FRY ON PROBABILITY

Probability and its Engineering Uses. By T. C. Fry. New York, Van Nostrand, 1928. xiv+476 pp. $7.50.

This textbook, one of the series called *Books from Bell Telephone Laboratories,* is the outgrowth of a set of notes originally prepared for one of the courses of Bell Telephone Laboratories, and subsequently revised for use in a course of lectures delivered at the Massachusetts Institute of Technology during the second term of the year 1926–27. The first six chapters contain an introduction into the mathematical theory of probability, and the five last, applications to different subjects in statistics. In eleven appendices useful tables are given.

The mathematical chapters deal with permutations and combinations, elementary principles of the theory of probability, Bernoulli's and Bayes' theorems, distribution functions and continuous variables. Chapter VII, on averages, compares the mathematical averages such as "expectation," "expected deviation" to the averages taken as results of experiment, as "mean," "mean deviation." Then follow the normal law, Poisson's law, Pearson's curves, Gram - Charlier series, Pearson's criteria of goodness of fit, applications of the theory of probability to problems of congestion and fluctuation phenomena in physics, especially the Schottky effect. Correlation and the theory of small samples are not discussed in this book.

This survey of the contents shows how much useful information is contained in this new book on probability. It is especially written for technical students. The mathematics is carefully selected and explained so as to be understandable to such a public of readers, and many examples are taken from the field of technical applications. Bayes' theorem, for instance, is illustrated by this problem showing its usefulness in an interesting way.

"A factory produces a certain type of screw as a standard product. The screws are collected at the machine in boxes of 1200 each. Long experience has shown that the proportion of these boxes which contain various percentages of bad screws is substantially as follows:

Per cent of bad screws in the box	0	1	2	3	4	5	6
Proportion of boxes observed to contain this percentage of bad screws	0.78	0.17	0.034	0.009	0.005	0.002	0.000

Two per cent badness has been adopted as a manufacturing standard; that is, any box, which contains 2 per cent or less of bad screws is regarded as satisfactory, the aim of the inspection process being to reject those which are poorer. The normal inspection consists in the examination of 50 screws out of each box. A particular box, produced at a time when there was no special reason to suspect that the machines were not operating properly, showed 6 bad screws under normal inspection. What is the probability that the manufacturing standard had not been maintained in the production of this box?"

This problem is to a certain extent typical for the book in its endeavour to connect good mathematics with technical applications. The chapter on problems of congestion, more than sixty pages, is the culmination point. There we are in the midst of the applications, mostly to telephone practice. Many mathematicians will here be astonished to see how many, how varied and how complicated, present day applications of the theory of probability to purely technical subjects are.

The author has also tried to vary the examples belonging to the ordinary theory of probability. Here he uses frequently variations of what he calls the "psychic research problem," in which a spiritualistic medium is supposed to be tested by an experiment with a certain number of black and red cards. The mathematical problem itself is simple enough. Continuous probabilities are illustrated with examples taken from the theory of gases. The Saint Petersburg problem has found its proper place in the chapter on averages.

Perhaps it is a pity that this "psychic research problem" has taken the place of some of the classical examples. De Moivre's problem, De Montmort's problem (or the problem of the drunken inhabitants of Chicago, in Coolidge's version), Fermat-Pascal's problem of the players, Bertrand's problem on the ruin of the players, cannot be omitted from a book on probability without a loss; the same holds for such a problem on continuous probabilities as the needle problem.

Sometimes there is an unsystematic arrangement of the material. Problem 8, p. 43, dealing with a test batch of lamps, leads to what Pearson has called the "hypergeometric law." This problem returns later as Ex. 26 in the psychic research problem. Then again as Ex. 28 is a "ball problem." Then at last the law is given as formula (25).

There is an excellent discussion of Bernoulli's theorem, with graphical representations that will make its meaning clear to all students. There is a clear distinction between the distribution curves dealing with the number of successes and those dealing with the ratio of the number of successes to the total number of events. Tchebycheff's generalizations are not discussed.

The treatment of continuous variables is not so good. There we have to meet the fact that the probability of hitting a set of zero measure in the continuum is zero. It has to be made clear that the theory of probabilities deals with a comparison of the measure of point sets and the corresponding theory of summation. These things are, of course, rather out of the way for the average technical student, but though the subject is touched, the idea of the set of measure zero is not clearly explained. In the same chapter we find an exposition of the change of variables in distribution functions of one and more variables, but the author does not point out clearly—as Bertrand and Poincaré do—that the choice of the variables is a matter of an a priori convention sometime urged upon us by the mechanics of the problem. We get other probabilities if we accept other variables, unless we change the weighting function.

The chapter on distribution functions contains not only an exposition of the normal law, but also of Poisson's law. The importance of this law has been recognized more and more in recent years. The deduction from the binomial law by a curious limiting process for small probabilities does certainly not account for its great use. Borel, in the last edition of his book on probabilities, has given another, more satisfactory, deduction. Fry gives still another, more general than Borel's, basing his deduction on the two conceptions of "events happening individually at random" and "events happening collectively at random." He also deduces a generalization of Poisson's law on a slightly modified basis. This introduction to Poisson's law is, we believe, a very valuable part of the book.

Pearson's curves are not very extensively discussed; only a few types are mentioned. Gram-Charlier series are introduced after a short introduction to Hermitian polynomials. There Fry's exposition can easily be supplemented by Arne Fisher's book. This book is, also generally speaking, a very valuable supplement to Fry's book, for instance in the historical part. Fry makes very few remarks of a historical character, too few perhaps; but here Arne Fisher's book gives plenty of information.

Several problems on curve fitting are elaborately worked out, fitting by means of the binomial law, by Pearson curves, by the Poisson law as well as by Gram-Charlier series. An appendix with the principal statistics of these different curves is helpful in fitting problems. On p. 305 Fry admits, with Pearson, as a valuable representation, the "binomial" distribution

$$p(n) = C_n^{12.135}(0.339325)^n(0.660675)^{12.135-n}$$

with a broken exponent 12.135 instead of 12. It is doubtful whether or not this has much of a meaning.

One of the most valuable parts of the book is the tenth chapter, on problems of congestion. It is a systematic treatment of the distributions that may occur in the practise of telephoning.

The last chapter deduces the integro-differential equation of statistical mechanics for ideal gases, which yields the H function, the Boltzmann distribution and Maxwell's law of velocities. The book ends with some of the author's investigations on the Schottky effect.

A number of exercises, some very interesting ones, give the student an opportunity to test his knowledge. For technical students we strongly recommend the book, and students of more theoretical interests will find it instructive to see how many new practical applications the old theory of Pascal and Laplace has found.

D. J. Struik