BULLETIN OF THE

AMERICAN MATHEMATICAL SOCIETY.

THE SEATTLE MEETING OF THE SAN FRANCISCO SECTION.

A SPECIAL meeting of the San Francisco Section of the American Mathematical Society was held at Seattle, on Thursday and Friday mornings, June 17–18, in connection with the meetings of the Pacific division of the American Association for the Advancement of Science. Professor Blichfeldt, chairman of the Section, presided at the Thursday morning session, Professor Moritz temporarily filling the chair during the presentation of Professor Blichfeldt's paper. At the Friday morning session Professor Moritz presided. In the absence of Professor Bernstein, Professor Bell acted as temporary secretary.

The attendance included the following twelve members of the Society:

Professor E. T. Bell, Professor H. F. Blichfeldt, Professor A. F. Carpenter, Professor E. C. Colpitts, Professor G. I. Gavett, Dr. G. F. McEwen, Professor W. E. Milne, Professor R. E. Moritz, Professor L. I. Neikirk, Dr. L. L. Smail, Dr. A. R. Williams, Professor R. M. Winger.

The following papers were presented at this meeting:

(1) Professor W. F. DURAND: "Some points of contact between mathematics and applied science."

(2) Professor H. F. BLICHFELDT: "On the approximate representation of irrational numbers, and the theory of geometry of numbers."

(3) Professor E. T. BELL: "An arithmetical dual of Kummer's quartic surface."

(4) Dr. T. H. GRONWALL: "On the Minkowski volume and surface theory."

(5) Professor A. F. CARPENTER: "Congruences associated with a ruled surface."

(6) Professor FLORIAN CAJORI: "Moritz Cantor, the historian of mathematics."

(7) Professor L. I. NEIKIRK: "The functional variable. Second and higher derivatives."

(8) Dr. L. L. SMAIL: "Notes on some points in the theory of summable series."

(9) Dr. L. L. SMAIL: "Bibliography of the theory of summable series."

(10) Professor E. T. BELL: "An image in four dimensional lattice space of the theory of the elliptic theta functions."

(11) Professor E. T. BELL: "Certain arithmetical consequences of the equation of three terms in elliptic functions."

(12) Professor E. T. BELL: "Systems of higher determinate equations."

(13) Professor W. E. MILNE: "Infinite systems of vectors."

Professor Durand was introduced by Professor Blichfeldt. In the absence of the authors, Dr. Gronwall's paper was read by title and Professor Cajori's was read by Professor Blichfeldt. Professor Bell's last three papers were also read by title. Abstracts of the papers follow below.

1. Professor Durand's paper first discusses the types of problem that arise in applied science with especial reference to their mathematical requirements, and suggests certain lines for the classification of such problems. Following this general view of the subject, mention is made of some specific problems for the treatment of which adequate mathematical means are still lacking. This section of the paper is intended to suggest to mathematicians needs for the further development of mathematical methods and means of analysis in order to treat adequately actual problems that arise in various fields of applied science.

2. Professor Blichfeldt discusses certain fundamental theorems in the geometry of numbers, after which the approximate solution in integers of a system of linear equations is taken up, in particular of the system $x_i + a_i - x_1 x_{n+1} + b_i = 0$ $(i = 1, 2, \cdots, n)$. The results of Kronecker, Hermite and Minkowski are compared, and a recent theorem on this system by the author is stated.

3. If E is a homogeneous algebraic equation in x, y, z, w, it represents a surface S when the variables are point coordi-

nates; if the variables are interpreted arithmetically, E can be made to represent a system S' of theorems concerning integers. When S implies S' and S' implies S, Professor Bell calls S, S' arithmetical duals of each other, and determines an S' for Kummer's S, such that all the geometrical properties of S imply and are implied by systems of arithmetical theorems, which in turn may be interpreted in terms of lattice configurations lying upon two concentric spheres in space of sixteen dimensions.

5. Paul Serret* has proved the theorem "the double-ratio of the four points in which any generator of a ruled surface intersects four fixed asymptotic curves is constant." Analogously, any four directrix curves of a ruled surface which cut all the generators in four points of constant double-ratio may be called a Serret set. Let C_y, C_z, C_μ, C_ν be any four directrix curves of a ruled surface S, P_y, P_z, P_μ, P_ν the points where they cut any generator g, and T_y, T_z, T_μ, T_ν the tangents to C_y, C_z, C_μ, C_ν at these points. T_y, T_z, T_μ determine a quadric $K\theta$. Professor Carpenter's paper proves that T_ν will lie upon $K\theta$ and be a ruling of the same kind as T_y, T_z, T_μ if and only if C_y, C_z, C_μ, C_ν constitute a Serret set, and that there is a one-parameter family of such quadrics $K\theta$ associated with each generator g of S. The generator g belongs to one regulus of each quadric $K\theta$. The paper next considers one of the quadrics $K\theta$ whose equation is invariant in form. One of these quadrics is associated with each generator of S, and this family of quadrics gives rise to two congruences Δ_1 and Δ_2 whose properties are investigated.

6. Professor Cajori gives a biographical sketch of Moritz Cantor and an estimate of his work as a historian.

7. Volterra and others have given implicit definitions for a function of a line and its first derivative. Professor Neikirk, following the method of one of his previous papers, gives explicit definitions of second and higher derivatives. Among other results, he gives the following classification of functions of a functional variable: (1) The first derivative is a point function. The second derivatives are zero or infinite. For functions of this class the methods of this paper may be

* P. Serret, Théorie nouvelle géométrique et mécanique des Lignes à double Courbure, Paris, Bachelier, 1860.

used to derive the second variation as used in the calculus of variations. (2) The first derivative is a function of a functional variable. The second derivatives are the results of repeated operations. (3) The first derivative is a mixed, point and functional variable, function. The second derivatives are the results of the repeated operations except for the case F_{ii}, which is infinite.

8. In Dr. Smail's first paper, a number of minor gaps in the systematic treatment of the theory of summable divergent series are filled in, and new proofs of some familiar theorems are given.

9. Dr. Smail's second paper gives a bibliography of the theory of summable series.

10. In this paper Professor Bell proves eleven geometrical theorems concerning point configurations upon a sphere in lattice space of four dimensions which imply, and are implied by, the theory of the elliptic theta functions.

11. Extending Liouville's theorems in the fifth of his memoirs on general formulas useful in the theory of numbers, Professor Bell shows that similar general formulas exist for a pair of bilinear forms each in four variables, and finds a complete set of 57 such formulas.

12. Professor Bell shows in this paper by two detailed examples how the general formulas of the kind given in the preceding paper may be used to derive information concerning the number of solutions of certain systems of indeterminate equations of any degree.

13. In this paper Professor Milne shows that corresponding to every infinite set of vectors of finite norm in infinitely many dimensions there exists an infinite set of constants λ_i ($i = 1, 2, \cdots$) with the following properties: The necessary and sufficient condition that (*a*) the system be orthogonal is that $\lambda_i = 1$ for every i; (*b*) the system have an adjoint is that $\lambda_i > 0$ for every i; (*c*) the system be essentially linearly dependent is that $\lambda_i = 0$ for some i. Applications are made to infinite matrices and to infinite sets of linear equations.

E. T. BELL,
Acting Secretary of the Section.