ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

323. R. G. Archibald: *Relatively highly composite numbers.*

Highly composite numbers have been defined and properties have been obtained by S. Ramanujan. The present paper defines a *relatively highly composite number* with reference to a sequence of integers containing an infinitude of primes. For such an arbitrary sequence it is shown that the series of reciprocals of the relatively highly composite numbers is convergent. Further results are obtained for relatively highly composite numbers with respect to arbitrary sequences, and also with respect to certain classes of sequences. (Received July 26, 1938.)

324. L. A. Aroian: *The moments of F and of z.*

R. A. Fisher has shown that the distribution of $z=1/2 \log s_1^2/s_2^2$ is $P(z) =(2n_1^{n_1/2}n_2^{n_2/2}e^{n_1z})dz/B(n_1/2,\ n_2/2)(n_1e^{2z}+n_2)^{(n_1+n_2)/2}$ and that of $F=e^{2z}$ is $P(F) = (n_1^{n_1/2}n_2^{n_2/2}F^{n_1/2-1})dF/B(n_1/2, n_2/2)\cdot(n_1F+n_2)^{(n_1+n_2)/2}$. From these, the moments of F and the semi-invariants of z are found, and the approach to normality of both distributions is studied. By use of the Type III curve, the results are applied to finding approximations of the 5% and 1% points of z. Other applications are indicated. (Received July 14, 1938.)

325. D. H. Ballou: *On the location of the roots of real polynomial equations when two roots are equal.*

If $a+ib$ is a complex root of the real polynomial equation $f(x)=\sum_{k=0}^{n} a_kx^k=0$, then the point (a, b) lies on the curve $\phi(x, y)\equiv f'(x)-y^2f'''(x)/3!+\cdots+(iy)^{\lambda-1}f^{(\lambda)}(x)/\lambda! =0$, where λ is the largest odd integer less than or equal to n. In this paper, equations $f(x)=0$ which have double roots are considered, and it is proved that if $a'+ib'$ is any root either real or complex, other than the double root, then the point (a', b') is a real focus of the curve $\phi(x, y)=0$. (Received July 25, 1938.)

326. W. D. Baten: *Concerning the distribution of the means of n independent chance variables when each is subject to a certain frequency law.*

The first four moments about the mean of the distribution of the sample averages were derived by employing results obtained from sampling theory. On applying Pearson's criteria for his types, it was found that the distribution of the sample averages was a Type I distribution. Pearson's criteria depend on the first four moments only. On examining the fifth moment about the mean of the distribution of the sample averages, it was found that the distribution of the sample averages did not satisfy the criteria for a Type I distribution. (Received July 21, 1938.)

327. P. O. Bell: *Integral invariants of projective differential geometry.*

In this paper, the author characterizes two well known integral invariants associated with an analytic surface, namely, the integral of the projective linear element and the integral of Fubini called the projective arc length. These characterizations afford simple geometric interpretations of the pangeodesics and the projective geodesics, the families of extremals of the above named integrals. The projective normal at a point of the surface, inasmuch as it is the cusp-axis of the projective geodesics passing through that point, is thus completely determined geometrically. Finally, a general class of integral invariants containing numerous special integrals of interest is characterized geometrically. Among these special integrals, perhaps the most notable are the integral of the projective linear element and the two integrals of Sullivan whose families of extremals have at a point the scroll directrices of Sullivan as cusp-axes. (Received July 16, 1938.)

328. Richard Brauer: *On $\mathfrak{p}$-adic and modular representations of semi-simple algebras.*

Let I be an order (not necessarily maximal) of a semi-simple algebra A of rank n over an algebraic number field K. Let $e_1, e_2, \cdots, e_n$ be an integral basis of I. Form the regular representation R of A with basis e_i. If $\mathfrak{p}$ is a prime ideal of K, then R can be taken as a modular representation (mod $\mathfrak{p}$) and split into indecomposable parts U_i and irreducible constituents F_i. The investigation of this question is closely tied up with the study of $\mathfrak{p}$ as an ideal of I. On the other hand, R may be considered as a representation of the $\mathfrak{p}$-adic algebra $A_\mathfrak{p}$ in the $\mathfrak{p}$-adic field $K_\mathfrak{p}$, and the irreducible parts $\overline{F_k}$ may be formed. Any U_i can be obtained from a $\mathfrak{p}$-adic representation by taking the latter modulo $\mathfrak{p}$. Therefore U_i can be split into $\overline{F_k}$'s. On the other hand, every $\overline{F_k}$ can be split into F_i's. A relation between these two kinds of decompositions is given. The results can also be extended to more general fields K. (Received July 26, 1938.)

329. R. V. Churchill: *On the problem of temperatures in a nonhomogeneous bar with discontinuous initial temperatures.*

The temperature function is required to satisfy the heat equation with variable coefficients, to vanish at $x=0$ and $x=1$, and to approach $F(x)$ as t approaches zero, where $F(x)$ is piecewise continuous. Discontinuities in $F(x)$, or the failure of this function to satisfy an end condition, add serious difficulties to the investigation of the uniqueness of a solution. A uniqueness theorem is established here. The additional conditions imposed by it upon the temperature function are of the type to be expected, physically, to eliminate the presence of instantaneous sources of heat. The theorem is applied to establish a solution as the only one which satisfies those natural restrictions. A series form as well as an integral form of the solution is found; both are necessary for the investigation. The asymptotic behavior at $t=0$ of the temperature function and its derivatives, at the end points, and at points of discontinuity of $F(x)$ in particular, is determined. (Received July 26, 1938.)

330. C. E. Clark: *Simultaneous invariants of a complex and subcomplex.*

Let P and Q be finite euclidean polyhedra with $Q \subset P$. Let P admit of a simplicial division K such that a subcomplex of K, say L, is a simplicial division of Q. Let K_2 and L_2 be the second barycentric subdivisions of K and L. The simplexes of K_2 which have at least one vertex in L_2 determine a complex N_2. Although N_2 depends on the simplicial division K, certain properties of N_2 are independent of K. These invariants include the Betti groups (as defined in Alexandroff and Hopf, *Topologie*, p. 205) of the complex B_2 consisting of the simplexes of N_2 that have no vertex in L_2, the Betti groups of the complex R_2 consisting of the simplexes of K_2 have no vertex in L_2, and groups which are generalizations of the groups $H^r(B_2, R_2)$ (ibid., p. 345) and $N^r(N_2, R_2)$ (ibid., p. 293). Also, relations among the invariant groups are obtained. (Received July 23, 1938.)

331. H. S. M. Coxeter: *A method for proving certain abstract groups to be infinite.*

Given an abstract definition for a finite group, we may determine its order by systematically enumerating the co-sets of a recognizable subgroup (Proceedings of the Edinburgh Mathematical Society, (2), vol. 5, pp. 26–34). In practice, however, the work becomes too tedious if the number of co-sets is more than one hundred or so. If it is suspected that the group is actually infinite, its infinite order can sometimes be established by enumerating the co-sets of a subgroup of unknown order, so chosen that the unfinished tables show, by some kind of regularity, that they will forever remain unfinished. This method is illustrated by the hitherto unknown group $P^4=Q^4=(PQ)^4=(P^{-1}Q)^4=(P^2Q)^4=(PQ^2)^4=1$, in which the subgroup $\{P^2, Q^2, (PQ)^2, (QP)^2\}$ is seen to be of infinite index. (Received July 5, 1938.)

