ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross-references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

184. Professor B. P. Gill: *Generalization of Lucas' u_n, v_n for recurrences of third and higher order.* Preliminary report.

Initial values are given for k solutions of a homogeneous recurrence relation of order k in such a way that the solutions have algebraic properties that generalize those of the functions of Lucas. For example, the coefficients in the addition theorems expressing the functions with subscript $m+n$ as bilinear forms in those with subscripts m and n are semi-invariants of the generating polynomial of the recurrence. (Received March 22, 1935.)

185. Professor Leonard Carlitz: *On a certain function connected with polynomials in a Galois field.*

This paper continues the study of the function defined in a previous paper (abstract 39-5-161, this Bulletin, vol. 39 (1933), p. 352). Applications are here made to the properties of a class of polynomials analogous to the cyclotomic polynomials; an application of a different nature is made to the theory of the congruence $t^P - t \equiv A \pmod{P}$, where A and P are polynomials. (Received March 23, 1935.)

186. Dr. Max Zorn: *The automorphisms of Cayley's non-associative algebra.* Preliminary report.

A normal form for the automorphisms of the Cayley numbers is established, by means of a 7-dimensional vector calculus; as a corollary we obtain a result of Brandt concerning the numbers a defining automorphisms by $x \rightarrow a^{-1}xa$. Furthermore we get a rational representation of all automorphisms by six iterated transformations with suitable Cayley numbers. (Received March 14, 1935.)

187. Mr. Marshall Hall: *Indices in cubic fields.*

Dedekind first showed that one cannot always find an algebraic integer of unit index in a given field, by exhibiting fields in which there is an index divisor common to all integers of the field. In particular the greatest common index divisor in a cubic field is at most two. This paper investigates the question: is there an algebraic integer in an arbitrary field whose index is the greatest common divisor of all indices of the field? The answer is in the negative. From a study of the forms which represent the indices, it is shown that there exist cubic fields in which the least index is arbitrarily large. (Received March 20, 1935.)

188. Dr. G. C. Webber (National Research Fellow): *Concerning transcendental numbers.*

Gelfond and Schneider have proved, independently, that in general ω^α is transcendental where both ω and α are algebraic numbers. By simple generalizations of Schneider's proof we obtain results such as the following: if ω is transcendental, θ_1 and θ_2 linearly independent non-rational algebraic numbers, one of ω^{θ_1}, ω^{θ_2} is transcendental. (Received March 21, 1935.)

189. Dr. H. S. M. Coxeter and Dr. Abraham Sinkov: *The groups determined by the relations* $S^l = T^m = (S^{-1}T^{-1}ST)^p = 1$.

The usual approach to the problem of groups generated by two operators has been to assign the periods of the generators and the period of their product. The present paper studies a slightly different approach in which the period of the product of the two generators is replaced by the period of their commutator. A method is presented for representing these groups geometrically; the representation is three-dimensional and has for its fundamental region a tetrahedron with dihedral angles $2\pi/l$, $2\pi/m$ at two opposite edges, the four remaining dihedral angles being $\pi/2p$. The space required is spherical, euclidean, or hyperbolic according as $\sin (\pi/l) \sin (\pi/m)$ is greater than, equal to, or less than $\cos (\pi/2p)$. The group is generated by rotations about the two special edges of the tetrahedron and is finite only when the space is spherical. A complete solution is given for all the spherical and euclidean groups; in the latter cases it is necessary to impose an additional defining relation upon the generators. This new condition determines the period of their product and suggests that the two different methods of approach be combined in the study of the hyperbolic groups. In this connection, it is shown that the four relations $S^l = T^m = (ST)^n = (S^{-1}T^{-1}ST) = 1$ are in general insufficient to determine a finite group. (Received March 22, 1935.)

190. Dr. Nathan Jacobson: *Locally compact totally disconnected rings.*

This paper is concerned with an investigation of locally compact, separable, totally disconnected rings F. We have divided it into two parts. In Part I we determine the structure of the additive group of a simple ring F (not necessarily associative or commutative). The results are as follows: If the characteristic $\chi(F) = p$, F is additively a direct sum of cyclic groups of order p. If $\chi(F) = 0$, F is additively a finite vector space over the field of p-adic numbers K_p. It follows in this case that F is a hypercomplex system with a finite basis over K_p. In Part II we restrict ourselves to associative fields F (not necessarily commutative). The methods here are arithmetic in nature. We prove the existence of a unique maximal compact and open subring R of F. By means of R a *valuation* (Bewertung) of F is defined. F is then shown to be a cyclic algebra over its centrum. This centrum may be determined as, indeed, it has been by v. Dantzig and by Hasse and Schmidt. (Received March 12, 1935.)

191. Mr. W. R. Church: *Theory of finite distributive free structures.*

A structure is a system of elements in which equality and two binary operations, cross-cut and union, are defined and for which the usual axioms (see Dedekind, Werke, II, pp. 236–271) are postulated. This paper is concerned with structures which satisfy the distributive laws for these operations. The existence of the free distributive structure generated by means of n elements is shown by obtaining a necessary and sufficient condition for the independence of n elements, and then giving an explicit realization of this condition by means of certain elements in a finite Boolean algebra of minimal dimension. Among other things this approach furnishes the means whereby some details of the description of the structures of this class are investigated. (Received March 23, 1935.)

192. Mr. Garrett Birkhoff: *Postulate systems for Lie groups.*

It is shown by an elementary argument that if in some neighborhood L of the origin of ordinary n-space there is defined an associative law of composition $c_i=\phi^i(a;b)$ of class C' such that $\phi^i(a;0)=\phi^i(0;a)=a_i$, then L is the nucleus of a Lie group. Related arguments lead to interesting topologically invariant postulate systems for Lie groups. (Received March 21, 1935.)

193. Professor Edward Kasner: *The duals of velocity families and natural families.*

A velocity family, in two dimensions, is defined by a differential equation of the form $y''=(A+By')(1+y'^2)$. The author obtains a dual type in hessian line coordinates $v''=A+Bv'$. In this case the osculating cycles (oriented circles) along any straight line touch another line, and this constitutes a complete characterization of the dual type. In a velocity family the osculating circles at a point have a second point in common. Natural families constitute a subtype, Characterized by a certain relationship between the functions A and B, which yields a certain hyperosculating property (see Kasner, Princeton Colloquium, Chapter 2). The dual sub-type, obtained from certain infinitesimal contact transformations, discussed in a previous paper read at the February meeting (see abstract 41-3-131), is also characterized geometrically. (Received March 19, 1935.)

194. Dr. S. B. Myers (National Research Fellow): *Connections between differential geometry and topology.* II: *Closed surfaces.*

In a previous paper (see abstract 41-1-40) the author introduced the notion of the locus of "minimum points" on a complete analytic 2-dimensional Riemannian space and studied this locus on simply connected surfaces. In this paper the locus is studied on a closed (compact) Riemannian surface S. A point M on a geodesic ray g issuing from A is said to be a minimum point with respect to A if M is the last point on g such that AM furnishes an absolute minimum (proper or improper) to the length of arcs joining A to M. The locus m of such points with respect to A is shown to be a continuous curve which is locally a tree; if S is analytic, m is a finite linear graph. The cyclomatic number of m equals the connectivity number mod 2 of S. The order of a point M of m as a vertex of m equals the number of shortest geodesics joining M to A. The end points of m are conjugate to A and are cusps of the locus of first conjugate points to A. The surface S is reduced to a single 2-cell a with m as its singular boundary; a is simply covered (except at A) by the geodesic rays through A cut off at their intersections with m. Thus a solution is given to the hitherto vaguely answered question as to when the field formed by the geodesic rays through A breaks down. (Received March 22, 1935.)

