THE OCTOBER MEETING OF THE AMERICAN MATHEMATICAL SOCIETY.

THE one hundred and ninety-third regular meeting of the Society was held in New York City on Saturday, October 27, 1917. The attendance at the morning and afternoon sessions included the following thirty-five members:

Professor M. J. Babb, Dr. Emily Coddington, Professor F. N. Cole, Dr. W. L. Crum, Professor Louise D. Cummings, Professor L. P. Eisenhart, Professor P. F. Field, Professor H. B. Fine, Dr. C. A. Fischer, Professor T. S. Fiske, Professor W. B. Fite, Dr. T. R. Hollcroft, Professor Dunham Jackson, Mr. S. A. Joffe, Professor Edward Kasner, Professor C. J. Keyser, Dr. J. R. Kline, Professor P. H. Linehan, Professor C. R. MacInnes, Professor H. B. Mitchell, Professor R. L. Moore, Dr. G. W. Mullins, Mr. L. S. Odell, Mr. George Paaswell, Professor H. W. Reddick, Dr. J. F. Ritt, Dr. Caroline E. Seely, Professor D. E. Smith, Professor P. F. Smith, Professor H. D. Thompson, Professor Oswald Veblen, Mr. H. E. Webb, Dr. Mary E. Wells, Professor H. S. White, Mr. J. K. Whittemore.

Professor Veblen occupied the chair, being relieved by Professor Eisenhart. The Council announced the election of the following persons to membership in the Society: Dr. J. V. DePorte, State College, Albany, N. Y.; Mr. J. W. Lasley, Jr., University of North Carolina; Mr. Vicente Mills, Philippine Bureau of Lands; Professor B. M. Woods, University of California. Five applications for membership in the Society were received.

A committee was appointed to audit the accounts of the Treasurer for the current year. The Council reported a list of nominations for officers and other members of the Council to be printed on the official ballot for the annual election. The Secretary was directed to procure insurance to the extent of $10,000 on the library of the Society, now deposited in the Columbia University Library.

The following papers were read at this meeting:

(1) Professor R. D. CARMICHAEL: "Elementary inequalities for the roots of an algebraic equation."

(2) Professor LOUISE D. CUMMINGS: "The two-column indices for triad systems on fifteen elements."

(3) Dr. G. A. PFEIFFER: "On the continuous mapping of regions bounded by simple closed curves."

(4) Dr. J. F. RITT: "On the differentiability of asymptotic series."

(5) Professor W. B. FITE: "Concerning the zeros of the solutions of certain linear differential equations."

(6) Professor J. E. ROWE: "Hexagons related to any plane cubic curve."

(7) Professor G. D. BIRKHOFF: "On a theorem concerning closed normalized orthogonal sets of functions with an application to Sturm-Liouville series."

(8) Professor EDWARD KASNER: "Systems of circles related to the theory of heat."

(9) Professor O. E. GLENN: "Systems of invariants and covariants of Einstein's transformations in the theory of relativity."

(10) Mr. J. K. WHITTEMORE: "Theorems on ruled surfaces."

(11) Professor R. L. MOORE: "On certain systems of equally continuous curves."

(12) Professor R. L. MOORE: "Continua that have no continua of condensation."

(13) Dr. J. R. KLINE: "Necessary and sufficient conditions, in terms of order, that it be possible to pass a simple continuous arc through a plane point set."

(14) Professor OSWALD VEBLEN: "On the deformation of n-cells."

(15) Professor OSWALD VEBLEN: "Deformations within an n-dimensional sphere."

Professor Birkhoff's paper was presented by Professor Dunham Jackson, and the papers of Professor Carmichael, Dr. Pfeiffer, Professor Rowe, and Professor Glenn were read by title. Abstracts of the papers follow below. The abstracts are numbered to correspond to the titles in the list above.

1. The principal object of Professor Carmichael's paper is to derive numerous elementary inequalities for the greatest absolute value X of a root of an algebraic equation. The methods employed are altogether elementary. The results generalize a number of known inequalities and contain several new ones of interest. In each case the inequality yields an upper bound to the value of X.

2. Professor Cummings' paper deals with the two-column indices as a method of comparison for triad systems. The two-column indices for the 80 systems on fifteen elements are obtained, and the supplemental theory necessary to make this method of comparison adequate is given. An important paper by Kirkman, which has apparently been overlooked by writers on triad systems up to the present time, is briefly discussed, and an inaccuracy in Netto's article on Tripelsysteme in the Encyklopädie der Mathematischen Wissenschaften is pointed out.