332. M. M. Day: *Regularity of function-to-function transformations.*

Hill (this Bulletin, vol. 42, p. 225) considered the transformation (1) $U_x(f) = \int_0^\eta K(x, y)f(y)dy$, $(0<x<\xi)$, on the class V_1 of bounded measurable functions f of one real variable y such that $\lim_{y\to\eta} f(y)$ exists. He derived conditions on $K(x, y)$ necessary and sufficient that (1) be regular on V_1, that is, that for each $f \varepsilon V_1$ and for each x, $U_x(f)$ exists and $\lim_{x\to\xi} U_x(f) = \lim_{y\to\eta} f(y)$. In this paper, the author first extends Hill's results by considering transformations of the form (1) on a class V_m of bounded measurable functions of m variables, and obtains analogous conditions for regularity. For each kernel $K(x, y)$, the domain of regularity K is defined as the largest class of functions on which (1) is regular. Conditions are found under which, for a function $f, f \varepsilon K$. Using these, necessary and sufficient conditions are determined that (1) be regular on certain classes of functions containing V_m. (Received July 25, 1938.)

333. R. P. Dilworth: *Non-commutative arithmetic.*

This paper is a continuation of that described in abstract 44-5-188. The multiplication is less general than that considered in the previous paper in that it has the semigroup property and is closely connected with the lattice division; however, the lattice is not assumed to be closed with respect to the multiplication. The Dedekind modular condition plays an important role in determining the arithmetic of the lattice. With a suitable definition of prime elements, it is shown that each element of the lattice Σ has a unique (up to similarity) decomposition into prime factors. Similarity is a reflexive, symmetric, transitive relation such that every element similar to a prime is also a prime. Of particular interest is the application to the arithmetic of a semigroup. Let Σ be a non-commutative semigroup with a G.C.D. and an L.C.M. Then (i) the ascending chain condition in Σ, (ii) the descending chain condition for the factors of each element, and (iii) the modular condition are necessary and sufficient that Σ have an arithmetic, that is, unique (up to similarity) decomposition into prime factors. (Received July 23, 1938.)

334. R. J. Duffin and A. C. Schaeffer: *Extension of a theorem of S. Bernstein to non-analytic functions.*

If $f(x)$ and its nth derivative are bounded by 1, $(-\infty < x < \infty)$, then it is known that the intermediate derivatives of $f(x)$ from 1 to $n-1$ are bounded, $(-\infty < x < \infty)$, by constants which are, however, greater than 1. By making stronger assumptions on the derivatives of $f(x)$, the authors obtain stronger results. Suppose $f(x)$ is real. If $(f(x))^2 \leqq 1$ and $(f^{(n)}(x))^2 + (f^{(n+1)}(x))^2 \leqq 1$ for $-\infty < x < \infty$, it results that $(f^{(k)}(x))^2 + (f^{(k+1)}(x))^2 \leqq 1$ for $-\infty < x < \infty$ and $0 \leqq k \leqq n$. It is seen that the equality is satisfied if $f(x)$ is $\cos x$. This is a generalization of a theorem of S. Bernstein on the derivative of a trigonometric polynomial. (Received July 25, 1938.)

335. Arnold Emch: *New point configurations and algebraic curves attached to them.*

In papers (Commentarii Mathematici Helvetici, vol. 4 (1932), pp. 65–73; Tôhoku Mathematical Journal, vol. 37 (1933), pp. 100–109) which appeared some time ago, the author treated a rather extensive class of Cremona transformations in S_r with the property that to a point $(x_1, x_2, \cdots, x_{r+1})$ corresponds a point $(x_1', x_2', \cdots, x_{r+1}')$, and, with the same coordinates, to a point $(x_1, x_2, \cdots, x_k', \cdots, x_l, x_{l+1}, \cdots, x_h', \cdots, x_{r+1})$ corresponds the point $(x_1', x_2', \cdots, x_k, \cdots, x_l', x_{l+1}', \cdots, x_h, \cdots, x_{r+1}')$, for any number of transpositions x_i, x_i'. Such a transformation T_r (involutorial) is $\rho x_i' = 1/x_i$, $(i = 1, 2, \cdots, r+1)$. The cases $r=2$ and $r=3$ give rise to the theorems: 1. On every diagonal triangle $A_1A_2A_3$ of a Steinerian quadrangle, there are $\infty^1 \Delta_8$ configurations on the elliptic cubic invariant under T_2, with the property that the eight points of a Δ_8 lie by twos on four lines through each A_i. 2. With the T_3 is associated a line complex of order three. On every point (generic) B, there is a cubic cone of the complex. Corresponding points of T_3 on this cone lie on a C_7 of genus three. On this C_7 there are $\infty^1 \Delta_{16}$ configurations, such that the 16 points lie by twos on eight lines through each of $A_1(1000)$, $A_2(0100)$, $A_3(0010)$, $A_4(0001)$, and B. (Received July 8, 1938.)

336. W. W. Flexner: *A class of singular 2-manifolds.*

Singular manifolds are defined by Veblen, *Analysis Situs*, American Mathematical Society Colloquium Publications, vol. 5_2, 2d edition, 1931, p. 95. A singular 2-manifold is here called "half-simple" if its singular complex (locus of points whose linked complexes are not spheres) consists of disjunct 1-manifolds (topological images of the circle) and isolated points. Half-simple 2-manifolds are classified by means of a matrix, topologically invariant except for permutations of rows and columns, which may be calculated from the incidence matrices of the manifold. The problem of finding all singular 2-manifolds with a given half-simple singularity is studied by means of the invariants appearing in the duality theorem for singular manifolds. (Received July 25, 1938.)

337. W. W. Flexner: *Duality theorems for singular generalized manifolds.*

Singular generalized n-manifolds M are defined by Veblen, *Analysis Situs*, American Mathematical Society Colloquium Publications, vol. 5_2, 2d edition, 1931, p. 95. Two extensions, applicable to them, of the Poincaré duality theorem are: $Y_p = Y_{n-p}$ and $Z_p = Z_{n-p}$, where $Y_p = R_p - \lambda_p - n_{p-1} + s_p$ and $Z_p = R_p - \lambda_p + \rho_p - n_p + s_{p-1}$. The number R_p is the pth Betti number of M; λ_p is the pth Betti number of the singular complex L of M; s_p is defined by Lefschetz, *Topology*, American Mathematical Society Colloquium Publications, vol. 12, 1930, p. 149; n_n is defined by Alexandroff and Hopf. *Topologie*, p. 299, where K_1, as here used, is the closure of a simplicial neighborhood of L, and $K_2 = \overline{M - K_1}$; $\rho_p = R_p(K_1 \cdot K_2)$. That n_p and ρ_p are independent of the neighborhood K_1 is a consequence of a recent theorem due to C. E. Clark. (Received July 25, 1938.)

338. Bernard Friedman: *Linear diophantine equations in quaternion analysis.* I.

It is well known that a necessary and sufficient condition for the existence of integral quaternion solutions of the diophantine equation $AX + BY = C$ is that C be divisible by the greatest common divisor of A and B. The analogous equation $AX + YB = C$ has not been previously considered. In this paper the case $A = B$ is studied and the following result is obtained: A necessary and sufficient condition that there exist integral quaternion solutions of the diophantine equation $AX + YA = C$ is that the trace of $\bar{A}C$ be divisible by the norm of A where $\bar{A}$ is the conjugate of A. (Received August 2, 1938.)

