

## VOLUME III OF LIE'S MEMOIRS

*Sophus Lie's Gesammelte Abhandlungen (Samlede Avhandlingar)*. Edited by Friedrich Engel and Poul Heegaard. Volume III. *Abhandlungen zur Theorie der Differentialgleichungen, erste Abteilung (Avhandlingar til Differentialligningernes Teori, første Avdeling)*, edited by Friedrich Engel. Leipzig, B. G. Teubner, and Kristiania, H. Aschehoug and Co., 1922. xvi + 789 pages.

Twenty-three years after the death of Sophus Lie appears the first volume to be printed of his collected memoirs. It is not that nothing has been done in the meantime towards making his work more readily available. A consideration of the matter was taken up soon after his death but dropped owing to the difficulties in the way of printing so large a collection as his memoirs will make. An early and unsuccessful effort to launch the enterprise was made by the officers of Videnskapselskapet i Kristiania; but plans did not take a definite form till 1912; then through the Mathematisch-physische Klasse der Leipziger Akademie and the publishing firm of B. G. Teubner steps were taken to launch the project. Teubner presented a plan for raising money by subscription to cover a part of the cost of the work and a little later invitations to subscribe were sent out. The responses were at first not encouraging; from Norway, the homeland of Lie, only three subscriptions were obtained in response to the first invitations.

In these circumstances, Engel, who was pressing the undertaking, resorted to an unusual means. He asked the help of the daily press of Norway. On March 9, 1913, the newspaper *TIDENS TEGN* of Christiania carried a short article by Engel with the title *Sophus Lies samlede Afhandlingar* in which was emphasized the failure of Lie's homeland to respond with assistance in the work of printing his collected memoirs. This attracted the attention of the editor and he took up the campaign: two important results came from this, namely, a list of subscriptions from Norway to support the undertaking and an appropriation by the Storting to assist in the work. By June the amount of support received and promised was sufficient to cause Teubner to announce that the work could be undertaken; and in November the memoirs for the first volume were sent to the printer, the notes and supplementary matter to be supplied later.

The Great War so interfered with the undertaking that it could not be continued, and by the close of the war circumstances were so altered that the work could not proceed on the basis of the original subscriptions and understandings and new means for continuing the work had to be sought. Up to this time the work had been under the charge of Engel as editor. But it now became apparent that the publication of the memoirs would have to become a Norwegian undertaking. Accordingly, Poul Heegaard became associated with Engel as an editor. The printing of the work became an enterprise not of the publishers but of the societies which support them in this undertaking. Under such circumstances the third volume of the series, but the first one to be printed, has now been put into our hands. "The printing of further volumes will be carried through gradually

as the necessary means are procured; more I cannot say about it," says Engel, "because the cost of printing continues to mount incessantly."

The general plan contemplates the publication of Lie's memoirs in six volumes while a seventh volume is to be devoted to the principal works found among Lie's literary remains. Volumes I and II are to contain the memoirs on geometry; volumes III and IV, those on differential equations; and volumes V and VI, those on transformation groups. Naturally these three divisions are not rigorously separated one from another; in fact, in most of the memoirs in a given part appears matter relating also to the other two parts. In each part the memoirs are arranged chronologically; for the most part, the material in the first of two related volumes is that which was first published at Christiania, while the material in the second is taken principally from the *MATHEMATISCHE ANNALEN* and the publications of the Leipzig Akademie. In this way an arrangement is effected by which two articles never appear in the same volume one of which is a reworking of the other. Every memoir already has its definite place assigned to it in the completed work and a definite number in the volume to which it belongs is given to it. In this way it becomes possible to give cross references from volume to volume without any possibility of confusion. In the (unexpected) event of the discovery of an article at first overlooked, it will be assigned to its chronological position, with a number followed by a star, so as not to disturb the numbering of the other articles.

To facilitate the examination of references by means of the collected works and without the use of the articles in their original place of publication, the exact paging of the original articles is indicated by means of numbers inserted in the lines at the appropriate places. This is a feature of convenience which is to be commended. By means of it the paging of the original is easily determined from the collected memoirs.