3. The theorem proved in Dr. Pfeiffer's paper is:

Given two simple plane curves J_1 and J_2 which are in $(1-1)$ continuous correspondence under a correspondence π. Let J_1 and J_2 be interior to the regions R_1 and R_2 respectively; then there exists a $(1-1)$ continuous correspondence between the points of R_1 and R_2 such that the points of J_1 correspond to points of J_2 and conversely as fixed by the correspondence π.

This theorem is essentially the theorem proved by Schoenflies in volume 62 (page 319) of the *Mathematische Annalen* to the effect that a $(1-1)$ continuous correspondence π between the interiors of two simple closed curves can be set up such that π is continuous with any given $(1-1)$ continuous correspondence between the boundaries (the given closed curves).

The important points of difference between the proof which Dr. Pfeiffer gives in this paper and that given by Schoenflies is first that the former is non-metric and second that no use is made therein of the Jordan theorem which states that a simple closed curve divides the plane into two regions. The Jordan theorem is thus an immediate corollary of the present proof when one of the curves J_i is taken as a triangle and the corresponding R_i as the whole plane.

4. Dr. Ritt shows that if $f(z)$ has, in a sector of the complex domain, the asymptotic development

$$f(z) \sim a_0 + a_1z + a_2z^2 + \cdots + a_nz^n + \cdots,$$

then, in any sector interior to the given one,

$$f'(z) \sim a_1 + 2a_2z + \cdots + na_nz^{n-1} + \cdots.$$

This fundamental result was known and had been used by Professor Birkhoff, who failed to publish his proof, being under the impression that such a proof had already been given by Professor W. B. Ford.

5. Certain conditions under which the solutions of linear homogeneous differential equations of the second order vanish an infinite number of times are determined by Professor Fite in this paper. A theorem of comparison for the solutions of two such equations under circumstances different from those considered by Sturm results from one set of these conditions. Related questions in connection with certain binomial equations of the nth order are also considered.

6. In Professor Rowe's paper Theorems I and II of the second paper read by him before the Society at the Cleveland Meeting, September 4, 1917, which relate to hexagons inscribed in or circumscribed about the rational plane cubic curve, have been extended to apply to any plane cubic curve. The method used consists in expressing parametrically the coordinates of a point on the curve in terms of the elliptic functions $\wp(u)$ and $\wp'(u)$ (Durège-Maurer, Theorie der Elliptischen Functionen, pages 246–252), after which the proofs are algebraically as simple as in the case of the rational plane cubic curve.

7. The theorem of Professor Birkhoff's note states that if a set of normalized orthogonal functions $u_1(x)$, $u_2(x)$, $\cdots$ $(0 \leqq x \leqq 1)$ is closed, and if $\bar{u}_1(x)$, $\bar{u}_2(x)$, $\cdots$ is a second set of normalized orthogonal functions such that the infinite series $\sum_{n=1}^{\infty}(u_n(x) - \bar{u}_n(x))u_n(y)$ converges to a function $H(x, y)$ less than 1 in absolute value, then, under a certain simple further condition, the second set $\bar{u}_1(x)$, $\bar{u}_2(x)$, $\cdots$ is also closed.

By means of this theorem it is readily proved that any set of characteristic functions of Sturm-Liouville type is closed. An elementary and simple proof of this important fact has hitherto been lacking.

The theorem is really one in the field of general analysis as defined by E. H. Moore and can be extended to biorthogonal sets of functions.

The note will appear in the *Proceedings of the National Academy of Sciences*.

8. In the propagation of heat in a plane there are in general ∞^2 isothermal curves, ∞^1 for each value of the time. Professor Kasner shows that these ∞^2 curves can never be circles. If, however, the system degenerates into ∞^1 curves (that is, if the curves are the same at all times), as discussed in an earlier paper, systems of circles are possible, and all such cases are determined.

9. Professor Glenn's paper deals with complete invariant systems under the Einstein formulas S for the transformation of space and time coordinates. Systems are derived for binary forms in t and x (t being the time), with coefficients constant or arbitrary functions of the quantities left fixed by S. Their general theory is that of concomitants which are functions of the coefficients of the transformation, but they are, in fact, free from the relative velocity of the moving systems of reference, while involving the constant c, the velocity of light. Included in the paper is the explicit system of relative concomitants for the form of general order m, and absolute systems for the orders to the fourth inclusive.

10. Mr. Whittemore's paper is a continuation of that presented to the Society at the April meeting. It is concerned chiefly with the determination of the "flat points" of any ruled surface, and with the discussion of the properties of orthogonal trajectories of the rulings at points where the total curvature of the surface vanishes.