339. K. O. Friedrichs: *Underdetermined systems of linear partial differential equations and Lagrange's multiplier method.*

An elementary procedure for obtaining solutions of underdetermined systems of linear partial differential equations is described. Without integration one obtains solutions vanishing with derivatives at the boundary of a prescribed region. This procedure is applied to Lagrange's multiplier method in the case of linear partial differential equations as accessory conditions. The justification of this method is reduced to the establishment of necessary and sufficient conditions for the solutions of overdetermined systems. (Received August 2, 1938.)

340. O. E. Glenn: *The modular covariants* (mod 2) *of formal type, of the quantics of orders less than eight.*

Since, by a theorem due to the author, the arbitrary binary form, $f_m = a_0 x_1^m + a_1 x_1^{m-1} x_2 + \cdots + a_m x_2^m$, is reducible, modulo 2, in terms of formal modular covariants of f_m, of orders 1, 2, 3, the transformations being those of the total group G_6 (mod 2), it follows, by the principle of copied forms, that a complete covariant system of the set (f_1, f_2, f_3) gives the respective systems of all single forms of orders one to seven, inclusive. In the present paper, the system of (f_1, f_2, f_3) is constructed. It is composed of 239 invariants and covariants. Moreover, certain covariant types do not increase in number after a minimum m is passed. Therefore it is possible to obtain, by theoretical considerations, a complete system of covariants of f_m when m is a positive, arbitrary integer. (Received July 26, 1938.)

341. Louis Green: *Twisted cubics associated with a space curve.*

The local geometry of an analytic curve immersed in a three-dimensional projective space is studied in this paper. The methods employed involve projection from and upon the osculating plane at a point of the curve, intersections of tangent developables with one another as well as with planes and quadrics, and certain dual considerations. Characterizations are obtained of various five-point twisted cubics and of associated tetrahedra, and new properties of higher order neighborhoods of the curve are found. (Received July 22, 1938.)

342. Marshall Hall: *A type of algebraic closure.*

If the scalar product AB of vectors A and B vanishes, A is said to annihilate B on the left and B is said to annihilate A on the right. The left closure of a set of vectors S is defined as the totality of left annihilators of the totality of right annihilators of S. The right closure is similarly defined. An algebra is said to be strongly closed if every vector ideal over it is its own closure, and weakly closed if every ideal is its own closure. It is shown that a weakly closed algebra has a unit and that weak closure does, in fact, imply strong closure. The closure of an algebra depends upon the closure of the ideals in the radical, and, as the author showed previously, a semi-simple algebra is closed. Various structure (lattice) properties of annihilators and closures are proved. (Received July 21, 1938.)

343. D. C. Harkin: *Matrix representation of algebraic units and the solution of algebraic equations.*

Matrix representation of certain algebraic units and factorization of certain symmetric determinants and their characteristic equations lead to a unified and simplified method for the algebraic solution of the general equations of the second, third, and fourth degrees. This method depends only on the inherent property of the coefficients as elementary symmetric functions of the roots. (Received July 18, 1938.)

344. T. R. Hollcroft: *Regular plane curve systems with more cusps than the number possessed by certain irregular systems of the same order.*

The first irregular plane curve system was discovered by B. Segre in 1929. It is of order 6α and genus $12\alpha^2-9\alpha+1$, with $6\alpha^2$ cusps; and it is represented by the symbol $(6\alpha, 12\alpha^2-9\alpha+1, 6\alpha^2)$. In the present paper, regular curve systems $(6\alpha, 12\alpha^2-9\alpha-2, 6\alpha^2+3)$ are shown to exist for $\alpha=1, 2, 4, 8, 16, \cdots$. The two following theorems are thereby established: 1. Irregular curve systems of certain orders exist which are of higher genus and have fewer cusps than existing regular curve systems of the same respective orders. 2. A plane curve system may be irregular because of relations among the positions of its cusps, and for that reason only. (Received July 26, 1938.)

345. Charles Hopkins: *Concerning nil-rings with minimal condition for admissible left ideals.*

The main theorem asserts that a nil-ring R is nilpotent provided that the minimal condition holds for left ideals which are admissible with respect to an operator system of a certain type. The following corollaries may be stated: Let O be any ring with minimal condition for ordinary left ideals. Then the radical R of O is nilpotent; O is semi-primary; any subring of O which contains only nilpotent elements is itself nilpotent. (Received July 25, 1938.)

346. Harold Hotelling: *Tubes and spheres in n-spaces and a class of statistical problems.*

A *tube* in a Riemannian space is defined as the locus of points at a fixed distance from a curve C, measured along geodesics perpendicular to C. Simple and exact formulas are found for the n-dimensional volumes enclosed by tubes in both spherical and euclidean spaces. In these spaces, local self-overlapping exists if and only if the radius of the tube exceeds the radius of geodesic curvature of C. The spherical case has important statistical applications in determining the significance of regression equations, non-linear in a disposable parameter. In general Riemannian space series of powers of the radius giving the volumes of both geodesic spheres and tubes are investigated. The first terms coincide with the formulas for euclidean space. The next non-vanishing terms are determined in this paper in terms of known invariants, and it is shown that alternate terms in the series vanish. To determine the invariant expressions for the tubular volume, a new system of coordinates related to an arbitrary curve in Riemannian space is set up which is found to have interesting properties. (Received July 26, 1938.)

347. D. H. Hyers: *Pseudo-norms in linear spaces.*

Let O be a partially ordered set with the Moore-Smith property that for a, b in O there is a c in O with $c>a, b$ (Garrett Birkhoff, Annals of Mathematics, (2), vol. 38 (1937), p. 40). Let there be ordered to each x of a linear space L and each d of O, a non-negative real number $n(x, d)$ subject to the following postulates: (1) $n(x, d)=0$ for all d in O implies that $x=\theta$; (2) $n(\alpha x, d)=|\alpha|\cdot n(x, d)$; (3) $n(x, e)\geqq n(x, d)$ for $e>d$; (4) given ϵ, e, there exist δ, d with $n(x+y, e)<\epsilon$ for $n(x, d)<\delta$, $n(y, d)<\delta$. The pseudo-norm of x with respect to d is called $n(x, d)$. It is found that the necessary and sufficient condition for a linear space to be pseudo-normable is that it be a Hausdorff space with continuous fundamental operations. This theorem makes possible a characterization of the abelian subgroups of a linear topological space. The above pseudo-norm is a generalization of that of von Neumann (Transactions of this Society, vol. 37 (1935), p. 18). (Received July 23, 1938.)

348. Dunham Jackson: *A class of orthogonal functions on plane curves.*

The notion of a system of polynomials in one variable orthogonal with respect to an arbitrary weight function is modified and, in a sense, generalized by substitution of trigonometric sums for the polynomials. Another generalization results from the consideration of polynomials in two variables orthogonal with respect to integration along an algebraic curve in the plane of the variables. These two ideas are combined in the present paper to define a still broader class of orthogonal functions, in the study of which attention is given both to formal properties and to questions of convergence of the resulting developments in series. The theory is simpler when the transition from polynomials to trigonometric sums is made with respect to just one of the variables than when the polynomials in two variables are replaced by double trigonometric sums. (Received July 23, 1938.)

349. Erich Kamke: *A new proof of Sturm's comparison theorems.*

The purpose of this paper is to develop a simple proof of Sturm's comparison theorems concerning the zeros of the solutions of differential equations. It is desired to compare the solutions of two systems each of which consists of a pair of first order linear homogeneous differential equations with variable coefficients. A transformation to polar coordinates produces two new systems which are readily compared. The inequalities thus obtained can now be interpreted in terms of the original systems. This device yields the desired comparisons between the locations of the roots and between the numbers of the roots of the original systems. (Received July 29, 1938.)

