Vorlesungen über Geschichte der Antiken Mathematischen Wissenschaften. Band I. Vorgriechische Mathematik mit 61 Figuren. By O. Neugebauer. Berlin, Springer, 1934. xii+242 pp.

The most amazing discoveries in the history of science are presented in this account of pre-Grecian mathematics. Up to this time, Egyptian papyri of considerable extent as to length and contents have been completely translated and explained. The Rhind or Ahmes Papyrus has been subjected to the most careful study as a result of advances in Egyptology and two treatises on this early historical record have appeared recently. The contents of an earlier document, the Moscow Papyrus whose existence was first really made known in 1917, have been made accessible to historians through translation only within five years. The records of Babylonian mathematics had revealed no more than certain facts about notation, a geometry of mensuration, the simplest arithmetical and geometric progressions, and other notions of the most elementary kind.

And now come these investigations with the revelation beyond the shadow of a doubt of a development of algebra by the Babylonians which raises them to the rank of real mathematicians. Fortunately records of different epochs are at hand so that this algebra is seen as a step in a growth through a long period of time. The author devotes four chapters to the role of numbers in Babylonian mathematics and to the complete number system on which their entire structure rests. It is a position system with 60 as the base and the arithmetical technique set up for whole numbers and fractions is as simple as the present day decimal system. The acceptance then of such a number system led easily to developments in other directions. Irrational numbers, and new facts in geometry appear. Among the topics treated so convincingly in this work are the solution of linear equations in more than one unknown, solution of quadratic equations, cubic equations, not yet completely understood, biquadratic equations. A single symbol is used for a concept or idea in general writing, and so it is a natural step to make use of such a symbol in algebraic problems.

Most impressive is the claim of the author with respect to the appearance of the "proof" in mathematics. To the Greeks has hitherto been given the great distinction of providing the first deductive proof in any science. This has, indeed, differentiated Grecian from pre-Grecian mathematics. This distinction must now be set aside. If a "proof" consists of a series of logical steps leading from one statement to another, then the Babylonians had a "proof." It is impossible to believe that such a complicated system of formulas as are used in the solution of certain problems (given in this work) could have been arrived at empirically.

Only passing reference can be made to the evidence that this mathematics is entirely distinct from that needed for temple and state and must have been taught in special schools; and to the growing conviction that the Arabian algebra was the outgrowth of a Babylonian algebra rather than Greek or Hindu.

This is the first time that an attempt has been made to give a complete presentation of the history of mathematics before the Greeks. Most of the chapters have already appeared in whole or in part in *Quellen und Studien zur Geschichte der Mathematik* published at intervals since 1928, and some of the

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facts presented are already well known through articles and books in English. These studies, however, have been woven together into a whole at the same time that the results of more recent investigations have increased their value. A study of language and script has been added. The chapter headings cover: Babylonische Rechentechnik; Allgemeine Geschichte Sprache und Schrift; Zahlensysteme; Ägyptische Mathematik; Babylonische Mathematik.

This work has broadened the world for the historian of mathematics. For this, as for much more, the historian is greatly indebted to the author.

LAO G. SIMONS

Aufgabensammlung zur höheren Algebra. By Helmut Hasse. Sammlung Göschen, vol. 1082. Berlin, de Gruyter, 1934. 175 pp.

The present collection of problems is a sequel to the author's two-volume treatise on algebra in the Sammlung Göschen. The main headings are: *Theory* of fields and rings, Groups, Linear algebra and determinants, Roots of algebraic equations including the solution of equations by radicals. Throughout the author takes the point of view of abstract algebra, trying to make the problems supplement and extend the theories of his textbook. Some problems are strikingly new. It is for instance rather surprising to find given as problems Zassenhaus' proof for the generalized Jordan-Hölder theorem and Witt's proof for Wedderburn's theorem that any field with a finite number of elements must be commutative.

Some problems are quite simple, but on the whole they become increasingly difficult until one reaches the last stage at the final problem where the author confesses that he cannot solve it himself. The book may be recommended to anyone interested in abstract algebra and it should form a suitable supplement to a more advanced course in higher algebra.

Oystein Ore

## Mathematische Grundlagenforschung. Intuitionismus. Beweistheorie. By A. Heyting. Berlin, Springer, 1934. iv+73 pp.

This pamphlet is one of the series *Ergebnisse der Mathematik* published by the editors of the Zentralblatt. It deals chiefly with the foundations of mathematics, and mathematical logic, from two points of view, the intuitionism of Brouwer and the formalism of Hilbert, and gives an able, clear, and concise account of the essentials of these two points of view and of the important results which have been obtained in connection with them. As explained in the introduction, no attempt is made to give an account of the logistic formulation of the foundations of mathematics, a subject which is to be treated in a later number of the series.

This work is recommended, not only to mathematical logicians, but also to mathematicians in general who desire an understandable survey of its field. The reviewer knows of no better such survey, indeed of none nearly so good.

The first section begins with a notice of Poincaré as historical forerunner of intuitionism, describes the point of view of the French semi-intuitionists as they are here called (Borel, Lebesgue, Baire), the first theory of Weyl, and