THE OCTOBER MEETING OF THE AMERICAN MATHEMATICAL SOCIETY.

THE two-hundredth regular meeting of the Society was held in New York City on Saturday, October 26, 1918. War conditions and the prevailing epidemic cooperated to reduce the attendance, which included the following eleven members:

Professor E. W. Brown, Professor F. N. Cole, Professor T. S. Fiske, Mr. T. C. Fry, Professor O. E. Glenn, Mr. S. A. Joffe, Professor Edward Kasner, Professor C. J. Keyser, Professor P. H. Linehan, Professor R. L. Moore, Professor H. W. Reddick.

Professor E. W. Brown presided at the morning session, and Professor O. E. Glenn at the afternoon session. The Council announced the election of the following persons to membership in the Society: Professor R. A. Arms, Juniata College; Professor M. D. Earle, Furman University; Professor Ernest Flammer, Queen's University; Professor Gillie A. Larew, Randolph-Macon Woman's College; Dr. Flora E. Le Stourgeon, Mt. Holyoke College; Professor John Matheson, Queen's University; Dr. F. R. Morris, University of California; Professor Susan M. Rambo, Smith College; Dr. W. G. Simon, Adelbert College. One application for membership in the Society was received.

A committee was appointed to audit the accounts of the Treasurer for the current year. A list of nominations for officers and other members of the Council was adopted and ordered printed on the official ballot for the annual election.

A committee was also appointed to collect subscriptions from members of the Society and others for the purpose of establishing at Harvard University a suitable memorial of the late Professor Maxime Bôcher, President of the Society in 1909–1910.

The Chicago meeting in the Christmas holidays was designated by the Council as the annual meeting of the Society for this year, the usual eastern meeting being cancelled. Members attending the Baltimore meeting of the American Association were invited to read papers before Section A, having previously registered titles and abstracts with the Secretary for record in the report of the annual meeting.

The Southwestern Section decided not to hold its usual meeting this year. The February, 1919, meeting of the Society will also be omitted. The usual meetings will be held in April.

The following papers were read at the October meeting:

(1) Dr. J. E. McAtee: "The transformation of a regular group into its conjoint."

(2) Professor E. W. Chittenden: "On the Heine-Borel property in the theory of abstract sets."

(3) Professor Gillie A. Larew: "Necessary conditions for the problem of Mayer in the calculus of variations."

(4) Professor D. M. Y. Somerville: "Quadratic systems of circles in non-euclidean geometry."

(5) Professor M. B. Porter: "Derivativeless continuous functions."

(6) Dr. G. H. Hallett, Jr.: "Concerning the definition of a simple continuous arc."

(7) Professor R. L. Moore: "A characterization of Jordan regions by properties having no reference to their boundaries."

(8) Professor R. L. Moore: "Concerning simple continuous curves."

(9) Professor Edward Kasner: "Fields of force and Monge equations."

Dr. Hallett's paper was communicated to the Society through Professor R. L. Moore. In the absence of the authors the papers of Dr. McAtee, Professor Chittenden, Professor Larew, Professor Somerville, Professor Porter, and Dr. Hallett were read by title. Abstracts of the papers follow below.

1. The statement that there is a transformation of order 2 that transforms a regular substitution group into its conjoint occurs in the literature. In this paper Dr. McAtee exhibits such a transformation.

2. Professor Chittenden's paper appeared in full in the November Bulletin.

3. Since A. Mayer formulated that problem of the calculus of variations usually called by his name, considerable attention has been given to the deduction of the Euler-Lagrange equations of the problem as a first necessary condition. Little attention, subsequent to the original paper of Mayer, has been

paid to the necessary conditions analogous to those of Legendre and Jacobi or to the deduction of a necessary condition analogous to that of Weierstrass. It is the purpose of Professor Larew's paper to inquire systematically into the question of necessary conditions. In the investigation of the corner-point condition for the so-called "discontinuous solutions," the theorem on the necessary condition of Euler is extended to include arcs which are continuous, but which may have a finite number of corners. A formulation and proof is supplied for the necessary condition of Weierstrass, and the Legendre condition follows directly. The Jacobi condition is deduced in much more simple fashion than usual by an application of the Euler equations and the corner-point condition to the second variation.

4. Professor Somerville's paper discusses the quadratic systems of circles with one or two parameters in non-euclidean geometry. The general one-parameter system consists of one of three systems of circles all having double contact with a fixed conic K. The limiting points are a pair of foci, and the limiting lines are a pair of focal lines of K. Coaxal and homocentric systems are degenerate cases, and a third degenerate case consists of a system of equal circles. The two-parameter quadratic system is found to be composed of one-parameter systems whose conic envelopes all have double contact with a fixed conic. A specialized system is noteworthy in which every circle cuts a fixed circle orthogonally; another system, reciprocal to this, has the property that every circle is tangentially distant a quadrant from a fixed circle.

