

## SHORTER NOTICES.

*Théorie générale des Nombres.* By E. DUMONT, Capitaine du Génie Belge. Paris, Gauthier-Villars, 1916. (No. 35 of the series of Scientia.) 93 pp. 2 francs.

IN spite of the distractions on the Yser in 1914-1915, Captain Dumont has been able to produce a very clear account of the fundamental notions of number which he had previously published in extended form in his book *Arithmétique générale* (1911). This small volume is one of the physico-mathematical series in the Scientia publications of Gauthier-Villars, which undertake to present in succinct form the ideas of science in process of development. Several others of the series have dealt with recent ideas in geometry, analysis, and algebra.

This volume is divided into two parts: *Des Nombres absolus*, and *Des Nombres relatifs*. In the first part is given a development of numbers as the ratios of segments of a straight line. The notion of segment of a line is taken as an undefined element. An absolute number is then any law of formation of a segment from a given segment. The product of a segment by a number (absolute number) is another segment. Captain Dumont in fact sets up a set of logical relatives of which the domain is the range of segments of a straight line, the converse domain is the same range, and the relative is considered only with reference to certain processes of addition, subtraction and multiplication of the segments. These processes are supposed to be known. This first part does not differ essentially, save in formal statements, from other derivations of the system of real numbers.

In the second part the fundamental entity is no longer a segment but a vector, by which is meant a directed segment. The notion of direction is assumed. Definitions are set up for free vectors, glissants, symmetric or opposite vectors, and nul vectors. The definition of relative number then follows as any law of determination of one vector from another; by which is really meant again a logical relative whose two domains are the same, namely the range of all vectors. We arrive thus at qualified numbers, which change a glissant vector into another on the same line, or support; versors, which turn glissant vectors around an axis perpendicular to

the line of the vectors; glisseurs, which move a glissant vector parallel to itself along an axis perpendicular to the line of the vector. Combinations of these give quaternions, in which are included complex numbers, vector-quaternions, and surcomplex numbers.

In this part the author insists on the distinction between the vector that is commonly associated with a complex number in the Argand representation, and the number itself. This distinction is, in the opinion of the reviewer also, vital in many places, and unless noticed leads to trouble. For instance, the Steinmetz treatment of alternating currents is handled by Cramp and Smith from this point of view to the improvement of the whole treatment. Hamilton was the first to define a quaternion as the ratio of two vectors, having arrived at this definition by a study of certain groups of linear substitutions, of which one was a form of the quaternion group as an abstract group. The later expositions given by Hamilton used this geometric method of approach. One might go farther and produce a system of numbers corresponding to the ratios of the members of any set of geometric elements, such as bipoints, biplanes, as Cailler and others have done. Some of these geometric elements have even received names, as *flèche*, *bouclier*, *drapeau*, *feuille*, *moulinet*. Captain Dumont, however, develops the notion only as far as bi-quaternions.

He insists rightly that the method of development is synthetic, and demands no postulates except those of euclidean space, while other methods of the arithmetico-logical type demand large lists of postulates. Further the development is not referred to any coordinate system, and to no fixed base of units. It becomes thus autonomous. Whether he is correct in thinking it could be taught easily from this intuitive point of view is a question which time and experience will settle. The intuitive method seems at least as good as any way to identify the fundamental elements that one starts with, and in a genuinely logical sense it is just as rigorous. When anyone thinks that by talking of the set of elements  $A, B, C, \dots$  he is any better able to identify an element than he is when he talks of the segments of the line,  $OA, OB, OC, \dots$  he is deceiving himself into a fancied rather than a real rigor of treatment. The really essential thing, it would seem, is whether or not some unstated quality of the element is

one that the deductions depend upon. To illustrate, one may prove the theorems of geometry relating to the circle by using wire circles just as well as by using a highly artificial definition of an abstract circle, provided that he does not depend upon the copper or silver, or their qualities, nor upon the thickness, or cross-section of the wire. In most work in mathematics we are engaged in finding the deductions that can be drawn from certain features only of a concrete existence and which are not in the least affected by other features of the object. In other words the most highly concrete object is just as good as the most highly abstract object for deductions that are limited to certain characters possessed in common by the two. From this point of view the definition from geometric objects is justifiable. On the other hand the recognition of the non-geometric character of all the numbers deduced is one that if more common would prevent much wasted ink and time. Such a notion as that of the product of two geometric vectors ought to disappear from the field, for instance, save as a phraseology perhaps. The fruitless discussions over the identification of the right quaternion and the vector (geometric) would no longer be heard of, and all vector systems would be recognized as algebras of hypernumbers.

JAMES BYRNIE SHAW.

*Napier Tercentenary Memorial Volume.* Edited by CARGILL GILSTON KNOTT. London, Longmans, Green and Company, 1915. xi+441 pp. Price, £1 1s.

THIS sumptuous volume compiled by Dr. Knott is made up of the addresses and essays communicated to the International Congress held in Edinburgh in 1914 to commemorate the tercentenary of the publication of Napier's epoch-making *Mirifici Logarithmorum Canonis Descriptio*.

The papers are both historical and mathematical, the former dealing with the life and works of Napier and of his contemporaries, immediate predecessors, or followers, and the latter part of the work treating of the modern progress in calculation, in the preparation of tables, and the like. There is also an account of the Edinburgh meeting, with the addresses of a formal congratulatory nature, a list of members, and two indexes, one of subjects and the other of names.

The historical papers are as follows: "The invention of logarithms," by Lord Moulton, a careful study of the working