350. D. H. Lehmer: *Semiregular continued cotangents.*

This paper is an extension of a previous investigation of a cotangent analog of continued fractions (Duke Mathematical Journal, vol. 4, pp. 323–340). It is concerned with expansions of the type $x = \cot \sum_{\nu=0} \alpha_\nu \text{ arc cot } n_\nu$, in which $\alpha_\nu^2 = 1$, the numbers n_ν are positive integers satisfying certain inequalities (to insure uniqueness when the α's are prescribed), and the series may be finite or infinite. Such an expansion is called a semiregular continued cotangent. In case $\alpha_\nu = (-1)^\nu$, the expansion becomes the regular continued cotangent previously considered. Analogs are obtained of most of the known theorems for semiregular continued fractions, except those having to do with the periodicity of the partial quotients. For example, by choosing different α's and n's, it is possible to obtain not more than Q different semiregular continued cotangent expansions of a fixed rational number $x = P/Q$. The extreme rapidity of convergence of the regular continued cotangent is also a feature of the semiregular case. (Received July 26, 1938.)

351. Walter Leighton and W. T. Scott: *A general continued fraction expansion.*

The general expansion of a power series into a continued fraction of the form $1 + K[a_i x^{c_i}/1]$ is discussed. A typical theorem is the following: A necessary and sufficient condition that a continued fraction of the above form converge uniformly in an open region T containing the origin to the analytic function $f(x)$ generating the continued fraction is that the nth approximant $A_n(x)/B_n(x)$ be uniformly bounded in T for n sufficiently large. (Received July 20, 1938.)

352. Howard Levi: *Prime and composite polynomials with coefficients in any infinite field.*

Composite polynomials with complex coefficients were treated by J. F. Ritt (Transactions of this Society, vol. 23 (1922), pp. 51–66). By function-theoretic methods, he analyzed the decompositions of composite polynomials into prime polynomials, obtaining extremely precise results. In the present paper, many of his results are derived for the case of polynomials with coefficients in an arbitrary field of characteristic zero. (Received July 28, 1938.)

353. R. G. Lubben: *The extension of homeomorphisms of normal spaces to topologically related spaces.*

Let S be a normal space. In abstracts 43-9-344 and 44-1-32, this Bulletin, the author discussed spaces whose points are composition points of S, amalgamation points of S, portions of S, and those which serve as fields for extensions of various topological properties of S. In this paper, it is shown that the elements of a certain class $E(S)$ of these spaces are universal spaces relative to the problem of extending the elements of the group $G(S)$ of all homeomorphisms of S into itself. If $T \in E(S)$, there exists a group $G(T)$ of homeomorphisms of T into itself such that each element f of $G(T)$ is an extension, which satisfies certain simple and important conditions, of an element f_T of $G(S)$, and such that the pairing of f_T and f determines a simple isomorphism of $G(S)$ and $G(T)$. Similar results hold for sets of homeomorphisms between two normal spaces. (Received July 25, 1938.)

354. W. H. McEwen: *On the degree of convergence of the derivatives of the Birkhoff series.*

Let $S_N(x)$ be the Nth order partial sum of the Birkhoff series associated with a given function $f(x)$ and a given regular nth order linear differential system $L(u)+\lambda u=0$, $W_i(u)=0$, defined on (0, 1). Milne (Transactions of this Society, vol. 19 (1918), pp. 143–156) has shown that if $f, f', \cdots, f^{(m-1)}$ vanish at 0 and 1, and if $f^{(m)}$ is continuous and of limited variation on (0, 1), then $f-S_N=O(N^{-m})$ uniformly on (0, 1). The purpose of the present paper is to show that, under the same hypotheses, $f^{(k)}-S_N{}^{(k)}=O(N^{-m+k})$ uniformly on (0, 1), with $k=0, 1, \cdots, m-1$, and also, for a restricted class of problems in which $f, f', \cdots, f^{(m-1)}$ vanish at 0 and 1 and $f^{(m-1)}$ satisfies a Lipschitz condition with parameter μ, that $|f^{(k)}-S_N{}^{(k)}| \leqq K\mu \log N/N^{m-k}$, where K is a constant independent of N, k. (Received July 20, 1938.)

355. J. K. L. MacDonald: *A note on Sturm-Liouville expansions and interpolations.*

Series of proper function solutions of Sturm-Liouville differential equations with two-signed coefficients are examined from the viewpoint of nodes and interpolation. It is proved that if the negative-proper-value expansion coefficients are not all zero, then the non-negative-proper-value expansion coefficients cannot all be zero for a piecewise continuous integrable function having a finite number of changes in sign in a certain interval and having the value zero elsewhere. (Received July 25, 1938.)

356. J. K. L. MacDonald: *On singularities and solutions of linear ordinary differential equations.*

Existence and comparison theorems and transformations are developed for use in determining the forms of solutions near singularities, in particular for second order linear ordinary differential equations. Simple integrability conditions for the coefficients permit classification of solutions, and the solutions are expressible through convergent iteration. (Received July 25, 1938.)

357. Robert MacKay: *Planarity of Peano spaces in terms of homology bases.*

A combinatorial condition for planar graphs was given by MacLane in terms of a basis of 1-circuits. It is the purpose of this paper to give a characterization of planar Peano spaces from the same point of view. It is first shown that a Peano continuum has a homology basis of simple closed curves whose diameters form a null sequence. By the use of such a null basis, a Peano space P may be made simply 1-connected in the sense of homology by the addition of a denumerable set Q of non-intersecting open 2-cells. Two 1-cycles Z_1, Z_2 of P with carriers C_1, C_2, respectively, are then called *partially dependent* relative to the above basis if there exist in $P+Q$ two irreducible membranes K_1, K_2 relative to Z_1, Z_2, respectively, such that $C_1 \cdot C_2 \not\supset K_1 \cdot K_2$ and $K_1 \neq K_1 \cdot K_2 \neq K_2$. Planarity is determined chiefly in terms of partial dependence of 1-cycles; for example, a necessary and sufficient condition that a cyclic Peano continuum be planar is that it have a null basis such that no two simple closed curves with only a connected set in common are partially dependent relative to this basis. (Received July 25, 1938.)

358. Saunders MacLane: *The structure of automorphism groups of p-adic fields.*

The theorem of Hasse and Schmidt that the structure of a discrete perfect field is determined by the structure of its residue-class field is here given a new proof based upon a simple construction of such perfect fields, not requiring the Witt vector analysis. If a p-adic field k has two p-adic extensions K_1 and K_2 with the same residue-class field $\mathfrak{K}$, then K_1 and K_2 are equivalent over k if some p-basis of the smaller residue-class field $\mathfrak{k}$ is p-independent in $\mathfrak{K}$. This includes the case when $\mathfrak{K}$ has a separating transcendence base over $\mathfrak{k}$. The group G of all analytic automorphisms of a p-adic field K has an inertial subgroup G_1 and certain ramification subgroups $G_2 \supset G_3 \supset \cdots$. The quotient group G/G_1 is isomorphic to the automorphism group of the residue-class field $\mathfrak{K}$, while the quotient groups G_n/G_{n+1} are abelian and representable as direct sums of the additive group $\mathfrak{K}$, the number of summands being the degree of imperfection of $\mathfrak{K}$. Finally, the subfield of K left elementwise fixed by G_1 is the uniquely determined p-adic subfield of K whose residue-class field is the maximal perfect subfield of the residue-class field $\mathfrak{K}$. (Received July 9, 1938.)