5. Professor Porter's paper deals with continuous functions which either have nowhere a derivative or only possess derivatives inside an interval or at a point. A simple method is developed for constructing such functions in a large variety and it is shown that in the case of Weierstrass's type the condition that a be an odd integer is unnecessary.

6. In a paper entitled "Curves in non-metrical analysis situs with an application in the calculus of variations," *American Journal of Mathematics*, volume 33 (1911), pages 285–326, N. J. Lennes gives the following definition of a simple continuous arc. "A continuous simple arc connecting two points

A and B, $A \neq B$, is a bounded, closed, connected set of points $[A]$ containing A and B such that no connected proper subset of $[A]$ contains A and B." The purpose of Dr. Hallett's paper is to show that the word "bounded" in this definition is superfluous.

7. Schoenflies* has formulated a set of conditions under which the common boundary of two domains will be a simple closed curve. A different set has been given by J. R. Kline (cf. this BULLETIN, July, 1918, page 471). Carathéodory† has secured conditions under which the boundary of a single domain will be such a curve. In each of these treatments conditions are imposed (1) on the boundary itself, (2) regarding the relation of the boundary to the domain or domains in question. In the present paper Professor Moore establishes the following theorem in which all the conditions imposed are on the domain R alone:

In order that a simply connected limited two-dimensional domain R should have a simple closed curve as its boundary it is necessary and sufficient that R should be uniformly connected im kleinen.

A point set M is said to be uniformly connected im kleinen‡ if for every positive number e there exists a positive number δ_e such that if P_1 and P_2 are two points of M at a distance apart less than δ_e they lie together in a connected subset of M every point of which is at a distance less than e from P_1.

8. Various definitions have been given of simple continuous arcs, simple closed curves and open continuous curves. In the present paper Professor Moore defines these terms from a point of view which, as far as he knows, is different from any hitherto employed for this purpose. He makes use of the notion of the boundary of a point set with respect to a point set that contains it. If M is a proper subset of N, a boundary point of M with respect to N is a point which is a point or a limit point of M and also a point or a limit point of $N - M$.

Definition 1. A simple continuous closed curve is a continuous§ point set M every continuous proper subset of which has two and only two boundary points with respect to M.

* *Göttinger Nachrichten*, 1902, p. 185.

† *Math. Annalen*, vol. 73 (1912–13), p. 366.

‡ Cf. H. Hahn, *Jahresbericht der Deutschen Mathematiker-Vereinigung*, vol. 23 (1914), pp. 318–322.

§ A set of points is said to be continuous if it is closed and connected.

Definition 2. A simple continuous open curve is a continuous point set M every continuous bounded subset of which has just two boundary points with respect to M.

Definition 3. If A and B are two distinct points, a simple continuous arc from A to B is a continuous point set M containing A and B such that (1) no continuous subset of M has more than two boundary points with respect to M, (2) a subset K of M has just two boundary points with respect to M if and only if K contains neither A nor B.

He shows that these point sets may also be defined in the following simple manner.

Definition 1′. A simple continuous closed curve is a continuous bounded point set which is disconnected by the omission of any two of its points.

Definition 2′. A simple continuous open curve is a continuous point set which is disconnected by the omission of any one of its points.

Definition 3′. If A and B are two distinct points, a simple continuous arc from A to B is a continuous bounded point set which contains A and B and is disconnected by the omission of any one of its points other than A or B.

Every simple closed curve as defined in 1 or 1′ is in one-to-one continuous correspondence with a circle. Every simple continuous open curve as defined in 2 or 2′ is in one-to-one continuous correspondence with a straight line and every simple continuous arc as defined in 3 or 3′ is in one-to-one continuous correspondence with a linear interval. As has been shown by Dr. G. H. Hallett, Jr., in Lennes's definition of a simple continuous arc the condition that the point set in question should be *bounded* is superfluous. In Definition 3′ this is not the case.

9. After observing that the ∞^5 trajectories defined by a general field of force in space cannot all satisfy a Monge equation, Professor Kasner raises the question under what conditions this is possible for ∞^4 of the trajectories. It is shown that the Monge equation must be linear (that is, Pfaffian) and that it must define a null system. The direction of the force vector at any point must be in the null system. All forces of this type are determined explicitly.

F. N. Cole,
Secretary.