

1. Elementary treatment of complex numbers as number pairs, treated arithmetically and geometrically. 2. Limits and series, containing the fundamental concepts and definitions. 3. Continuity, region, derivatives, series of functions, conformal mapping. 4. The elementary functions of a complex variable. 5. Integration. 6. Cauchy's integral formula. 7. Theory of residues. 8. Analytic continuation. 9 and 10. Algebraic functions. 11. Elliptic functions. 12. Simple periodic functions. 13. Expansion of analytic functions in infinite series and products. 14. The Gamma function.

The printing is very good, unusually free from errors. The reviewer wishes to point out that on page 34 in the paragraph on the inverse function defined by  $w = f(z)$  the requirement  $f'(z_0) \neq 0$  should be added. Again on page 35 in the paragraph in which Laplace's differential equation is derived, the condition necessary to make the equation  $v_{xy} = v_{yx}$  valid is not stated.

The eighty figures in the book are illuminating and helpful. Perhaps the chief contribution is contained in the use of the modern point-set theory in the fundamental definitions. This is a logical development of the fundamental importance which this theory is rapidly assuming in all branches of mathematics, especially in function theory. Two important theorems are to be noted, the first an extension of Weierstrass's theorems on a uniformly convergent series of analytic functions and an extension of Mittag-Leffler's theorem. The first, on page 165, is due to Vitali, and the second, on page 292, is due to Runge. The book as a whole is a welcome addition to complex function theory and marks a distinct contribution.

The second book is a simplified summary of the Lehrbuch and possesses the same merits as the larger text. The last two chapters contain applications to potential theory and hydrodynamics.

H. J. ETTLINGER

*Vorlesungen über Höhere Mathematik.* By Hermann Rothe. Vienna, L. W. Seidel und Sohn, 1921. xi + 691 pp.

This book is strikingly like the first volume of Pierpont's *Theory of Functions of a Real Variable*, both in scope and in the spirit of rigor which pervades it. Its larger, more closely printed pages contain much more detail of proof, but beyond a considerable wealth of illustrative examples and some geometric applications of the calculus, the material and the degree of generality of the theorems are almost identical. At many points, the reader will find the proofs easier to study than those of Pierpont because every step is supplied. But at other points, he will find this same trait wearying and obscuring to the real essence of the demonstration. It is not likely that the author had any knowledge of the *Theory of Functions of a Real Variable*, for in supposing that he might be offering the first valid proof of the theorem on the derivative of a function of a function (see the preface), the only place in which he claims novelty of results, he overlooks the satisfactory proof given in that book.

The fact that these *Vorlesungen* are an elaboration of lectures given in a prescribed course in the Technische Hochschule at Vienna will mislead

no one in this country into thinking that they can be of large service to technical students here. We appreciate rigor, to be sure, and feel the need of more of it. But we want also some analytical skill, we want some treatment of series, including Fourier series, and of approximation processes, and we want practice in the application of the calculus to problems of physics and engineering. And we do not care particularly for unusual generality at the expense of simplicity of proofs.

For the student of pure mathematics, on the other hand, the book has value. He may consult it with confidence, and will find it useful in supplying alternative proofs, or in supplying details of proofs given too succinctly elsewhere. He will find it well printed, well arranged, and supplied with figures which are models of clearness, if exception be made of a few, which, like the text, are overloaded with detail.

It is regrettable that so many mathematical texts are written without any clear purpose of stimulating self-activity. In the *Vorlesungen* there is not a single problem to be worked, and no detail of proof to be supplied. Many a mathematician owes his interest in the science to early bouts with problems, and the writer who gives careful attention to the selection of problems, or who includes as exercises valuable theoretical results, renders a high service. Both of these merits characterize, for instance, Appell's *Mécanique Rationnelle*, and Goursat's *Cours d'Analyse*. Had our present author followed some such course, he might have saved space, and produced a much more stimulating book.

O. D. KELLOGG

*Plane Geometry.* By L. B. Benny. Glasgow and Bombay, Blackie and Son, Ltd., 1922. vi + 336 pp.

The title of this book is misleading to American readers. The sub-title is as follows: "An account of the more elementary properties of conic sections treated by the methods of coordinate geometry and of modern projective geometry with applications to practical drawing." The contents will appeal to the American reader as a rather unusual mixture of elementary analytic geometry and projective geometry. In addition to the ordinary analytic treatment of the straight line and conic sections, the book contains chapters on ranges and pencils, harmonic properties of circles, inversion, projection, confocal conics, cross ratios and ends with a treatment of Pascal's and Brianchon's theorems. While the selection of contents is an unusual one in this country, the reader will find much of interest. Like most English texts, it is well supplied with exercises and problems of a type and difficulty unusual on this side of the Atlantic. The book also contains a number of portraits of mathematicians who have contributed to the subjects under discussion with an appendix giving biographical notes concerning them. These include Cayley, Riemann, Cremona, Descartes and Pascal.

J. W. YOUNG