359. G. M. Merriman: *Concerning sets of polynomials orthogonal simultaneously on several ellipses.*

In a previous paper (this Bulletin, abstract 42-7-279), the author announced two results: (1) if a set of polynomials in the complex variable is orthogonal (with respect to a suitably chosen norm function) simultaneously on two or more concentric circles, then the set is orthogonal on all circles concentric with these (that is, on all "level curves" of the family); (2) an analogous proposition is true if the circles are replaced by confocal ellipses. The former result has recently been published (this Bulletin, vol. 44 (1938), pp. 57–69); the latter proposition is false. In the present paper, this second question is again treated. It is found that the hypothesis of orthogonality, with respect to a suitable norm function, simultaneously on a finite number (greater than unity) of confocal ellipses is not sufficient to induce orthogonality simultaneously on all level curves of the family; but that a hypothesis of orthogonality simultaneously on a denumerable infinity of confocal ellipses is sufficient. Thus, there may exist sets of polynomials orthogonal simultaneously on and only on a given finite number of confocal ellipses, in contrast to the case of orthogonality on circles. (Received July 27, 1938.)

360. Marston Morse: *A solution of the problem of infinite play in chess.*

The rules of chess include a description of admissible moves, together with rules which define a victory or a draw. Among the latter rules is one which proclaims a draw if any given sequence of moves is repeated twice in succession and is immediately followed by the first move of a third repetition. The question at issue is that of the possibility of an *unending* game of chess, that is a game which never leads to a victory or a draw. One of the conditions on the game is that it never lead to an undisputed superiority of one side, for in such a case the game is ordinarily required to end in a prescribed number of moves, or else become a draw. An unending sequence of symbols, first used by the author to establish the existence of recurrent motions in dynamics, is here used to establish the possibility of infinite play in chess. (Received July 9, 1938.)

361. David Moskovitz and L. L. Dines: *On convexity in a linear space with an inner product.*

In euclidean space of n dimensions (1) if a point set is *linearly connected*, it is *completely supported at its boundary points*; and (2) if a set is closed, possesses inner points, and is completely supported at its boundary points, it is linearly connected (American Mathematical Monthly, vol. 45 (1938), p. 202). This paper investigates the extent to which the validity of these theorems persists in a general linear space in which the notions distance, convergence, and hyperplane are defined in terms of a generalized inner product. It is found that (2) is valid; but (1) is not valid without qualifications. The necessary qualifications are determined and expressed in several different forms. (Received July 26, 1938.)

362. Tadasi Nakayama: *On symmetric algebras and Galois moduli over modular fields.*

Let A be a symmetric algebra (cf. R. Brauer and C. Nesbitt, Proceedings of the National Academy of Sciences, vol. 23; Nakayama and Nesbitt, this Bulletin, vol. 44 (1938), p. 346), and let $r(S)$ and $l(S)$ denote respectively the right and the left ideal of A consisting of all right and left annihilators of a set S of elements in A. If L is a left ideal of A, then $l(r(L))=L$, and moreover, if L_1 is a left subideal of L, then the representation of A defined by L/L_1 is equivalent to the one defined by $r(L_1)/r(L)$. Furthermore, the representations of A defined by Ac and cA are equivalent to each other, where c is any element of A. The proof begins with a general consideration concerning left and right representation moduli of an algebra which define equivalent representations. As an application of the theorem, M. Deuring's theorem on Galois moduli can be proved (Mathematische Annalen, vol. 113, §3) without the assumption of semi-simplicity of group algebras under consideration. (Received July 23, 1938.)

363. E. N. Oberg: *The approximate solution of certain linear functional equations.*

Let $T(u)=x(x)-\lambda U(u)=f(x)$ be a linear functional equation having a unique solution $u(x)$, and let $\Phi_n(x)$ be a linear combination of a set of $n+1$ linearly independent functions $\{\phi_n\}$. The present paper is concerned with the approximate representation of the solution of the equation $T(u)=f(x)$ by means of a function $\Phi_n(x)$ which is defined so that the integral of the mth power of $|f(x)-T(\Phi_n)|$ shall be the least. The problems involved center around the existence and uniqueness of such an approximating function for arbitrary values of n, and its convergence as n becomes infinite. These problems are made to depend upon the class and properties of the function $u(x), f(x)$, the transformation $U(u)$, and the set $\{\phi_n\}$. Analogous problems of this nature have been reported upon by the author and others in previous papers, but none have reached the generality of the present discussion. (Received July 20, 1938.)

364. R. E. O'Connor and Gordon Pall: *The quaternion congruence* $\bar{t}at\equiv b \pmod{g}$.

This paper includes and extends that of abstract 43-9-371. The congruence $\bar{t}at\equiv b\ (p^n)$ is solvable, in case $p\nmid Na$ and $p\nmid b$, if and only if the norm condition $Na(Nt)^2\equiv Nb\ (p^n)$ is solvable for Nt; and, if $p|Na$ but $p\nmid a$ and b, if and only if in addition the congruence is solvable for $n=1$ (discussed in abstract 43-9-371). There are equally many solutions t with Nt in any residue class (mod p^n) consistent with the norm condition. This is unlike the case of modulus 2^n (abstract 43-7-303), where all solutions satisfy one and only one of $Nt\equiv 1$ or -1 (mod 4). The symbol $\epsilon(a, b)$ is set equal to 1 or -1 according as $Nt\equiv 1$ or -1 (4); $[a, b]_p = 1$ or -1 according as $tat\equiv b$ (p) is solvable or not. If a and b are of the same norm h, the product of the values $[a, b]_p$ for the odd primes dividing h is equal to $\epsilon(a, b)$. Properties of these symbols are developed and a simple generalization is obtained of the theorem of abstract 43-7-303 concerning $h(8n+1)=x_1^2+x_2^2+x_3^2$. Other applications involve the Legendre symbol $(a_1b_1+a_2b_2+a_3b_3|p)$. (Received July 18, 1938.)

365. Rufus Oldenburger: *On decomposition of elements in abelian groups.*

To treat the decomposability of an element of an abelian group G into a product of a given number of distinct elements of G, it is necessary only to distinguish between the unit element I, the idempotent elements different from I, and the non-idempotent elements of G. If G has two or more idempotent elements, these elements form a subgroup G' of G. Making use of pairs of elements of G whose products equal idempotent elements, one can prove that a non-idempotent element can be decomposed into a product of n distinct elements of G provided that n is less than the order of G. An idempotent element different from I can be so decomposed unless n is the order of G and the order of the subgroup G' of G is greater than two. Like conditions hold for the decomposability of I which depend only on n, and the orders of G, G'. These results have applications in the theory of forms. (Received July 26, 1938.)

366. A. J. Palermo: *Asymptotic expansions for certain Bessel functions.*

Certain convergent series converge rather rapidly when the argument is of relatively low value; but if the argument takes on high values, the convergence of these series is so slow as to entail prohibitively laborious, if not practically impossible, mathematical drudgery. In the present paper, a number of such series are transformed into asymptotic series in which the summation is very simple for high values of the argument. (The concise character of the paper fosters brevity and also permits the reader to try some of his own ingenuity on the transformations.) The author has applied the results of the present paper to high frequency phenomena. (Received July 25, 1938.)

