

*Automorphic Functions.* By Lester R. Ford. New York, McGraw-Hill, 1929. vii+333 pp.

This book will be welcomed by students and teachers of function theory. It is a distinct acquisition to the text-book literature in analysis.

Workers in analysis know well how hard it has been to secure a respectable knowledge of the automorphic functions. To plod through Poincaré's memoirs, or the two-volume work of Klein-Fricke is hardly an appropriate task for a mathematician not specializing directly in the subject. Recent expositions of conformal mapping and uniformization, that in Bieberbach's *Lehrbuch*, for instance, give an easy approach to certain aspects of the theory, but a short and direct treatment of properly discontinuous groups, and the functions to which they lead, has hitherto been lacking.

In the first half of his book, Professor Ford gives a very simple and elegant treatment of groups of linear transformations, their fundamental regions and the functions invariant under the groups. The account given, while not exhaustive, is quite comprehensive. All that is presupposed of the reader is a knowledge of the elements of complex variable theory. It is interesting to notice that non-euclidean geometry, which figures conspicuously in all previous treatments of the subject, is not used at all.

Professor Ford employs a method of investigation developed by himself, which is based on the systematic use of the *isometric* circles of the linear transformations, that is, the circles at which the linear functions in the transformations have derivatives of modulus unity. This method seems to have had its origin in determinations made of the fundamental region of a properly discontinuous group by J. I. Hutchinson and by Humbert. The proofs assume a surprisingly simple form. This matter deserves special emphasis. Professor Ford's work, in the first part of his book, is not mere exposition. The methods which he creates are original, and of permanent scientific value.

The exposition is remarkably clear and explicit. A possible exception to this will be found in the discussion of the theta-series for groups in which infinity is a fixed point for certain transformations.

The second half of the book gives a detailed account of conformal mapping and uniformization, and considers some of the relations of automorphic functions to differential equations. These chapters can be read with great profit. As the author remarks himself, his treatment of conformal mapping is not as brief as those in which normal families of functions are used. In the uniformization chapter, the topological work is intuitive, rather than arithmetic.

An excellent second course in complex variables can be based upon this book. Teachers who have been following up first courses in complex variables by a semester of elliptic functions will perhaps welcome a chance to give a semester of automorphic functions instead.

The firm of McGraw-Hill is to be congratulated on having published this work. Together with the author, they have done students and teachers of analysis a brilliant service.

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