The publication is started with the third volume because the letters of Lie furnish such an abundant material for the explanation of the memoirs in it; and this material has been used freely. The letters to A. Mayer have been particularly useful in this respect. Besides the material gathered from the letters there is a rich copiousness of notes and explanations by Engel. These have been prepared with the purpose of making the memoirs in this volume more readily understood by the reader. The editor anticipates that the reader may sometimes find these too extensive; but, in case this evil is found to exist in them, he has thought that it is much less than the possible opposite evil of notes which are not sufficiently full. The theory of partial differential equations of the first order and of Pfaff's problem was greatly modified by the researches of Lie. Concerning the elucidation of these researches in the notes Engel speaks as follows in his preface: "Ich habe nun versucht auseinanderzusetzen, wie Lie etwa ursprünglich zu seinen Sätzen gelangt ist, namentlich habe ich mich bemüht, für die Theorie der partiellen Differentialgleichungen 1.O. nach Möglichkeit den Standpunkt wiederherzustellen, auf dem Lie war, als er die ersten Abhandlungen des Bandes (Nr. I-IV) veröffentlichte. Obwohl ich mir bewusst war, mir damit eine äusserst schwierige, fast unlösbare Aufgabe gestellt zu haben, habe ich doch den Versuch gewagt, und ich

hoffe, in den Anmerkungen zu Abhandlung I [covering 38 pages] wenigstens zum Teil das Richtige getroffen zu haben; vollständig befriedigt bin ich auch selber noch nicht, obwohl ich gerade diese Anmerkungen ein halbdutzendmal umgearbeitet habe. Sicherer bin ich meiner Sache beim Pfaffschen Probleme; ich darf wohl behaupten, dass meine Anmerkungen zu Abhandlung XI [16 pages] den ursprünglichen Gedankengang Lies im wesentlichen richtig wiedergeben."

The memoirs of Lie reprinted in the present volume cover 562 of the total 805 pages. But the notes and supplementary matter are printed much more compactly than the memoirs themselves, so that the former make up distinctly more than one third of the whole volume. This additional material has been prepared with great care and with the convenience of the reader always in mind. The articles originally printed in Norwegian appear now in a German translation. The few notes in French are printed in that language except in the case of notes which Lie himself had published both in French and in German, these latter being reprinted in German with a reference also to the place of publication in French. All of Lie's earlier work placed by the editors under differential equations is therefore readily available to all who read German (the material in the French notes being only brief abstracts); and this is placed before the reader in convenient form with an ample richness of explanatory material.

In his preface Engel insists upon certain qualities of Lie's work—qualities which partly explain the copiousness of the notes which he has inserted in the present volume. The ideas and points of view which Lie possessed and never adequately made known to others were irretrievably lost with his death. In his printed works there are many suggestions which were not followed up by him. These are scattered, and are largely inaccessible. When his complete works are before us in convenient form, these fragmentary suggestions can be seen in their totality and they will throw light one upon another. In this way Lie will become his own interpreter. This will be particularly true when the material in the letters is brought to bear upon the researches to which its parts refer. It seems important therefore to bring the scattered suggestions of Lie together in connection with the memoirs to which they may properly be attached.

The editor's careful work in preparing this volume for the printer and in seeing it through the press has evidently been for him a labor of love; in performing it he has rendered a great service to all those who will have occasion to use the memoirs of Lie which are reprinted in this volume. At present the intention is that the fifth volume shall follow the third; but no definite promises are being made owing to the difficulties which we have already named. The memoirs for the fifth volume are now ready for the printer; but the notes for it will be prepared while the volume is in course of being printed. It is impossible yet to tell what the extent of these notes will be; but the editor states that they will be far less extensive than those in the present volume. All mathematical libraries will of course procure copies of the volume now printed. It is to be hoped also that every individual who can utilize a private copy of the volume effectively will purchase one so as to facilitate the publication of the next volumes.

R. D. CARMICHAEL