367. Gordon Pall: *On the factorization of generalized quaternions.*

Quaternions are treated whose norm form is $Q=t_0^2+F(t_1, t_2, t_3)$, where F is the adjoint of a ternary form $\sum a_{\alpha\beta}x_\alpha x_\beta$. If the $a_{\alpha\beta}$ are integers, consider the ring of integral quaternions $t=t_0+i_1t_1+i_2t_2+i_3t_3$ with rational integer coefficient t_i. If $v=ut$ in integer quaternions, t is called a right-divisor of v; necessarily, $Nt|Nv$. Let p be an odd prime dividing Nv, but not $|a_{\alpha\beta}|$, nor all of $v_0, \cdots, v_3$; no right-divisor of v having norm p exists unless p is consistent with the genus of Q; if it is so consistent, each right-divisor is connected with the representations of the number 1 in a certain quaternary quadratic class in the same genus as Q; if this genus contains but one class, the number of right-divisors is equal to the number of units (solutions of $Q=1$). A partial extension is made to divisors of composite norm. (Received July 18, 1938.)

368. T. P. Palmer: *Procedural rules in the propositional calculus.*

A system is deduced from one postulate (*DDpDqrDDDsrDDpsDpsDpDpq*, suggested by Wajsberg in 1927) by means of four rules of procedure, two of which are parts of the usual blanket rule of substitution, and two of which are parts of the usual blanket rule of inference. The equivalence of this system to the propositional calculus of *Principia Mathematica* is shown by introducing the familiar definitions of the operators of *Principia Mathematica* in terms of the operator D, and by proving that any substitution or inference performable by the rules of *Principia Mathematica* is performable in this system. The originality of this paper lies in the restriction of the procedural rules to a minimum. This restriction makes possible the complete avoidance of any reference to significance or meaningfulness of expressions. Wajsberg's postulate was used rather than that of Nicod or of Łukasiewicz because of the better phrasing of the rules of inference which it made possible. (Received July 26, 1938.)

369. G. B. Price: *Definitions and properties of monotone functions.*

Let $x(t)$ be a function defined on a general range with values in an abstract space. We say $x(t)$ is monotone if and only if $x(t_2)$ is between $x(t_1)$ and $x(t_3)$ whenever t_2 is between t_1 and t_3. In this paper, monotone functions defined in this manner in terms of the notion of betweenness are studied. Four definitions of betweenness are employed, one in partially ordered spaces, two in metric spaces, and one in linear spaces. For real-valued functions of a single variable, each type of monotoneity reduces to ordinary monotoneity. The discontinuities of the various types of monotone functions are determined, and the relation of monotone functions to functions of bounded variation is examined. The paper ends with a detailed study of a new class of real-valued monotone functions of two variables. A complete analysis of the nature and distribution of the points of discontinuity of these functions is given. These monotone functions form a subclass of the monotonic functions of Lebesgue and McShane. (Received July 23, 1938.)

370. H. A. Rademacher: *Some further problems of partitions.*

Hardy and Ramanujan (1917) in their paper on $p(n)$ mentioned also asymptotic formulas for the numbers $q(n)$ and $q_0(n)$ of partitions of n into odd integers with and without repetition, respectively. They gave only the first few terms of the asymptotic formulas which close with error terms $O(1)$. The generating functions of $q(n)$ and $q_0(n)$ are essentially modular forms of dimension zero ("modular functions" in the strict sense), belonging to congruence subgroups modulo 2 and modulo 4 of the modular group. The method which the author applied to the coefficients of the modular function $J(\tau)$ (American Journal of Mathematics, vol. 60 (1938), pp. 501–512) leads to convergent series for $q(n)$ and $q_0(n)$. (Received July 26, 1938.)

371. W. T. Reid: *Some remarks on linear differential systems.*

Bliss (Transactions of this Society, vol. 28 (1926), pp. 561–584; see also abstract 42-5-156) has considered self-adjoint and definitely self-adjoint systems of ordinary linear differential equations of the first order, with two-point boundary conditions, for which the coefficients are real. In this note, linear differential systems with complex-valued coefficients are considered and so-called self-conjugate adjoint and definitely self-conjugate adjoint systems are defined. It is shown that definitely self-conjugate adjoint systems have only real characteristic values, and that such a system may be reduced to a definitely self-adjoint system with real coefficients. An associated matrix differential equation is solved, and the question of self-conjugate adjointness is reduced to the determination of a non-singular constant matrix of a certain type. Finally, there is given an application of this matrix differential equation to the theory of matrix differential equations of Riccati type. (Received July 26, 1938.)

372. D. Riabouchinsky: *Galileo's antinomy and the class of numbers defined by their value and origin.*

Galileo's and Cantor's ideas on infinite sets are discussed. (See E. Kasner, *Galileo and the modern concept of infinity*, this Bulletin, vol. 11 (1904), pp. 499–501.) To follow the development of any representation in the unlimited, one must first of all establish it strictly in the finite and only then project it into the infinite. Failing this precaution, one often encounters contradictions or indeterminate relations. The following topics are treated: geometrical interpretation of transfinite numbers; the origin of a number; the class of numbers defined by their value and origin; the operation of passing to the limit and the inverse operation of returning from the limit; applications of these concepts to the theory of sets. (Received July 26, 1938.)

373. H. E. Robbins: *On the classification of the mappings of a 2-complex into a space.*

Necessary and sufficient conditions are given for the homotopy of two mappings of a 2-complex into a space. The conditions are of a combinatorial character, involving chains with coefficients from the 2-dimensional homotopy group and fundamental group of the space. Such classification has been carried through previously only where the fundamental group or the higher dimensional homotopy groups of the space are assumed to vanish (Hopf, Hurewicz). The present work is closely related to recent (unpublished) work by Whitney, classifying mappings into projective spaces. (Received July 20, 1938.)

374. G. E. Schweigert: *Concerning non-alternating interior transformations.*

The space under consideration is a locally connected continuum M; any A-set in M is denoted by A. It is shown that a non-alternating transformation $T(M)=N$ preserves A-sets; that is, $T(A)$ is also an A-set. Under the hypothesis that $T(M)=N$ is interior and non-alternating, it is established that $T^{-1}T(A)=A$ when A and N are non-degenerate. Implications concerning cyclic extensibility and reducibility of the non-alternating and interiority properties for a transformation follow this result immediately. Concerning an A-set K in N, it is shown that there are but a finite number of components C in $T^{-1}(K)$ and, when T is a light transformation, that each C is locally connected. Although not directly in line with the above results, a sufficient condition on M in order that T be topological is given. (Received July 22, 1938.)

375. Stephan Serghiesco: *On the subdivision of a mechanical system of corpuscules in equilibrium.*

The most recent theories of the mass of a particle, especially of the photon, include certain conceptions of the subdivision of this mass. In this paper, it is shown in general lines that it is possible to arrive at these conceptions of subdivisions of mass through a purely analytical mechanics method which is presented as a continuation of a special corpuscular theory of the author (Comptes Rendus (Paris), vol. 202 (1936), p. 1563 and p. 1761). As one cannot draw very advantageous conclusions about the mass of the photon from the relativity formula, a special theoretical expression for its mass, during a short interval of time, is given in this paper. (Received July 22, 1938.)

376. Max Shiffman: *The automorphisms of an abelian group.*

Let A be a primary abelian group all of whose elements have a finite height; let σ be the ring of all automorphisms of A. In this paper, a one-to-one correspondence is established between certain subgroups of A and certain (two-sided) ideals of σ. A regular characteristic subgroup R of A is one such that $R\alpha = R$ or $R\alpha \subset R$ for all automorphisms α of σ. A right (left) characteristic ideal $c(c')$ of σ is one such that there exists an ideal i for which $i \cdot c = 0$, and $i \cdot d = 0$ implies $d = c$ or $d \subset c$ ($c' \cdot i = 0$, and $d \cdot i = 0$ implies $d = c'$ or $d \subset c'$). The one-to-one correspondence is between regular characteristic subgroups and right (left) characteristic ideals. To R corresponds the right characteristic ideal $u(R)$ which is the totality of automorphisms α such that $R\alpha = 0$. To c corresponds the regular characteristic subgroup $r(c)$ which is the totality of elements r such that $rc = 0$. It is shown that the correspondences $u(R)$, $r(c)$ are inverse to one another, $r[u(R)] = R$, $u[r(c)] = c$. Similar correspondences can be established between the regular characteristic subgroups R and the left characteristic ideals c'. (Received July 26, 1938.)