195. Professor H. L. Black: *Systems of curves and surfaces related under cyclic involutions.*

A class of involutions I_n of prime order, on algebraic curves or surfaces with but a finite number of invariant points, is studied. These exist only on loci having particular moduli; special cases have been examined previously by Hutcherson. It is shown in S_2 that all the invariant curves factor. We apply the term "related" to systems of invariant surfaces in S_3 which enable us to map the involution in turn on distinct loci in a common hyperspace $\sum_{(n^2+6n+11)/6}$. By this means correspondences $(1, m)$ one way and $(1, k)$ the other are established between loci in hyperspaces of different dimensions. (Received March 12, 1935.)

196. Professor J. H. Roberts: *Collections filling a plane.*

The author has shown that there exists an upper semi-continuous collection G filling the plane such that every element of G is a bounded continuum of diameter greater than 1 which does not separate the plane. In the present paper it is proved that there exists such a collection G each element of which is an arc. In view of a theorem of R. L. Moore this result may be stated as follows: there exists a continuous single-valued transformation T of the plane into itself such that for each point y the set of all points x for which $T(x)=y$ is an arc of diameter greater than 1. (Received March 22, 1935.)

197. Dr. D. C. Lewis (National Research Fellow): *The formal theory of conservative transformations in 2n-dimensional space.*

A transformation carrying a point (x) in $2n$-dimensional space into a point (x') is said to be conservative if there exists a set of functions $X_i(x)$, such that the skew-symmetric determinant $|\partial X_i/\partial x_j - \partial X_j/\partial x_i| \neq 0$ and such that $\sum_{i=1}^{2n} X_i(x')dx_i' - \sum_{i=1}^{2n} X_i(x)dx_i$ is an exact differential, the unprimed x_i being the independent variables. The paper gives a formal theory of invariant points generalizing the work of Birkhoff who treated the case $n=1$ (Acta Mathematica, vol. 43). Analyticity of the transformation and the X's is assumed. (Received March 15, 1935.)

198. Mr. I. N. Kagno: *Irreducible non-toral graphs.*

Irreducible graphs, which cannot be mapped on a torus are found, corresponding to Kuratowski's irreducible non-planar graphs (Fundamenta Mathematicae, vol. 15 (1930), pp. 271–282). It is shown that there are a large number of such graphs, falling into eleven classes, and that no such graph has more than twenty-two, nor fewer than eight vertices. Among the preliminary theorems it is shown that: (1) if a graph when mapped on an orientable surface of genus p fails to separate that surface, then it can be mapped on a surface of lower genus; (2) a necessary and sufficient condition that a graph be on a surface of least possible genus is that the graph separate the surface into 2-cells, no matter how mapped on the surface. The existence of irreducible graphs which cannot be mapped on an orientable surface of any assigned genus is established. (Received March 8, 1935.)

199. Mr. Nelson Dunford: *Integration in general analysis.*

The known relationships between convergence almost everywhere, approximate convergence, and almost uniform convergence hold when the functions have as domain a metric space and as range a complete linear vector space (Banach's type B), provided, of course, the existence of a measure function is assumed. Starting with a class S_0 of uniformly continuous and bounded functions of the above type and defining a norm in terms of the Riemann Stieltjes integral $\|f\| \equiv \int_E \|f(P)\| d\beta$, where β is the total variation of the measure function, we show that to every class of equivalent Cauchy sequences of functions in S_0 corresponds uniquely (except for a set of measure zero) a function which we call summable. The limit of the Riemann integrals of the functions forming a Cauchy sequence defines the integral of a summable function. This space of summable functions is complete. The integral is a completely additive and absolutely continuous function of sets. This formulation of the theory of integration is equivalent to that given by Bochner (Fundamenta Mathematicae, vol. 20). (Received March 21, 1935.)

200. Professor C. N. Moore: *On the multiplication of series summable by Nörlund means.*

Cesàro's theorem regarding the summability of the Cauchy product of two series which are convergent or summable by arithmetic means of a given order was one of the first contributions to the modern theory of summable series. This theorem can be generalized to the case of Nörlund means in the following fashion. Given two power series $\sum a_n z^n$, $\sum b_n z^n$, converging within the unit circle, we set $c_n = a_0 b_n + \cdots + a_n b_0$. Any series $\sum u_n$ will be said to be summable $(N; d)$ to U if $\sum_{i=0}^{i=n} s_i d_{n-i} / \sum_{i=0}^{i=n} d_i$, where $s_n = \sum_{i=0}^{i=n} u_i$, approaches U as a limit as n becomes infinite. We may now establish the following theorem: if $\sum u_n$ is summable $(N; a)$ to U and $\sum v_n$ is summable $(N; b)$ to V, where the methods $(N; a)$ and $(N; b)$ satisfy the conditions of regularity, then the Cauchy product of the two series will be summable $(N; c)$ to UV. This theorem includes Cesàro's result and some of its later generalizations as special cases. (Received March 22, 1935.)

201. Professor Salomon Bochner: *Summation of double Fourier series by circles.*

If $\sum a_{m,n}e^{i(mx+ny)}$ is the Fourier series of a double periodic function $f(x, y)$ of class L, we denote by $S_N(x, y)$ the sum of those terms of its series for which $m^2+n^2 \leqq N^2$. In every point of continuity (in a generalized sense) the sequence $S_N(x, y)$ is summable $(R, 1/2+\epsilon)$, for any $\epsilon>0$; and a corresponding result holds for functions of more than two variables. (Received March 23, 1935.)

202. Professor I. M. Sheffer: *A differential equation for Appell polynomials.*

Let $\{P_n(x)\}$ be a *set* of polynomials; that is, an infinite sequence in which P_n is of degree not exceeding n. There exist infinitely many linear differential equations of infinite order, of type $\sum L_n(x)y^{(n)}(x) = \lambda y(x)$, which for appropriate characteristic values $\{\lambda = \lambda_n\}$ have the sequence $\{P_n\}$ as solutions. Here L_n is a polynomial of degree not exceeding n. This type of differential equation is, then, a *universal form* for sets of polynomials. A natural question arises: what equations of this type are characteristic of certain known polynomial sets? In the present note the question is answered for Appell polynomials. There is obtained a peculiarly simple equation that characterizes Appell sets. (Received March 18, 1935.)

203. Mr. Garrett Birkhoff: *On the lattice theory of linear dependence.*

It is shown that the theory of "matroids" developed by Whitney to describe the abstract properties of linear dependence can be put into correspondence with a type of lattice (in fact, complemented semi-modular lattices), and that this relates certain properties of matroids to properties discovered in other connections. (Received March 21, 1935.)

204. Professor Alonzo Church and Dr. J. B. Rosser: *Some properties of conversion.*

The process of *conversion* is defined by Church in the Annals of Mathematics, (2), vol. 33 (1932), p. 357. A *normal form* of a formula is defined by Kleene in the Annals of Mathematics, (2), vol. 35 (1934), p. 535. Let a *reduction* be defined as a conversion which consists of applications of Rules I and II only and contains one and only one application of Rule II. It is proved that, if a formula has a normal form, the normal form is unique to within applications of Rule I, and any sequence of reductions of the formula must, if continued, terminate in the normal form. Also that if a formula B, not in normal form, is obtainable by conversion from a formula A, not in normal form, then, whether or not A and B have a normal form, it is possible to find a formula C which is obtainable by a sequence of reductions from A and likewise from B. Analogous theorems are proved about certain other processes which are similar in their character to conversion. (Received March 22, 1935.)

205. Professor Alonzo Church: *An unsolvable problem of elementary number theory.* Preliminary report.

Following a suggestion of Herbrand, but modifying it in an important respect, Gödel has proposed (in a set of lectures at Princeton, N. J., 1934) a definition of the term *recursive function*, in a very general sense. In this paper a definition of *recursive function of positive integers* which is essentially Gödel's is adopted. And it is maintained that the notion of an effectively calculable function of positive integers should be identified with that of a recursive function, since other plausible definitions of effective calculability turn out to yield notions which are either equivalent to or weaker than recursiveness. There are many problems of elementary number theory in which it is required to find an effectively calculable function of positive integers satisfying certain conditions, as well as a large number of problems in other fields which are known to be reducible to problems in number theory of this type. A problem of this class is the problem to find a complete set of invariants of formulas under the operation of conversion (see abstract 41-5-204). It is proved that this problem is unsolvable, in the sense that there is no complete set of effectively calculable invariants. (Received March 22, 1935.)

