an exhaustive treatment, but to bring the reader up to and a little beyond the fundamental theorem of Dedekind that the separation of an ideal into the product of prime ideals is unique.

The subjects considered beyond the work necessary for the proof of the theorem mentioned are: the theory of the classification of ideals showing that the number of classes is finite, and the theory of units in an algebraic number field, with the proof of the existence of a fundamental system of units.

The second part is devoted to the analytic theory of ideals. The first chapter gives a clear introduction to the functions $\zeta(s)$ and $\zeta(s;\kappa)$, where κ is a class of the field. It is shown that, except for a pole of the first order in $s = 1$, the functions are regular in the entire plane, and that the residue of $\zeta(s;\kappa)$ in $s = 1$ is a number λ, independent of the class κ, while that of $\zeta(s)$ is λh, where h is the number of classes. Hecke's functional equations for $\zeta(s)$ and $\zeta(s;\kappa)$ are also developed.

The second chapter is a study of the distribution of the zeros of $\xi(s)$, and the third chapter leads to the proof of the author's remarkable theorem that, asymptotically, the number of prime ideals is the same in all fields.

The last chapter contains the result of the author's researches regarding the number of ideals, in a field or a class, whose norms are less than x. If this number be denoted by $H(x)$ for the field and $H(x;\kappa)$ for a class, it is shown that

$$H(x;\kappa) = \lambda x + O(x^{\vartheta})$$
$$H(x) = \lambda h x + O(x^{\vartheta})$$

where λ and h have the meaning given above,

$$\vartheta = 1 - \frac{2}{n+1}$$

and $O(x^{\vartheta})$ is a function whose quotient by x^{ϑ} is limited for sufficiently large x. It is shown that the exponent ϑ cannot be less than

$$\frac{1}{2} - \frac{1}{2n}.$$

The last four pages contain a brief historical survey with examples from a quadratic field, as well as notes of reference to the literature bearing on the various sections.

G. E. Wahlin

Lehrbuch der Funktionentheorie. By L. Bieberbach. Band I. *Elemente der Funktionentheorie.* Leipzig, Teubner, 1921. 6 + 314 pp.

Funktionentheorie. By L. Bieberbach. Teubner's Technische Leitfaden, Bd. 14. Leipzig, Teubner, 1922. 118 pp.

The author in his preface to the first book pleads guilty to entertaining the hope that he has written a text-book on complex function theory. He proceeds to set forth the qualities that such a text-book should have, viz: completeness, clarity, simplicity and unity. That the author succeeds in giving a clear, elementary presentation of the fundamental principles of the theory of functions of a complex variable cannot be gainsaid.

The table of contents indicates the usual order of topics.

1. Elementary treatment of complex numbers as number pairs, treated arithmetically and geometrically. 2. Limits and series, containing the fundamental concepts and definitions. 3. Continuity, region, derivatives, series of functions, conformal mapping. 4. The elementary functions of a complex variable. 5. Integration. 6. Cauchy's integral formula. 7. Theory of residues. 8. Analytic continuation. 9 and 10. Algebraic functions. 11. Elliptic functions. 12. Simple periodic functions. 13. Expansion of analytic functions in infinite series and products. 14. The Gamma function.

The printing is very good, unusually free from errors. The reviewer wishes to point out that on page 34 in the paragraph on the inverse function defined by $w = f(z)$ the requirement $f'(z_0) \neq 0$ should be added. Again on page 35 in the paragraph in which Laplace's differential equation is derived, the condition necessary to make the equation $v_{xy} = v_{yx}$ valid is not stated.

The eighty figures in the book are illuminating and helpful. Perhaps the chief contribution is contained in the use of the modern point-set theory in the fundamental definitions. This is a logical development of the fundamental importance which this theory is rapidly assuming in all branches of mathematics, especially in function theory. Two important theorems are to be noted, the first an extension of Weierstrass's theorems on a uniformly convergent series of analytic functions and an extension of Mittag-Leffler's theorem. The first, on page 165, is due to Vitali, and the second, on page 292, is due to Runge. The book as a whole is a welcome addition to complex function theory and marks a distinct contribution.

The second book is a simplified summary of the Lehrbuch and possesses the same merits as the larger text. The last two chapters contain applications to potential theory and hydrodynamics.

H. J. Ettlinger

Vorlesungen über Höhere Mathematik. By Hermann Rothe. Vienna, L. W. Seidel und Sohn, 1921. xi + 691 pp.

This book is strikingly like the first volume of Pierpont's *Theory of Functions of a Real Variable,* both in scope and in the spirit of rigor which pervades it. Its larger, more closely printed pages contain much more detail of proof, but beyond a considerable wealth of illustrative examples and some geometric applications of the calculus, the material and the degree of generality of the theorems are almost identical. At many points, the reader will find the proofs easier to study than those of Pierpont because every step is supplied. But at other points, he will find this same trait wearying and obscuring to the real essence of the demonstration. It is not likely that the author had any knowledge of the *Theory of Functions of a Real Variable,* for in supposing that he might be offering the first valid proof of the theorem on the derivative of a function of a function (see the preface), the only place in which he claims novelty of results, he overlooks the satisfactory proof given in that book.

The fact that these *Vorlesungen* are an elaboration of lectures given in a prescribed course in the Technische Hochschule at Vienna will mislead