377. Max Shiffman: *The Plateau problem for minimal surfaces which are not relative minima.*

In this paper it is proved that a curve Γ which bounds two minimal surfaces which are relative minima also bounds a minimal surface which is not a relative minimum. Let γ_1, γ_2 be the two minimal surfaces which are relative minima, and consider the space of all surfaces bounded by Γ. Let S be any connected set of surfaces containing γ_1 and γ_2, and let $d(S)$ be the least upper bound of the Dirichlet function $D(\gamma)$ for all γ belonging to S. Let d be the greatest lower bound of $d(S)$ for all such S. It is first shown that d is attained for some S. It is then shown that on S there is a minimal surface γ which is not a relative minimum and for which $D(\gamma) = d$. If this were not so, a deformation of S into S' could be defined for which $d(S') < d$, contrary to the definition of d. The method is applicable to the most general case of several boundaries and higher topological structures. Furthermore, it permits the application of the Morse theory to the Plateau problem. (Received July 26, 1938.)

378. C. D. Smith: *Three-circle problems in modern geometry.*

The radical axes of a system of three circles meet in a point called the radical center. If three circles have a point P in common, P is the radical center of this system. Let the circles meet again two by two at A_1, B_1, C_1, so that angles A_1PC_1, C_1PB_1, B_1PA_1 are each less than a straight angle. Inscribe a quadrilateral A_1PC_1B in circle A_1PC_1 with B restricted to arc A_1BC_1. In like manner for each of the other two circles draw inscribed quadrilaterals C_1PB_1A and B_1PA_1C. The following fundamental theorem is proved: The three inscribed angles opposite the radical center are together equal to a straight angle. The theorem of Miquel for a triangle is a special case of this theorem. The theorems of Ceva and Menelaus, the Simson line, the Lemoine point, the Gergonne point, the Nagel point, and the Brocard points are demonstrated as special cases of the three-circle theorem. The following new results are obtained: a Miquel correspondence between points of the plane, the Miquel line of a triangle with the Simson line as a special case, and other interesting theorems. The geometry of the notable points of the triangle is connected and the proofs simplified by this method. (Received July 5, 1938.)

379. H. S. Thurston: *On the number of sets conjugate to a matrix with linear elementary divisors.*

Any n-rowed matrix M having only linear elementary divisors is expressible as a polynomial in a matrix A with simple latent roots. If M possesses r distinct latent roots, then M has, in the ring $R(A)$, $[(r-1)!]^{n-1}$ sets of conjugates with respect to its minimum equation, the matrices of $[(r-1)!]^{r-1}$ of these sets being expressible as integral rational functions of M. (Received July 23, 1938.)

380. H. S. Thurston: *On factoring a matric polynomial with scalar coefficients.*

The following theorem is proved: If μ is the number of characteristic values of a square matrix A, then any polynomial of degree t, with simple zeros, breaks up in $(t!)^{\mu-1}$ ways into a product of t linear factors in the ring $R(A)$. Two conjugate sets $G=\{X_0, X_1, \cdots, X_{t-1}\}$ and $H=\{Y_0, Y_1, \cdots, Y_{t-1}\}$, obtained from distinct factorizations, are called *similar* if there exists a non-singular matrix T such that $Y_i=T^{-1}X_iT$, the same T serving for every i. A sufficient condition for the existence of similar sets is that the matrix A have a Segre characteristic of the form $[k\ k \cdots k]$. (Received July 23, 1938.)

381. Esther M. Torrance: *Superposition on monotonic functions.* II.

Let $y=g(x)$ be a monotonic function. A necessary and sufficient condition that $f[g(x)]$ be measurable for every measurable function $f(y)$ is that the inverse monotonic function $x=g^{-1}(y)$ be absolutely continuous on the G_δ' over which the function defines a homeomorphism. If $f(y)$ is a Baire function of class α, $(\alpha>1)$, then $f[g(x)]$ and $f[g^{-1}(x)]$ are Baire functions of class α. (Received July 25, 1938.)

382. W. J. Trjitzinsky: *Theory of non-linear q-difference equations.*

In this paper the analytic (asymptotic) theory of equations of the indicated type is developed. At the basis of the work lies the theory of linear q-difference equations as developed by the author (Acta Mathematica, 1933, pp. 1–38). Use is made of a method, due to the author, specifically applicable to a type of non-linear problems and previously employed by him in the fields of differential and difference equations (see Transactions of this Society, vol. 42 (1937), pp. 225–321; Compositio Mathematica, 1937, pp. 1–66; Mémorial des Sciences Mathématiques, no. 90, 1938). (Received July 15, 1938.)

383. A. H. Wheeler: *Groups of cubes and groups of regular tetrahedra inscriptible in a cube.*

In this paper it is shown that in a given cube there can be inscribed an infinite number of groups of six equal cubes each, all of which are concentric with it. Furthermore, there can be inscribed an infinite number of groups of twelve equal cubes each, and these twelve cubes are not concentric one with another, or with the given cube. Two congruent regular tetrahedra can be inscribed in a cube with their vertices at the vertices of the cube. Accordingly, there can be inscribed in a given cube an infinite number of groups of twelve congruent, regular tetrahedra each; and these tetrahedra are concentric, one with another, and with the given cube. There can also be inscribed in a given cube, an infinite number of groups of twenty-four congruent, regular tetrahedra each; and these tetrahedra are not concentric, one with another, or with the given cube. (Received July 19, 1938.)

384. Margarete C. Wolf and Louise A. Wolf: *The linear equation in matrices with elements in a division algebra.*

To determine necessary and sufficient conditions for the existence of solutions of the equations $\sum_{k=1}^{m} A_k X B_k = C$ in $r \times r$ matrices and to determine the number of solutions, each matrix M may be written as $\sum_{i=1}^{n} u_i M^{(i)}$, where the u_i, $(i=1, 2, \cdots, n)$, are the basis of the algebra and the $M^{(i)}$ are matrices with elements in the centrum. This reduces the problem to solving nr^2 linear equations in nr^2 elements whose coefficients are in the centrum. The matrix of the coefficients of this system of equations can be expressed as a sum of direct products of matrices whose elements are in the centrum. For $AX = I$ this matrix of the coefficients can be written as $\sum_{i=1}^{n} R^{(i)} \cdot x A^{(i)} \cdot x I_r$, where I_r is the $r \times r$ identity, $R^{(i)}$, $(i=1, 2, \cdots, n)$, is the $n \times n$ first matric representation of u_i, and $A = \sum_{i=1}^{n} u_i A^{(i)}$. A necessary and sufficient condition that $AX = I$ have a unique solution is that $\left|\sum_{i=1}^{n} R^{(i)} \cdot x A^{(i)}\right| \neq 0$, which may also be obtained by other methods. To study the equation $XA = I$, it is more convenient to use the second matric representation of the basal elements. (If the algebra is a field, see C. C. MacDuffee, *The Theory of Matrices.*) (Received July 25, 1938.)