206. Dr. F. G. Dressel: *Green's theorem for generalized harmonic functionals.*

The paper presents a more general Green's theorem than that given in a preceding paper entitled *A generalization of harmonic functionals.* (Received March 15, 1935.)

207. Mr. Nelson Dunford: *On a theorem of Plessner.*

Assuming $f(x)$ to be (1) of period 2π and (2) of bounded variation and (3) that the total variation in t of $f(t+u)-f(t)$ tends to zero with u, Plessner (Journal für Mathematik, vol. 160 (1929), pp. 26–32) has shown that $f(t)$ is absolutely continuous (also proved by Wiener and Young, Transactions of this Society, vol. 35 (1933), pp. 327–340). Ursell (Proceedings of the London Mathematical Society, (2), vol. 37 (1934), pp. 402–415) has shown that the theorem is still true when hypothesis (2) is weakened to mere measurability. From one of his inequalities it follows that the total variation of $f(t+u)-f(t)$ is continuous in u for all u. By a theorem of Lusin we conclude that $f(t)$ is continuous. These two facts combined with Plessner's proof provide a short proof of Ursell's extension. (Received March 21, 1935.)

208. Mr. A. H. Fox: *Differential equations with continuous spectra.*

The differential equation $u''(x)+[l-V(x)]u(x)=0$ is studied by the use of the theory of transformations in Hilbert space. The case in which $V(x)$ is a real step-function with a finite number of steps gives certain results concerning the nature of the continuous and point spectra of the equation. A sequence of such equations is formed in which the step-functions converge to a continuous function with a finite number of maximum and minimum points. For this limiting equation the properties of the spectrum are determined by considering the manner in which the spectrum is affected by the limiting process. (Received March 23, 1935.)

209. Mr. Marshall Hall: *Prime divisors of second order sequences.*

Dr. Morgan Ward obtained a partial solution of the problem: given a second order recurrence and the initial terms of a sequence satisfying it, to determine whether or not a given prime will divide any term of the sequence (this Bulletin, vol. 40 (1934), pp. 825–828). This paper gives a complete solution of the problem, reducing the question, as did Ward, to the determination of the reduced period of a sequence. (Received March 20, 1935.)

210. Professor G. A. Hedlund: *A metrically transitive group defined by the modular group.*

It has been proved (Annals of Mathematics, 1934, pp. 787–808) that the geodesics on certain closed orientable surfaces obtained by identifying points congruent under certain Fuchsian groups constitute metrically transitive systems. The groups considered have a fundamental region lying interior to the unit circle. If the region under consideration is the upper half-plane and the group is the modular group, the existence of transitive geodesics is known (Artin and Herglotz). It can be shown that this system is metrically transitive. The method of proof is of necessity different from that in the cases previously considered due to the fact that the characterization of the geodesics by means of infinitive symbols is obtained in the present case by means of continued fractions. The proof in this case is somewhat simpler. (Received March 21, 1935.)

211. Mr. J. D. Hill: *Some theorems on double limits.*

In this note is considered the problem of determining, for an arbitrary function $f(x, y)$, whether or not the existence of the double limit $\lim_{x,y\to 0}f(x, y)$ is implied by the existence of a unique limit for f as (x, y) tends to $(0, 0)$ on every curve of a given class of curves passing through $(0, 0)$. A general criterion is formulated and by its application are obtained definite answers for five classes of curves; these classes include (a) the class of curves having analytic parametric representations, (b) the class of curves having parametric representations of class C^∞, and (c) the class of curves having continuously turning tangents at $(0, 0)$. (Received March 18, 1935.)

212. Professor E. V. Huntington: *Inter-relations among the four principal types of order.*

The four types of order whose inter-relations are considered in this paper are: (1) *serial order* (order of points on a directed straight line); (2) *betweenness* (order of points on an undirected straight line); (3) *cyclic order* (order of points on a directed closed line); and (4) *separation of pairs* (order of points on an undirected closed line). After recapitulating all the known sets of postulates which define each of these types as an abstract system the author establishes the following theorems. (A) Without restriction, 2, 3, and 4 may be defined in terms of 1; and 4 may be defined in terms of either 1 or 2 or 3. (B) With respect to an arbitrarily selected element of the given system, 1 and 2 may be defined in terms of 3; and 2 in terms of 4. (C) With respect to an arbitrarily selected pair of elements, 1 and 3 may be defined in terms of 2. (D) With respect to an arbitrarily selected triad of elements, 1 and 3 may be defined in terms of 4. The paper will appear in the Transactions of the American Mathematical Society. (Received March 11, 1935.)

213. Professor E. V. Huntington: *The mathematical structure of Lewis's theory of strict implication.*

The formal postulates (or symbolic expressions) and the informal postulates (or rules of procedure) which occur in C. I. Lewis's theory of strict implication (Lewis and Langford, *Symbolic Logic*, 1932) are here rephrased in terms of a "subclass T," as an example of an abstract mathematical system. A comparison between "strict implication" and "material implication" is facilitated by numerical examples which illustrate the distinction between a "proposition" and the "assertion of a proposition"; and the connection between Lewis's system and Boolean algebra (cf. this Bulletin, vol. 40 (1934), pp. 729–735) is discussed. In particular, it is pointed out that a concept which may be called "effective implication" ($a=ab$) is definable in terms of "effective equality" and simple conjunction, without the use of negation or truth values. The paper will appear by invitation in the jubilee volume (vol. 25) of Fundamenta Mathematicae. (Received March 11, 1935.)

214. Professor R. L. Jeffery: *The inversion of approximate derivatives.*

Let $f(x)$ be a function which is finite almost everywhere on the linear interval (a, b). If there exists a sequence of summable functions $s_n(x)$ tending to $f(x)$ almost everywhere and such that $F(x)=\lim_{n\to\infty}\int_a^x s_n(x)dx$, then $f(x)$ is said to be integrable in the sequence sense to $F(x)$. The following results are proved: If $f(x)$ is finite almost everywhere on (a, b) and is almost everywhere equal to the approximate derivative of a continuous function $F(x)$, then $F(x)-F(a)=\int_a^x f(x)dx$, where the integration is in the sequence sense. Integration in the sequence sense includes the generalized Denjoy integral. The proofs of these results do not involve transfinite induction. Consequently integration in the sequence sense can be looked upon as a process of constructing the most general type of non-absolutely convergent integral without the use of transfinite numbers. (Received March 13, 1935.)

215. Professor Edward Kasner: *Characterization of the equilong group and a larger group.*

The only transformations which convert velocity families into velocity families form the conformal group. This characterization of conformal transformations was given by the author in 1906 (American Journal of Mathematics, 1906, p. 213). Later the same theorem was discovered by Scheffers, 1922. The present paper deals with the corresponding problem for line transformations preserving the dual-velocity type of doubly-infinite families. The solution is not the equilong group, as one might expect, but a larger group of the form $U=\phi(u)$, $V=v\chi(u)+\psi(u)$. This contains the equilong group as the special case $\chi=\phi'$. This type preserves the dual of the natural family type, studied in a paper read at the February meeting (abstract 41-3-131), obtained in connection with certain infinitesimal contact transformations. (Received March 21, 1935.)

216. Professor B. O. Koopman: *On distributions admitting a sufficient statistic.*

It is shown in this paper that those univariate statistical distributions of analytical frequency function which admit of the application of R. A. Fisher's conception of a sufficient statistic have a very special exponential form. (Received March 15, 1935.)

217. Dr. E. R. Lorch: *Functions of self-adjoint transformations in Hilbert space.*

The subject of functions of self-adjoint transformations in an abstract Hilbert space has been treated principally by Haar, Neumann, F. Riesz, and Stone. We give a new approach to the general theory, developing for this purpose a theory of measure and integration for operators patterned upon that of Lebesgue for numerical functions. Starting with a resolution of the identity, we define in terms of closed linear manifolds the notion of measure of a linear set. Measurable functions are then introduced and the definition of the function of an operator follows immediately without introducing bilinear forms. The paper concludes with a demonstration of the principal properties of such functions of operators. (Received March 7, 1935.)

