THE AMHERST COLLOQUIUM

A short program, five lectures on one subject instead of the usual two series by different speakers, proved nevertheless decidedly attractive. The list of those registering for Professor A. B. Coble's Colloquium numbered seventy-seven, nearly all being present on Tuesday, September 4th, at the opening lecture. The college opened the large lecture-room in Appleton Hall, with ample blackboard space and excellent acoustic properties. Professor H. L. Rietz, Vice-president of the Society, presided and introduced the lecturer. As it is expected that the lectures will be published promptly in the Society's Colloquium Series, no full account of the contents is here necessary. Excerpts from the quite full syllabus and from a supplementary sketch kindly furnished by Dr. Coble will suggest the extensive résumé and new researches, which sum up a line of the lecturer's publications extending back at least to 1915. Other authors cited were principally Stahl, Schottky, Picard, Cayley, Frobenius, and the Encyklopädie.

The title of the series was *The determination of the tritangent planes of the space sextic of genus four.* The standard definition of a function theta of p variables is given, by the aid of a square array which determines its (quasi) periods. The zero points of the variables are shifted by half-periods, leading to definitions of $2^{2p}-1$ odd and even thetas and their characteristics. This is ground-work for the proper subject of the lectures, as follows.

If the original periods are replaced by new periods (integer combinations of the old and vice versa) the $2^{2p}-1$ half periods are permuted, and the 2^{2p} odd and even functions also, according to a finite group G_{NC}. A geometric representation of this group is found by taking the half periods as points in a finite space S_{2p-1}, modulo 2. The collineation group G_{NC} has then an invariant null system N and the 2^{2p} theta characteristics behave under it like quadrics whose polar systems coincide with N. The finite geometry indicates a *basis notation* in which the points (or half periods) and quadrics (or odd and even thetas) are expressed by means of $2p+2$ subscripts (Annals, 1916, p. 101).....

Products and powers of these first-order thetas aid in defining thetas of the second and higher orders, whose relations form a bridge to the algebraic and abelian functions. Not these, however, but the groups concerned and their invariants were the central object, and a variety of geometric pictures were invoked to make the theory vivid.

The geometric theory of the theta functions of genus two is in the main related to figures determined by six points. We use the three six-points: P_6^1, six points on a line; P_6^2, six points in a plane (usually on a conic); and P_6^3, six points in space on a norm cubic curve C^3. The P_6^1 and P_6^3 are *associated sets* of points,..... The P_6^2, if on a conic, is self-associated; otherwise it determines projectively an associated Q_6^2.

With this geometric background there is a projective invariant theory.....A first object is to show that the theta-relations are satisfied by virtue of the projective relations among the invariants and covariants. The invariant relations and the modular relations ($\breve{u}=0$) are identical; the same is true of the covariant relations and the theta-relations ($\breve{u}\neq 0$). A second object is to show that the period transformations of the functions appear in the geometry as the passage from one set P_6 to a congruent set P_6' under Cremona transformation.....

The functions of genus three are intimately connected with the figure of seven points P_7^2 of the plane, and the eight base points P_8^3 of a net of quadrics in space. These eight base points P_8^3 are congruent under Cremona transformation to 36 such 8-points and give rise to a Cremona group of order $8!\cdot 36$. Also the 8-point can be expressed parametrically in terms of modular functions, and period transformations (mod 2) produce the same Cremona group. In this case both modes of approach are unified by the actual existence of a curve of genus three which defines the functions: namely, the locus of nodes of the quadrics of the net. Similarly in the case $p=4$ the set of 10 nodes of a Cayley symmetroid are congruent to $2^8 \cdot 51$ such 10-points and give rise to a Cremona group of order $2^8 \cdot 51 \cdot 10!$. Also Schottky has found that these ten nodes can be expressed parametrically in terms of modular functions of genus four. Again the period transformations (mod 2) produce the Cremona group. Thus far however a curve of genus four which defines the functions has not been discovered. The isolation of the tritangent planes of the normal space sextic curve of genus four is then dependent upon the detection of a curve of genus four attached to a symmetroid which will degenerate to a curve of genus three if either two nodes of the symmetroid coincide or four nodes are coplanar.

With two sessions on each of the first two days, the course was finished on Thursday. On Tuesday afternoon many of the participants motored to Mount Holyoke College, where the local mathematical faculty served tea and escorted the visitors to points of interest. Wednesday afternoon, Smith College colleagues were similarly hospitable. For Amherst, Professors Olds and Esty, with President Pease, were indefatigable in arrangements and personal courtesies; and the reception by President and Mrs. Pease at the Lord Jeffery Inn Tuesday evening created a nucleus for the memories of natural beauty, serene intellectuality, and pervasive friendliness which formed the setting of this twelfth Colloquium.

H. S. WHITE