

RAMSEY ON FOUNDATIONS OF MATHEMATICS

The Foundations of Mathematics, and other Logical Essays. By Frank Plumpton Ramsey. Edited by R. B. Braithwaite. Preface by G. E. Moore. (International Library of Psychology, Philosophy, and Scientific Method.) New York, Harcourt, Brace and Company; London, Kegan Paul, Trench, Trubner and Company, Ltd., 1931. xviii+292 pp. \$4.50.

Ramsey's *The Foundations of Mathematics* is a collection of eighteen papers all bearing in some way on logic. The papers range in date from 1923 to 1929, and have been edited after the death of the author in January, 1930, at age 27. Besides these writings and the Preface, the book also contains the editor's Introduction, his Bibliography of Ramsey, and his Note on Symbolism. The author was Lecturer in Mathematics in the University of Cambridge; the editor is Fellow of King's College, Cambridge; and Professor Moore, who wrote the preface, is Professor of Mental Philosophy and Logic at Cambridge.

Of Ramsey's eighteen papers, the first sixteen are divided into nine groups, I-IX; the remaining two are in the form of an Appendix and an Epilogue. Some idea of the nature of the papers in groups I-IX can be had from a mere inspection of their titles: (I) The Foundations of Mathematics; (II) Mathematical Logic; (III) On a Problem of Formal Logic; (IV) Universals; (V) Note on the Preceding Paper; (VI) Facts and Propositions; (VII) Truth and Probability; (VIII) Further Considerations—consisting of (A) Reasonable Degree of Belief, (B) Statistics, (C) Chance; (IX) Last Papers—consisting of (A) Theories, (B) General Propositions and Causality, (C) Probability and Partial Belief, (D) Knowledge, (E) Causal Qualities, (F) Philosophy.

The Appendix is a reprint of a critical review of Wittgenstein's *Tractatus Logico-Philosophicus*. The Epilogue touches lightly on a variety of subjects.

The papers are very uneven as to length and as to clearness and literary finish. Paper (B) of group (VIII) for instance, is scarcely more than a page long, and consists merely of a few rather disconnected remarks on statistics. Paper (I), on the other hand, is 61 pages in length, and is an admirably organized discussion of the logic of *Principia Mathematica* as a basis for mathematics. This unevenness in the length and in the literary quality of the papers is due to the fact that the editor has included in the book papers previously published, as well as papers never before published. It is among the hitherto unpublished writings that one finds the very short and somewhat obscure papers. Some of these are mere notes which were not intended by the author to be published in their present form, but were meant to be developed into publishable forms later.

To the mathematical reader, the papers that appear to be of special interest are (I), (II), (III), (VII), and (A) of (IX). In (I), whose title forms the principal title of the book, the author aims "to give a satisfactory account of the Foundations of Mathematics in accordance with the general method of Frege, Whitehead, and Russell. Following these authorities, I hold that mathematics is part of logic . . . I have therefore taken *Principia Mathematica* as a basis for

discussion and amendment; and believe myself to have discovered how, by using the work of Mr. Ludwig Wittgenstein, it can be rendered free from the serious objections which have caused its rejection by the majority of German authorities." The parts of the *Principia* which thus receive "discussion and amendment" concern the definition of *class*, the Axiom of Reducibility, the definition of *identity*, the Multiplicative Axiom, and the Axiom of Infinity.

In paper (II), the author discusses in outline the logical work of Weyl, Brouwer, and Hilbert, which has appeared since the publication of the *Principia* and which "claims to supersede altogether the position taken up by Whitehead and Russell as to the nature of mathematics and its logical foundations."

In (III), the author is concerned with the solution of a special case of Hilbert's *Entscheidungsproblem* in mathematical logic.

In (VII), there is developed "a purely psychological method of measuring belief," and "the theory of probability is taken as a branch of logic, the logic of partial belief and inconclusive argument."

In (A) of (IX), a mathematical theory is regarded "simply as a language for discussing the facts the theory is said to explain."

To the worker in Philosophy, the papers that should prove most interesting are those that have not hitherto been published—those in groups VII–IX. These are, in the main, the rough notes of a fertile mind on fundamental problems in philosophy.

All readers will feel, throughout the book, that they are in the presence of a penetrating intellect, a clear and effective writer, and a charming personality. It is true that Ramsey himself regards his work in connection with the *Principia*—probably his most important work—only as an essay: "although my attempted reconstruction of the view of Whitehead and Russell overcomes, I think, many of the difficulties, it is impossible to regard it as altogether satisfactory." But he has gone far in making clear to us the nature of *Principia Mathematica*. And so, we are grateful to Mr. Braithwaite for bringing close to us the writings and the personality of F. P. Ramsey.

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