218. Professor J. A. Shohat: *The relation of the classical orthogonal polynomials to the polynomials of Appell.*

The fact that the derivatives of the classical orthogonal polynomials are also orthogonal polynomials (of the same kind) enables us, starting with such a polynomial of a given degree, to form a sequence of Appell polynomials of degree 0, 1, 2, $\cdots$. They are orthogonal in an extended sense (in the ordinary sense, only, in case of Hermite polynomials). Thus the theory of Appell polynomials becomes applicable, for example, to the discussion of the zeros of the polynomials in question, to the expansion of x^n in a polynomial series, etc. (Received March 21, 1935.)

219. Professor G. T. Whyburn: *Concerning sequences and limiting sets.*

A convergent sequence $[A_n]$ of closed sets in a compact metric space is said to converge regularly relative to r-cycles provided that for each $\epsilon>0$ there exist positive numbers δ and N such that if $n>N$ then any r-dimensional complete cycle in A_n of diameter $<\delta$ is ~ 0 in a subset of A_n of diameter $<\epsilon$. It is shown that regular convergence relative to 0-cycles for sequences of arcs and simple closed curves gives limiting sets of the same type. For sequences of topological spheres it gives cactoids as limiting sets, and in case the convergence is regular also relative to 1-cycles, these cactoids reduce to topological spheres. For a sequence of 2-cells whose boundaries also converge, regular convergence relative to 0-cycles gives as a limiting set a hemicactoid (in the sense of C. B. Morrey) whose base set is bounded by a boundary curve which is the limit of the boundaries of the 2-cells. The hemicactoid reduces to its base set in case the convergence is regular also relative to 1-cycles, and this base set reduces to a 2-cell in case the boundaries of the 2-cells in the sequence converge regularly relative to 0-cycles. (Received March 23, 1935.)

220. Dr. S. L. Robinson: *Homogeneous sets which are disconnected by the removal of a finite number of points.*

The abstract spaces considered are not required to be Hausdorff topological spaces but only to satisfy the second property of Riesz. If the space P is totally disconnected and the group G of all homeomorphisms is transitive, G is k-fold transitive; if P is disconnected by the removal of one point and G is pseudo-3-fold transitive, G is pseudo-k-fold transitive for every k; while if P is disconnected by the removal of two points and G is pseudo-4-fold transitive, G is pseudo-k-fold transitive for every k and is doubly transitive. If G has pseudo-multiple transitivity of order k and of all lower orders and P is disconnected by the removal of $k-2$ points, then P is also disconnected by the removal of two points. If $k-2$ points are removed from P, the remaining space is connected if G is k-fold transitive and has less than k components if G is pseudo-k-fold transitive. (Received March 25, 1935.)

221. Dr. Saunders MacLane: *Abstract absolute values and polygonal irreducibility criteria.*

This paper investigates the connection between a certain type of irreducibility criterion for polynomials and the problem of determining all possible non-archimedean absolute values (*Bewertungen*) in a given polynomial ring. The irreducibility criteria concerned are the generalizations of Eisenstein's theorem which Dumas, Ore, and Kürschak have obtained by constructing Newtonian polygons corresponding to given polynomials. As Rella has pointed out (Journal für Mathematik, vol. 158), these generalizations depend on the use of an abstract absolute value for the coefficients in certain developments of the polynomials. The present paper shows that the same developments can be used to determine several successive absolute values in the corresponding polynomial ring. A suitable sequence of these absolute values will in turn give another type of absolute value for the ring, and these two types exhaust all the possible non-archimedean absolute values for the ring. The new absolute values thus obtained may be applied in turn to give new irreducibility criteria. The generalization consists essentially in going from the two moduli (a prime p and a polynomial $\phi(x)$, irreducible modulo p) used in the ordinary case, to a sequence of suitably chosen moduli, p, $\phi_1(x)$, $\phi_2(x)$, $\cdots$, $\phi_n(x)$. (Received March 25, 1935.)

222. Professor Hassler Whitney: *Sphere-spaces, with applications.*

To each point p of a complex K let there correspond an r-sphere $S_r(p)$ so that, roughly, the points on all spheres corresponding to a neighborhood U of p form the product of U and an r-sphere S_r. Then $S(K)$ is a *sphere-space*. Examples are the *tangent* and *normal* spaces to a manifold M_m in euclidean E_n, formed by the unit spheres in the tangent or normal planes through points p of M, with p as center. Invariants are defined which show the structure in the large of $S(K)$: to each i-cycle Z (mod μ) corresponds a number $\gamma(Z)$ which is a topological invariant (mod μ); in certain cases, $\gamma(Z)$ is defined only mod 2. If the points of $S(K)$ form the product of K and S_r, these invariants all vanish. Conversely, the invariants characterize the space, at least for low dimensions. The well known invariants for vector distributions on a manifold M are the highest invariants of the tangent space. The invariants of the normal space may be defined as "local intersections"; they vanish in a number of (but not all) cases. In particular, the normal space to an orientable surface is a product space. (Received April 19, 1935.)

223. Dr. W. V. Quine: *A unified calculus of propositions, classes, and relations.*

Propositions (or truth-values), classes, and relations of all degrees are taken collectively as undiscriminated elements of the system; rather than being artificially segregated as usual by distinctive styles of variables and restricted ranges of operations, the above categories become distinguishable only through formal properties (as odd and even are distinguished within arithmetic). Operations traditionally restricted to propositions, to classes, to dyadic relations, etc., become so generalized as to apply to all elements. Simplification attends this generalization in that many traditional operations become cases of one unrestricted operation. Like Boolean algebra, the system avoids bound variables, types, and functions such as $\iota`x$, $x\epsilon\alpha$, etc., involving types; but it covers all else, e.g., truth-functions, Boolean functions, and the totality of relational functions such as converse, domain, relative product, perordinate, distributive referent, and their unfamiliar analogues for higher relational degrees. All are definable in terms of two binary operations, $\alpha * \beta$ and $\alpha \dagger \beta$, describable respectively as generalizations of the $\overline{\alpha\beta}$ and $(\alpha \uparrow \beta \uparrow \alpha) \cap (\epsilon_a)_\iota$ of my *System of logistic*. Theorems issue from the formal postulates by two mechanical rules: (1) substitute for variables; (2) granted $X*(Y*Z)$ and $Y*(X*(W*W))$, put X for Y anywhere. (Received March 25, 1935.)

224. Professor D. V. Widder: *An application of Laguerre polynomials.*

By use of the Laguerre polynomials we give a new proof of the theorem of S. Bernstein to the effect that a function completely monotonic in $(0, \infty)$ can be represented by a Laplace-Stieltjes integral with non-decreasing determining function. The author obtains a new inversion formula for such an integral by a series of Laguerre polynomials. The method shows automatically that this series is summable in the sense of Abel to the determining function. It can be seen by other considerations that the series actually converges. (Received March 26, 1935.)

225. Dr. C. W. MacGregor: *The potential function method for the solution of two-dimensional stress problems.*

The potential function method for the determination of stresses and displacements in a body under plane stress or plane strain is developed further and in a somewhat different manner than heretofore. The expressions for the stress components are derived in terms of potential functions and also in terms of functions $W(z)$ of a complex variable z for both the general two-dimensional problem and for special cases of loading on the semi-plane boundary. It is found that a large number of interesting solutions can be obtained in the case of the semi-plane from the complex function $W(z)=f(z)\cdot\log(z/c)$ where c is a unit of distance and $f(z)$ determines the distribution of loading along the boundary. The stress components are given explicitly for the cases where $f(z)=z^n$ in which the straight boundary of the semi-plane is acted upon either by shear or normal stresses varying as r^n. Expressions for the displacement components are also derived for both the general and special cases mentioned. (Received March 26, 1935.)