11. Suppose that G is a set of simple continuous arcs each of which has one end point on the side AD and the other end point on the side BC of the square $ABCD$. Suppose that each arc of the set G lies, except for its end points, entirely within $ABCD$, that no two arcs of G have any point in common and that every point within or on $ABCD$ belongs to some arc of the set G. In this paper Professor Moore shows that a necessary and sufficient condition that such a set of arcs G may be mapped continuously on a set of straight segments possessing these same properties is that the arcs of the set G should be equally continuous.

12. In this paper Professor Moore shows that in two-dimensional space every continuum that has no continuum

of condensation is a continuous curve. The theorem that every such continuum which is irreducible between two points is a simple continuous arc is due to S. Janiszewski. Cf. his paper "Sur les continus irréductibles entre deux points," *Journal de l'Ecole Polytechnique*, 2e série, volume 16 (1911–12), pages 79–170.

13. Dr. Kline proves that, in order that it may be possible to pass at least one simple continuous arc through a given bounded closed point set K, it is both necessary and sufficient that the points of K may be ordered subject to the following conditions:

(1) If $A \neq B$, then either A precedes B or B precedes A.

(2) If A precedes B, then B does not precede A.

(3) If A precedes B and B precedes C, then A precedes C.

(4) If the point P of K is a geometrical limit point of some subset M of K, then every segment of K containing P (or every segment one of whose end points is P, if P is either the first or last point of K) contains at least one point of M, different from P.

If K is not closed, these conditions are not sufficient. In this case, he proves that a necessary and sufficient set of conditions is obtained if the points of the open set K can be ordered subject to conditions (1) to (4) and the following two additional conditions:

(5) If $A_1, A_2, A_3, \cdots$ is a countable infinity of distinct points of K such that either A_n precedes A_{n+1} for all values of n or A_{n+1} precedes A_n for all values of n, then the point set $A_1, A_2, A_3, \cdots$ has a single limit point A.

(6) If $A_1, A_2, A_3, \cdots$ and $B_1, B_2, B_3, \cdots$ are ordered sequences, approaching A and B, respectively, such that, for all values of n and k, A_n precedes B_k, then, if $A = B$, A_n precedes A_{n+1} $(n = 1, 2, 3, \cdots)$, B_{k+1} precedes B_k $(k = 1, 2, \cdots)$ and there is no point D $(D \neq A)$ such that D follows every A and precedes every B.

14. Professor Veblen's first paper contains a proof of the theorem that any $(1-1)$ continuous transformation of an n-cell and its boundary into themselves, which leaves all points of the boundary invariant, is a deformation within an n-dimensional euclidean space containing the n-cell.

15. In Professor Veblen's second paper it is proved that there does not exist a deformation on an n-sphere which carries each point of an $(n - 1)$-sphere into itself and one of the n-cells bounded by the $(n - 1)$-sphere into the other.

F. N. COLE,
Secretary.

THE OCTOBER MEETING OF THE SAN FRANCISCO SECTION.

THE thirtieth regular meeting of the San Francisco Section was held at the University of California, on Saturday, October 27. The attendance included the following seventeen members of the Society:

Professor R. E. Allardice, Dr. B. A. Bernstein, Professor H. F. Blichfeldt, Dr. Thomas Buck, Professor L. E. Dickson, Professor G. C. Edwards, Professor M. W. Haskell, Professor L. M. Hoskins, Dr. Frank Irwin, Professor D. N. Lehmer, Professor W. A. Manning, Professor H. C. Moreno, Professor C. A. Noble, Professor E. W. Ponzer, Professor T. M. Putnam, Dr. Pauline Sperry, Mr. J. S. Taylor.

Professor Manning and Dr. Bernstein were elected chairman and secretary, respectively, for the ensuing year. Professors Lehmer and Blichfeldt and Dr. Bernstein were elected members of the programme committee.

The next meeting of the Section will be held at Stanford University, April 6, 1918. The succeeding fall meeting will be held at the University of California on October 26.

The following papers were presented at this meeting:

(1) Dr. B. A. BERNSTEIN: "On a numero-logical foundation for the theory of probability."

(2) Professor L. E. DICKSON: "Some unsolved problems in the theory of numbers."

(3) Professor L. M. HOSKINS: "The strain of a gravitating, compressible sphere of variable density."

(4) Professor W. A. MANNING: "On the order of primitive groups, IV."

(5) Professor R. M. WINGER: "The rational plane cubic as an application of the theory of involution."

Professor Winger's paper was read by title. Abstracts of the papers follow in the order given above.