SHORTER NOTICES.

Die Quadratur des Kreises. By Eugen Beutel. Leipzig and Berlin, Teubner, 1920. 56 pp.

After stating clearly what is meant by a ruler and compass construction, the author gives a very complete history of this famous problem, which baffled mathematicians for twenty-five hundred years. He divides the history into three periods, the *geometric* period, the period of infinite series, the algebraic period. The first era, in which men tried to solve the problem by purely geometric methods, includes the work done by the Egyptians, Babylonians, Greeks, Romans, Hindus, Chinese, and the Christian nations up to the time of Newton. The work of Archimedes and of van Ceulen are the outstanding contributions in this period. When we stop to think that Archimedes was unfamiliar with the Arabic numerals and the decimal notation, we appreciate the magnitude of his task of computing the perimeter of the inscribed and circumscribed polygon of 96-sides in a circle of radius R. He found $3\frac{1}{7} < \pi < 3\frac{10}{71}$. Ludolf van Ceulen (1539–1610) carried Archimedes process of approximation still further and obtained the value of π correct to 35 decimal places. It is in honor of this contribution that the Germans today often refer to π as the Ludolfian number.

The second period contains the work of such men as Newton, Leibnitz, Gregorius von St. Vincentius, Kepler, Fermat and John Wallis. By means of infinite series these men were able to compute the value of π to several hundred decimal places, but they were unable to solve the problem. However, their investigations led them to believe that the problem was incapable of solution.

It was then shown that if the problem is solvable, π must not be transcendental. In 1873 Hermite proved that e was transcendental. Making use of this fact and of Hermite's method of proof, Lindemann, in 1882 proved that π was transcendental, settling forever the famous problem.

The book is well written and is well worth being read by every teacher of plane geometry.

FRANK M. MORGAN.