226. Dr. C. W. Vickery: *Sets of independent axioms for complete Moore space and complete metric space.*

A certain set of four axioms has been obtained. These axioms are shown to be independent and equivalent, in the topological sense, to axioms 0 and 1 of R. L. Moore's *Foundation of Point-Set Theory*. Axioms 0 and 2 of this set are the same as the corresponding axioms of Part IV (pp. 12–13) of the author's *Spaces in which there exist uncountable convergent sequence of points* (Tôhoku Mathematical Journal, vol. 40, part I, pp. 1–26); axiom 1 is a modification of axiom $1(\aleph_0)$ of that paper; axiom 3 is a modification of axiom 4 of that paper. By substituting for axiom 1 a different modification of axiom $1(\aleph_0)$ a set of independent axioms for complete space $(\mathcal{D})$ of Fréchet is obtained. (Received March 28, 1935.)

227. Professor E. V. Huntington: *Postulates for effective equality and effective implication in formal logic.*

The purpose of this paper is to present an abstract mathematical theory $(K, T, \times, \doteq)$ which is capable of being interpreted as that part of the calculus of propositions which is expressible in terms of conjunction and equality alone, without the use of negation or truth-values. A set of independent postulates for "$a\times b$," "$a\doteq b$," and "a in T" is given, and a relation called "effective implication" is defined by $(a<b)=(a\doteq ab)$. This relation proves to resemble "strict implication" more closely than "material implication." The "general rules of procedure" usually implicit in the proofs of theorems are listed explicitly. The paper will appear in the Proceedings of the National Academy of Sciences. (Received March 29, 1935.)

228. Professor H. A. Simmons and Mr. William Block: *Classes of maximum numbers associated with certain symmetric equations in n reciprocals.*

As in two previous papers of Simmons (Transactions of this Society, vol. 34, pp. 876–907) and Stelford and Simmons (this Bulletin, vol. 40, no. 12), we let $\Sigma_{\lambda,\mu}(x)$ stand for the elementary symmetric function of the μth order of the variables x_i $(i=1,\cdots,\lambda)$. The equations relative to which the above authors have identified classes of maximum numbers are of the forms (1) $\Sigma_{n,r}(1/x)$

$+\lambda\Sigma_{n,r+1}(1/x)=b/a$, $a\equiv(c+1)b-1$, $n>r$; (2) $\Sigma_{i=r}^{s}\Sigma_{n,i}(1/x)=b/a$, $n\geqq s>r$. In the present paper we find that by means of numerous modifications and extensions of methods which were employed in the two articles mentioned above we are able to identify (our usual) classes of maximum numbers relative to the equation (3) $\Sigma_{i=r}^{s}\lambda_i\Sigma_{n,i}(1/x)=b/a$, $n\geqq s>r$, where the λ's are such that (3) includes both (1) and (2) as (rather) special cases. If $(x_1\cdots x_n)^{-1}$ be added to the left member of (3) and $n\geqq s+2$, the new equation with coefficients as we restrict them to be for (3) is still amenable to our method of attack. (Received March 11, 1935.)

229. Professor H. R. Brahana: *Metabelian groups and trilinear forms.*

The metabelian group $G=\{H, U_1, U_2, \cdots, U_m\}$, with central of order p^{n-k} and commutator subgroup of order p^l, which is a subgroup of the holomorph of the abelian group H of order p^n and type $1, 1, \cdots$ determines a trilinear form $F(x, y, z)=\Sigma a_{hij}x_hy_iz_j$, $h=1, 2, \cdots, k$; $i=1, 2, \cdots, l$; $j=1, 2, \cdots, m$; a_{hij} in a $GF(p)$. Conversely, a form F determines a group G. A necessary and sufficient condition that G and G' be simply isomorphic is that F and F' be conjugate under linear transformations in $GF(p)$ on the x's, the y's, and the z's. Modular invariants of F are immediately expressible in terms of properties of G. In the definition of F by means of G the variables x, y, and z have special roles; in the classification of the forms F they are indistinguishable. Hence, a set of properties which determines G determines F which in turn determines two more groups G' and G''. The three groups G, G', and G'' are in general distinct. Thus the work of classification of the groups G is reduced to approximately one-third. (Received March 18, 1935.)

230. Professor I. A. Barnett, Dr. C. W. Mendel, Mr. Haim Reingold: *Generalized determinants of Vandermonde.*

This paper is concerned with determinants whose elements are the elements of different powers of a square matrix. Let G_{ij}, $i, j=1, 2, \cdots, n$, be a square matrix the determinant of which is denoted by g; let $G_{rs}^{(m)}$ stand for the element in the rth row and sth column of the mth power of the matrix G_{ij}, and let $G_{ij}^{(o)}$ denote the Kronecker δ. Consider the $n\times n$ determinant $\left|G_{r_1s_1}^{(m_i)}G_{r_2s_2}^{(m_i)}\cdots G_{r_ns_n}^{(m_i)}\right|$, $i=1, 2, \cdots, n$, in which the ith row is exhibited, where $r_1, r_2, \cdots, r_n$ and $s_1, s_2, \cdots, s_n$ are arbitrary permutations of the integers $1, 2, \cdots, n$, and $m_1<m_2<\cdots<m_n$; we denote this determinant by $\Delta(m_1, m_2, \cdots, m_n)$. The following results are proved: (1) $\Delta(1, 2, \cdots, n)=g\Delta(0, 1, \cdots, n-1)$; (2) $\Delta(k+1, k+2, \cdots, k+n)=g^{k+1}\Delta(0, 1, \cdots, n-1)$; (3) $\Delta(m_1, m_2, \cdots, m_n)=f(a_0, a_1, \cdots; a_n)g^{m_1}\Delta(0, 1, \cdots, n-1)$, where f is a rational integral function of the coefficients of the characteristic equation of the matrix G_{ij}. For the particular square matrix $G_{ij}=\Sigma_{i=1}^{n}x_i\delta_{ij}=x_j$ the determinant $\Delta(0, 1, \cdots, n-1)$ reduces to the determinant of Vandermonde. (Received March 21, 1935.)

231. Dr. E. D. Jenkins: *On the composition of quadratic forms.*

The problem considered is to find a computational method for determining a compound of binary quadratic forms. Dedekind established a correspondence between the composition of forms and the multiplication of moduls of an algebraic field. Chatelet and MacDuffee have shown that the theory of matrices is very effective in the treatment of algebraic moduls and ideals. It is possible to define a class of integral algebraic moduls, not as general as those considered by Dedekind, but such that the existing theorems concerning the correspondence between ideals and matrices with rational integral elements will carry over to this particular type of modul. The result is a matric method for the composition of forms. The characteristics which serve to recommend this method are first, its simplicity, and, second, the saving of time involved in its use for computational purposes. (Received March 20, 1935.)

232. Dr. W. T. Reid: *A boundary value problem associated with the calculus of variations.* II.

The author has treated a boundary problem linear in a parameter, and associated with the problem of Bolza in the calculus of variations (American Journal of Mathematics, vol. 54 (1932), pp. 769–790). The purpose of the present paper is two-fold: (1) to indicate for the theorems of the previous paper concerning the existence of characteristic numbers a proof which is simpler in detail and more closely related to classical methods of the calculus of variations; (2) to treat a related boundary problem which is in general non-linear in the parameter. The latter boundary problem is assumed to satisfy hypotheses similar to those used by Morse in a recent treatment of a self-adjoint second-order differential system (American Mathematical Society Colloquium Publications, vol. 18, chapter IV), but the method used here differs from that of Morse. There is associated with the given boundary problem an auxiliary problem which is linear in a second parameter, and the characteristic numbers of the associated linear problem are considered as functions of the original parameter. Comparison and oscillation theorems for the given problem are consequences of corresponding theorems for the associated problem, and for this latter problem these theorems follow from the extremizing properties of the characteristic numbers. (Received March 22, 1935.)

