that their subject can not discriminate them, it appears to involve a contradiction in terms or a verbal quibble or a verbal confusion. It seems sound to say that two sensations that are sensed as the same are the same, *as* sensations.

Then follow two lectures devoted to the notion of infinity—"the problem of infinity considered historically" and "the positive theory of infinity." Though these lectures are intensely interesting, they contain but little, except some novelty of exposition, that is likely to be new to the student acquainted with the literature of Cantorism. This is not to imply that they have not very great value as exhibiting the nature and power of logical atomism as a method and type of philosophy.

The work closes with a lecture treating "The notion of cause, with applications to the free-will problem." The discussion is very acute, the finest in the book. Hume's classical analysis is resumed and shown to require extension and this is undertaken. The vulgar notion of a cause as compelling its effect must be abandoned as having no warrant in logic and no essential rôle in natural science. A cause may as well accompany or follow its effect as precede it. Far more important than the notion of cause is that of causal law, which has been above stated in one form. It is acutely contended that, as causes do not compel, the acts of will may be caused without being externally coerced, and that omiscience, including knowledge of the entire future, is consistent with every thing in freedom that is worth preserving.

The book as a whole must be judged as an important contribution to the science of philosophy even if the reader must remain convinced that much that is destined to continue to be called philosophy will not, through logical analysis or other means, yield solid, scientific, objective knowledge. Personal idiosyncrasies are themselves facts and they are often more interesting than, and quite as important as, generic results that ignore them. In the future, as in the past, the value of philosophy will consist, not wholly in propositions established by it, but largely in philosophizing. Let the two kinds flourish side by side, but let them not be confounded.

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SHORTER NOTICES.

Combinatory Analysis. By Major PERCY A. MACMAHON.
Volume 1. Cambridge University Press, 1915. 300 +
xix pp.

THE author states that "the object of this work is, in the main, to present to mathematicians an account of theorems in combinatory analysis which are of a perfectly general character, and to show the connection between them by as far as possible bringing them together as parts of a general doctrine. Little attempt has been hitherto made either to make a general attack upon the territory to be won or to coordinate and arrange the ground that has been already gained. The combinatory analysis as considered in this work occupies the ground between algebra, properly so