233. Dr. M. S. Robertson (National Research Fellow): *Analytic functions star-like in one direction.*

Analytic functions $f(z)=z+\sum_2^\infty a_nz^n$ are considered which are regular for $|z|<1$ and which map each circle $|z|=r<1$, for values of r near 1, on contours C_r having the property that there exists at least one straight line through the origin which cuts each C_r in not more than two points. Then $|a_n|\leqq n^2$ for all n and equality is attained for essentially only one function of the class. If $f(z)$ is odd then $|a_n|\leqq n$. If the straight line in question is the real axis and if the coefficients are real, then $|a_n|\leqq n$ and $f(z)$ is then typically-real. Further, $\lim_{r\to 1}f(re^{i\theta})$ exists as a finite limit for almost all θ. If $g(z)$ is univalent for $|z|<1$, $g'(0)=1$, and maps $|z|=r<1$, for values of r near 1, on a contour convex in one direction (coefficients complex and denoted by c_n) then $|c_n|\leqq n$ for all n and equality is attained for a fixed n for only one function of this class. This constitutes a proof of the Bieberbach conjecture for this class of univalent functions. (Received March 15, 1935.)

234. Professor Wayne Dancer: *Concerning symmetrical cut-sets.*

The notion of symmetrical cut-point (= *s. c-p.*) of a connected point set, introduced by Gehman, may be extended in an obvious manner to the notion of symmetrical cut-set (= *s. c-s.*). Gehman showed how to characterize the simple arc in terms of *s. c-p.* In the present paper it is shown, for instance, that the simple closed curve and surface may be characterized in terms of *s. c-s.* Thus, the simple closed curve is a connected and locally connected space of which every pair of distinct points is an *s. c-s.* The strong *s. c-s.* is defined to be one which, under the transformation defining the notion of *s. c-s.*, remains point-wise invariant. It is shown that the only strong *s. c-s.* of the simple closed surface is the simple closed curve, and that the simple closed surface may be characterized as a Jordan continuum which is not disconnected by any pair of points, and of which every simple closed curve is a strong *s. c-s.* (Received March 22, 1935.)

235. Professors Otto Laporte and G. Y. Rainich: *Pseudominimal hypersurfaces and harmonic functions.*

This paper develops the subject studied in an earlier paper (see abstract 41-1-6) generalized to any number of dimensions. The integral representation of a *general* hypersurface is replaced by a representation in terms of a potential-like function ϕ and its derivatives and involving the same stereographic parameters used before. Due to the restricted transformation group a simplified form of tensor analysis results. Any conditions imposed on the curvature may be expressed as a differential equation on ϕ. For *pseudominimal* hypersurfaces this becomes a linear differential equation of second order. By a method analogous to that of separation of variables in mathematical physics an infinity of particular solutions is obtained which is of the form: "solid" harmonic times hypergeometric function of the radius. This establishes between pseudominimal hypersurfaces and harmonic functions a relationship generalizing that between minimal surfaces and analytic functions. In spaces of even dimensions this relationship can be given a closed form which, for example, for four dimensions becomes: $\phi=6(1-r^2)H+6(1+r^2)x_\rho\partial H/\partial x_\rho+(1+r^2)^2x_\rho x_\sigma\partial^2 H/\partial x_\rho\partial x_\sigma$. (Received March 22, 1935.)

236. Mr. C. B. Tompkins: *Linear connections in normal space.*

The author considers the formulas which express the derivatives of the coordinate vectors of an m-dimensional variety in an n-dimensional euclidean space. At each point of the variety there exists an $(n-m)$-dimensional normal space in which is chosen a set of mutually orthogonal unit vectors. The derivatives of the coordinate vectors and of these vectors are expressed as linear combinations of the set of vectors. Directions of reference in the normal space are not fixed. The normal components of the derivatives of the normal vectors are given by antisymmetric coefficients N_k^{pq} somewhat similar to the Christoffel symbols; they may be used as connection coefficients. From them the analogue of the Riemann tensor, S_{ik}^{pq}, is constructed and it is expressed in terms of the analogues of the coefficients of the second differential form of surfaces.

The vanishing of this tensor implies that there exists in the tangent space a set of mutually orthogonal vectors along which lie all the determinate principal directions of the indicating quadrics of the variety. Curves lying along the principal directions of these quadrics possess many of the properties of lines of curvature. (Received March 22, 1935.)

237. Mr. L. R. Wilcox: *Pairs of surfaces in five-dimensional space.*

The purpose of this paper is to construct a projective differential geometry of a pair of analytic surfaces immersed in a linear space of five dimensions, with an analytic one-to-one point correspondence between them. The theory is based on a system of partial differential equations and a set of power series expansions representing the surfaces in a neighborhood of a pair of corresponding points. Several families of curves on the surfaces and a number of covariant configurations associated with the surfaces are defined, and projective interrelations among them are studied. The analytic basis for a theory of envelopes and loci of the covariant configurations is constructed, and a portion of this theory is developed. (Received March 18, 1935.)

238. Professor A. A. Albert: *Normal division algebras of degree p^e over F of characteristic p.*

The author recently proved that if A is a normal division algebra of degree p over a field F of characteristic not p then A is cyclic if and only if A contains a sub-field $F(y)$, $y^p = g$ in F. In this paper the author considers algebras over F of characteristic p and proves that A of degree $p^e = n$ over F is cyclic if and only if A has a maximal sub-field $F(y)$, $y^n = g$ in F. (Received March 22, 1935.)

239. Professor C. C. MacDuffee: *Covariants of r-parameter groups.*

A covariant (contravariant) of an r-parameter group is defined as an invariant of the group and its first (second) parameter group. A general replacement theorem is proved showing that every absolute or relative covariant is a function of certain elementary covariants. It is shown that any set of equations defining an invariantive property can be put into covariant form. It is shown how the principal theorems of both algebraic and differential covariant theory appear as special instances. From this standpoint the tensor analysis is plainly a too narrow formulation of the covariant theory of differential forms. (Received March 20, 1935.)

240. Professor G. A. Bliss: *Abnormality in the calculus of variations.*

For a problem of Bolza in the calculus of variations Graves recently proved an analogue of the necessary condition of Weierstrass, and Hestenes generalized the Mayer condition and made an ingenious sufficiency proof, for an arc without normality restrictions. In the present paper the author analyzes the significance of abnormality. The papers of Graves and Hestenes for the first time bring to a satisfactory close an important chapter in the theory of arcs which are normal but not necessarily normal on sub-intervals. The abnormal accessory minimum problems recently studied by Carathéodory can be reduced to normal ones, but in general an abnormal minimizing arc is analogous to a singular point of implicit functions, and a complete theory of such arcs is likely to be difficult and complicated. The sufficient conditions described by Hestenes are not in general necessary, but they are the first ones established for such arcs. A statement in a recent paper by Morse might easily be interpreted to the contrary. In this paper of Morse an interesting lemma, first announced by Reid but earlier proved by Morse, is applied to extend to abnormal cases the effectiveness of a theorem of Bliss with the help of the results of Hestenes. (Received March 21, 1935.)

241. Professor A. A. Albert: *On the Hilbert irreducibility theorem.*

A field F is called an H.I. field if the Hilbert irreducibility theorem holds in F. W. Franz proved that every separable algebraic extension K of finite degree over an H.I. field F is an H.I. field, but was unable to treat the case where K is inseparable over F. The author now proves that every algebraic extension K of finite degree over an H.I. field F is an H.I. field. (Received March 22, 1935.)

242. Professor A. A. Albert: *Simple Lie algebras over a non-modular field.*

A Lie algebra L over a non-modular field F will be called normal simple over F if H is an algebraically closed extension of F and L_H is a simple algebra. In this paper the author proves that the adjoint enveloping associative algebra A of L is a total matric algebra of degree s over a field K of degree t over F and that the order of L over F is $n=st$. Moreover the basal units of L are linearly dependent in K and L is a normal simple Lie algebra of order s over K with A over K as adjoint enveloping algebra. (Received March 22, 1935.)

243. Mr. E. B. Escott: *Amicable numbers.*

This paper is an extension of a paper with the same title read at a meeting of the Chicago section on April 19, 1930. By means of extensions of Euler's methods, the author has been able to extend the number of amicable pairs of numbers to over 360 pairs. The paper will be published in Scripta Mathematicae. (Received March 22, 1935.)

244. Dr. Gertrude S. Ketchum: *On certain generalizations of the Cauchy-Taylor expansion theory.*

Okada and others have established the existence of expansions for functions regular at the origin in a set of functions $F_n(x)=x^n+\sum_{s=1}^{\infty} a_s^{(n)}x^{n+s}$ where $F_n(x)$ is analytic and is bounded with respect to n in some region about zero. We extend the region of convergence previously obtained and in important classes of such expansions establish the maximum region of convergence. A class of sets of integral functions is defined for which the expansion of functions analytic for $|x|<\rho$ is valid in the region $|x|<\rho$. The Bessel coefficients belong to this class. We also develop a very general theorem on expansions in products of functions of the form $F_n(x)$. When restricted to Bessel functions this theorem gives expansions of the form $x^\kappa f(x)=\sum_{n=0}^{\infty} c_n J_{\sigma_n^{(1)}}(x) J_{\sigma_n^{(2)}}(x) \cdots J_{\sigma_n^{(k)}}(x)$ where σ is restricted to an unbounded region of the complex plane such that $n/|\sigma+n|$ is bounded, $\kappa+n=\sigma_n^{(1)}+\sigma^{(2)}+\cdots+\sigma_n^{(k)}$, and $\lim_{n\to\infty}|\sigma_n^{(r)}|=\infty$, $r=1, 2, 3, \cdots, k$. (Received March 20, 1935.)

245. Dr. P. W. Ketchum: *Expansions of functions holomorphic in distinct regions.*

Let $f(x)$ be any function holomorphic in each of a set of non-overlapping regions R_k, $(k=1, 2, \cdots, q)$, and let a_k be any point interior to R_k. Let $F_{n,m}(x)$, $(n=0, 1, \cdots; m=1, 2, \cdots, q)$, have a zero of order n at a_k if $m=k$ and of order higher than n if $m\neq k$. Suppose a function $\theta_k(x)$ exists such that (1) $\theta_k(x)$ has a simple zero at a_k, (2) $\lim_{x\to a_k} F_{n,k}(x)[\theta_k(x)]^{-n}=1$, and (3) $|F_{n,m}(x)[\theta_k(x)]^{-n}|$ is bounded in R_k by a constant M independent of n, m, and k. Then there exists a unique set of numbers $\alpha_{n,m}$ such that $\Sigma_{n,m}\alpha_{n,m}F_{n,m}(x)$ converges absolutely and uniformly to $f(x)$ in each of a set of regions R_k', $k=1, 2, \cdots, q$, where R_k' contains a_k in its interior. (Received March 21, 1935.)

246. Professor W. J. Trjitzinsky: *Linear difference equations containing a parameter.*

The equation under consideration is of order n. Its coefficients are analytic functions of x and of a parameter λ, asymptotic in these variables to series in negative integral powers of x and of λ, with possibly a few positive powers of x and of λ present. The emphasis of the paper being on the investigation of the asymptotic properties of solutions in so far as the complex parameter λ is concerned, the equation is assumed to be formally of Fuchsian type in x. On the other hand, no further restrictions are made with regard to the parameter. The main result is embodied in the existence theorem, according to which there always exists a fundamental set of solutions asymptotic in x and in λ to certain formal series. (Received March 21, 1935.)

247. Dr. E. W. Miller: *Concerning polar singularities of an analytic function.*

Consider the analytic function $f(z)$ defined by the power series $\Sigma a_n z^n$ whose circle of convergence, C, has a radius 1. It is shown that $f(z)$ has $z=1$ as a pole of order m and has no further singularity on C if and only if there exists a polynomial $g(x)$ of degree $m-1$ such that $\overline{\lim}_{n\to\infty}|A_n-A_{n+1}|^{1/n}<1$, where $A_n=a_n/g(n)$. It is also shown that $f(z)$ has a pole of order m and no further singularity on C if and only if there exists a polynomial $g(x)$ of degree $m-1$ such that $\overline{\lim}_{n\to\infty}|A_n^2-A_{n-1}A_{n+1}|^{1/n}<1$, where $A_n=a_n/g(n)$. These theorems are simple generalizations of well known theorems of Pringsheim and Hadamard. A few applications are made and a theorem on limits which is closely connected with these matters is proved. (Received March 25, 1935.)

248. Dr. E. W. Miller: *On certain properties of Fréchet L-spaces.*

In different articles in Fundamenta Mathematicae W. Sierpinski, C. Kuratowski, and B. Dushnik have considered relations among the following properties in L-space: (A) the closure of every set is closed; (B) every nondenumerable set contains an element of condensation; (C) every well-ordered series of decreasing closed sets is denumerable; (D) every well-ordered series of increasing closed sets is denumerable; (E) every set Q contains a denumerable set D such that $Q\subset D+D'$. Dushnik raises the following questions. (1) Do conditions B and D together imply E? (2) Do conditions C and E together imply B? In the present note two L-spaces are constructed which show that these questions are to be answered in the negative. That the spaces in question fulfill the desired requirements appears as a consequence of two theorems which are proved for the linear continuum and which are concerned with properties somewhat stronger than C and D. (Received March 25, 1935.)

249. Professor H. P. Thielman: *Note on the factorization o Fredholm kernels.*

This paper establishes the theorem: if $G(x, y)$ is a real, positive, definite, symmetric kernel, there exist an infinite number of real, positive, definite, symmetric kernels $K(x, y)$ such that $G(x, y)=K(x, y)+K(y, x)+\int_a^b K(x, t)K(t,y)dt$, where $G(x, y)$ and $K(x, y)$ are such that their squares are integrable over $a\leqq x\leqq b$, $a\leqq y\leqq b$. This theorem is an analogue of a well known theorem on finite matrices (see C. C. MacDuffee, *The Theory of Matrices*, 1933). The result is used to prove that every Riemannian function space of a certain type can be immersed in a euclidean function space. (Received March 27, 1935.)

250. Professor Einar Hille: *Notes on linear transformations.* I.

We consider linear transformations of form $K_\alpha[f]=\alpha\int_{-\infty}^{+\infty}K(\alpha u)f(u+x)du$, in particular the following problems: (i) solutions of $K_\alpha[f]=0$, (ii) solutions of $K_\alpha[f]=f$, (iii) functional equations satisfied by $K_\alpha[K_\beta[f]]$, (iv) metric properties, including properties of contraction and degree of approximation of f by $K_\alpha[f]$. The analysis is carried out in detail for the kernels associated with the names of Dirichlet, Picard, Poisson, and Wierstrass. (Received April 2, 1935.)

251. Professor W. J. Trjitzinsky: *Linear functionals in the theory of quasi-analytic and D-analytic classes.*

Applications are made of the properties of linear functionals, relating to certain metric function spaces, to the theory of quasi-analytic and D-analytic classes. These developments are along lines similar to those of the author's recent work on quasi-analytic functions (Mathematische Zeitschrift, (1935), pp. 560–590). (Received April 16, 1935.)

252. Professor Oscar Zariski: *A topological proof of the Riemann-Roch theorem.*

The proof is based upon the consideration of the symmetric n-th product

V_n of a Riemann surface of genus p. Fundamental for the proof is the expression of a linear series $g_n{}^r$ on an algebraic curve of genus p as a cycle Γ_{2r} on V_n in terms of the cycles of a base for weak homologies on V_n. Recourse to algebra or to function theory is confined to the following elementary and least significant part of the Riemann-Roch theorem: if the $g_n{}^r$ is complete, then $r \geqq n-p$. The remainder of the theorem ($r=n-p$, if $n>2p-2$; the uniqueness of the canonical series g_{2p-2}^{p-1}; the formula $r=n-p+i$ for series of index of speciality i) is proved topologically by means of the intersection properties of the cycles on V_n. (Received April 15, 1935.)

253. Mr. J. F. Wardwell: *Continuous transformations preserving all topological properties.*

If A and B are any compact metric spaces and $T(A)=B$ is a continuous transformation, let G_0 denote the collection of all non-degenerate sets of the collection $[T^{-1}(b)]$, for all points b of B. Let G_i denote the collection of all sets of G_{i-1} which intersect $L_{i-1}=$ limit superior of G_{i-1}, for $i=1, 2, 3, \cdots$. If the following conditions are satisfied: (1) for any $\epsilon>0$, any set g of G_0, and any point x of g, there exists a homeomorphism $W(A-x)=A-g$ which is the identity outside of the ϵ-neighborhood of g in A, (2) there exists some number α of the first or second number class such that $G_\alpha=0$, and (3) $\Pi_1^\infty L_i$ is a zero-dimensional set, it is shown that there exists a topological transformation $S(A)=B$. (Received April 19, 1935.)

254. Professor Ervand Kogbetliantz: *On the jump of a function determined by its Hermite or Laguerre series.*

Using some previous results concerning the asymptotic behavior of the "singular" Hermite or Laguerre series, the author gives in the present paper a complete treatment of Gibbs' phenomenon for Hermite and Laguerre series. The situation turns out to be quite different from that familiar in the theory of trigonometric Fourier series. (Received April 26, 1935.)

255. Professor Wilhelm Maier: *Addition theorem of Euler's Γ-function.*

Hölder's classical statement concerning the non-existence of differential equations of finite order and algebraic coefficients satisfied by $\Gamma(u)$ suggests the construction of transcendent identities, comparable to the algebraic addition theorems of doubly periodic functions. Generalizing Newton's binomial theorem for fractional marks of summation $\nu \equiv x \pmod 1$, we find for certain values of a, b, λ the convergent expansion:

$$(a+b)^\lambda/\Gamma(1+\lambda)=\sum_{\pm\infty}^{\nu\,\overset{\nu\equiv x(1)}{}} (a^\nu b^{\lambda-\nu})/[\Gamma(1+\nu)\Gamma(1+\lambda-\nu)].$$

If besides convergence of the above, two initial conditions for $\Gamma(1+\nu)$ are granted, we are led to a new characterization of $\Gamma(u)$. (Received April 9, 1935.)

256. Mr. H. F. S. Jonah: *A functional equation of certain Vandiver polynomials.*

By means of a study of the generating function of the Bernoulli polynomials, the author has obtained a quadratic functional equation of a function closely related to the $h_n^{(k,l)}(x)$ of Vandiver (Annals of Mathematics, vol. 27 (1926), pp. 171–176). The functions used in this work are slightly more general than Vandiver's but can be expressed in terms of the $h_n^{(k,l)}(x)$. Using this functional equation we are able to obtain a transformation of the Kummer criteria, in particular, by an appropriate choice of our parameters, we are able to transform the congruences given by Mirimanoff (Journal für Mathematik, vol. 128 (1903), pp. 45–48); also the congruences given by Vandiver (Proceedings of the International Congress at Toronto, (1924), p. 679); also the congruences given by Vandiver (this Bulletin, vol. 28 (1922), p. 258); finally, the more general congruences given by Vandiver (Annals of Mathematics, vol. 27 (1926), pp. 171–176.) (Received April 9, 1935.)

257. Mr. Henry Scheffé: *Asymptotic solutions of certain linear differential equations in which the coefficient of the parameter may have a zero*. Preliminary report.

Asymptotic solutions for large values of the parameter ρ of the equation $u^{(n)}(z, \rho)+0+p_2(z)u^{(n-2)}(z, \rho)+ \cdots +\{p_n(z)-\rho^n\phi^n(z)\}u(z, \rho)=0$, where $p_j(z)$ and $\phi^n(z)$ are analytic, may be found from Birkhoff's paper (Transactions of this Society, vol. 9 (1908), p. 219) for z on a finite interval on the real axis on which $\phi^n(z)\neq 0$. In this paper are obtained forms valid when z is permitted to range over a finite region of the complex plane in which $\phi^n(z)$ has a zero of order ν. The second order case with lighter restrictions has been treated by Langer (Transactions of this Society, vol. 34 (1932), p. 447). As in his case, the forms depend primarily on an auxiliary variable $\xi=\rho\int_0^z\phi(z)dz$. The method, suggested by Langer, yields for each of certain configurations of z and ρ, and when $|\xi|>N$, formulas for n solutions of independent asymptotic form. Also, two sets of formulas are found for the principal solutions at the origin, for $|\xi|>N$ and $|\xi|<N$, respectively. All the results are explicit to factors $\{1+O(\xi^{-1})+O(\rho^{-\alpha})\}$ for $|\xi|>N$, and $\{\sigma(\xi)+O(\rho^{-\alpha})\}$ for $|\xi|<N$, where $\alpha=n/(n+\nu)$, and $\sigma(\xi)$ is a known power series. (Received April 18, 1935.)

258. Professor L. E. Dickson: *Cyclotomy when e is composite*.

This paper will appear in the Transactions of this Society, as a sequel to a paper in its current volume. There the author treated the case when e is a prime or double a prime. In this paper the author treats new cases when e is composite, and meets new difficulties. (Received April 19, 1935.)

259. Professor G. T. Whyburn: *Regular convergence and monotone transformations*.

In this paper it is first shown that if, in a compact metric space S, the sequence of closed sets $[A_n]$ converges to the limiting set A regularly relative to k-cycles (see the abstract of the author's paper *Concerning sequences and limiting sets*, No. 41-5-219), for every $k\leqq r$, then A is locally γ^r-connected. A continuous transformation $T(A)=B$ is said to be r-monotone provided that for each $b\epsilon B$ and each $k\leqq r$, the kth connectivity number of $T^{-1}(b)$ is zero.

Let the sequence of r-monotone transformations $T_n(A)=B_n$ converge uniformly to the limit transformation $T(A)=B$, where A, B, and B_n are closed and contained in S. Then in order that B_n converge to B regularly relative to r-cycles it is necessary and sufficient that B be locally γ^r-connected for every $k\leqq r$ and that T be r-monotone. (Received May 3, 1935.)

260. Mr. I. N. Kagno: *Irreducible non-projective planar graphs*. Preliminary report.

The author's methods of studying and classifying irreducible non-toral graphs are applied to irreducible graphs which cannot be mapped on a projective plane. It is shown that an irreducible non-projective planar graph has at least seven and at most fourteen vertices. The existence of irreducible graphs which cannot be mapped on a non-orientable surface of any given genus is established. (Received April 30, 1935.)

261. Professor T. Y. Thomas: *Algebraic characterizations in complex differential geometry*.

In this paper the idea of the algebraic characterization is introduced and the characterizations of the metric representations of affinely connected spaces are considered. The methods used are quite general and permit a wide range of applications. (Received April 30, 1935.)

262. Professor J. W. Campbell: *On the principles of Hamilton and Cartan*.

A holonomic dynamical system can be characterized by Hamilton's stationary action or by a Cartan integral invariant, and recently A. E. Taylor has extended Cartan's principle to non-holonomic systems. In this paper an equivalence theorem is proved and is then used to show more clearly the correspondence between the principles of Hamilton and Cartan. Both holonomic and non-holonomic systems are discussed; in the non-holonomic case the form of Cartan's integral invariant given by Taylor is obtained by a different and simpler method and the corresponding stationary integral of Hamilton is also obtained. From the latter there results an extension of Hamilton's principle to non-holonomic systems in the ordinary sense, viz. $\delta\int_{t_0}^{t_1}Ldt=0$, when comparison is made, with fixed end points, between the path followed and neighbouring varied paths which are kinematically possible. (Received May 1, 1935.)

263. Mr. Garrett Birkhoff: *On the combination of topologies*.

The notion of the relative strength of different topologies over a fixed set of points is well known. One can, however, obtain a sharper characterization of relative topologies by introducing two binary operations ("meet" and "join"). The existence and comparison of these operations with respect to the primitive ideas of distance, neighborhood, convergence, and closure occupies most of the paper. (Received May 6, 1935.)