385. R. P. Agnew: *On cores of bounded divergent complex sequences and of their transforms by square matrices.*

This paper characterizes matrices $\|a_{nk}\|$ of complex constants such that the transform $\sigma_n = \sum_{k=1}^{\infty} a_{nk} s_k$ of each real bounded divergent sequence s_n has a core Γ which is a nonempty subset of the core C of s_n. The same problem is solved for the class of complex bounded divergent sequences s_n. Analogous problems are solved for sequence-to-function transformations $\sigma(t) = \sum_{k=1}^{\infty} a_k(t) s_k$. (Received August 9, 1938.)

386. G. E. Albert: *Asymptotic forms for a general class of hypergeometric functions with applications to the generalized Legendre functions.*

The developments reported in abstract 44-7-288 have been extended to the solutions of the differential equation $(1-z^2)y'' + [\beta - \alpha - (\alpha+\beta+2)z]y' + n(n+\alpha+\beta+1)y = 0$. Solutions are developed asymptotically relative to n and z which are uniform in α and β in any fixed bounded domain. The results of the previous abstract are special cases; in addition, the Jacobi, Gegenbauer, and ultraspherical functions are included. (Received August 19, 1938.)

387. Henry Blumberg: *On the general point transformation.*

The author considers transformations $X' = f(X)$ of the points X of a euclidean n-space E into the points X' of a euclidean n-space E', f unconditioned except to be "one-valued" which, however, is not essential. Various theorems are proved stating properties which f has at every point of E if certain sets are disregarded. Such types of negligible sets are the denumerable, the exhaustible, the *sparse* set (sparse equals ridé of W. H. Young, see author's forthcoming paper, *Exceptional sets*, in Fundamenta Mathematicae), and the set of zero measure. Notably, an analog is derived of the central theorem in the author's paper, *The measurable boundaries of an arbitrary function* (Acta Mathematica, vol. 65 (1935), p. 263). Generalizations are obtained. (Received August 20, 1938.)

388. Jesse Douglas: *A Jordan space curve having the infinite area property at each of its points.*

This paper appears in full in the Proceedings of the National Academy of Sciences, September, 1938. (Received July 29, 1938.)

389. Jesse Douglas: *The problem of Plateau.*

This paper, giving an exposition in as nontechnical a style as seems possible of some of the essential features of the author's work of the last twelve years on the problem of Plateau, is published in Scripta Mathematica, July, 1938. (Received July 29, 1938.)

390. Jesse Douglas: *Geometry of polygons in the complex plane.*

If, on each side of any given triangle as base, an isosceles triangle with 120° vertex-angle is constructed (always outward or always inward), then the vertices of these isosceles triangles form an equilateral triangle. This theorem of elementary geometry is here generalized by simple linear algebraic calculations to polygons of n sides, supposed to lie in the complex plane. Definitions: (i) An n-gon Π is a system of n points in a given cyclic order $(P_1, P_2, \cdots, P_n)$. (ii) The construction S_a (a any complex number) means the formation from Π of the n-gon $\Pi' = (P_1', P_2', \cdots, P_n')$, where each triangle $P_kP_{k+1}P_k'$ is directly similar to $(a, 1, 0)$. (iii) A regular n-gon of ω^p-type, $(\omega = e^{2\pi i/n})$, is one directly similar to $(\omega^p, \omega^{2p}, \cdots, \omega^{np})$. Theorems: (I) If, starting with any n-gon Π, one performs successively, in any order, the $n-2$ operations S_{ω^m}, $(m=1, 2, \cdots, n-1$, except $n-p)$, so getting $\Pi', \Pi'', \cdots, \Pi^{(n-2)}$, then $\Pi^{(n-2)}$ is a regular n-gon of ω^p-type, and identically the same one, including position, regardless of the order of the operations S_{ω^m}. (II) If also $m=p$ is omitted, as well as $m=n-p$, then $\Pi^{(n-3)}$ is affinely equivalent to a regular polygon of ω^p-type. (Received July 29, 1938.)

391. W. D. Duthie: *Pseudo-valuation for Boolean rings.*

The definitions of a pseudo-valuation for an arbitrary ring as given by Mahler (*Über Pseudobewertungen*, I, Acta Mathematica, vol. 36, pp. 79–119) and by Deuring (*Algebren*, Ergebnisse der Mathematik, vol. 4, no. 1), when applied to a Boolean ring, give rise to pseudo-valuations which are incapable of assuming any value between 0 and 1. It is the purpose of this note to point out a simple modification in the definitions cited which eliminates this difficulty and at the same time retains most, if not all, of the properties of the original definitions. The modified definitions are distinct in type. An example of each type is given. It is also shown that these definitions impose a relation on the pseudo-valuations of the ordered elements of the Boolean ring. (Received August 19, 1938.)

392. Edward Kasner and J. J. De Cicco: *Conformal geometry of horn angles of second order contact.*

The object of this paper is to develop the conformal geometry of a horn angle of second order contact. A horn-set (γ) is defined as the totality of all curves (or fifth order differentiable elements) which pass through a given point in a common direction and which possess the same curvature γ. Let $x = d\gamma/ds$, $y = d^2\gamma/ds^2$, $z = d^3\gamma/ds^3$, where γ is the curvature and s is the arc length, of any curve C of the horn-set (γ) at the fixed point. A *linear series* is the totality of curves of a horn-set (γ) which satisfy the two linear equations $y = px + r$, $z = qx + s$, where p, q, r, s are constants. A *flat con-*

gruence is the totality of curves of a horn-set (γ) which satisfy the linear equation $z=ax+by+c$, where a, b, c are constants. It is found that the group of conformal transformations induces a fundamental five-parameter group G_5 between the curves of a horn-set (γ) and the curves of a horn-set (Γ). Under G_5 it results that linear series are carried into linear series and flat congruences are converted into flat congruences. The elementary invariants between the curves, the linear series, and the flat congruences of a horn-set (γ) are obtained. (Received August 4, 1938.)

393. F. L. Lamoreau: *Temperature distribution in a long rectangular bar of two layers.*

The problem is that of finding the temperature distribution at any instant in a long rectangular bar composed of two layers, where each layer is of different material. The outer lateral surface of the bar is held at zero temperature, and the initial temperature distribution in a cross section of the two layers is arbitrarily given. The procedure is similar to that used by Churchill, (*Temperature distribution in a slab of two layers*, Duke Mathematical Journal, vol. 2 (1936), pp. 405–414) in solving the analogous one-dimensional problem. The treatment of the formal work has been simplified by the introduction of a bi-orthogonal set of functions. (Received August 19, 1938.)

394. K. W. Lamson: *Linear families of linear S_2-complexes in S_5.*

An S_2-complex in S_5 is given by the equation $a_{ijk}p^{ijk}=0$ wherein p^{ijk} are the coordinates of an S_2. In this paper, the family $(\lambda a_{ijk}+\mu b_{ijk})p^{ijk}=0$ is studied. The following properties are found: in the general case, the family is determined by four arbitrary planes; if the invariant of the complex (of the fourth degree in λ, μ) vanishes, the complex has a directrix; if two of the four directrices intersect, the other two also intersect. (Received August 13, 1938.)

395. A. N. Lowan: *On the computation of a certain infinite series.*

In a previous paper (*Cooling of a radioactive sphere*, Physical Review, vol. 44 (1933)), the author derived the formula $T(r, t)=2R^3/(\pi^2Kr)\cdot\sum_{s=1}^{\infty}(1-e^{-s^2c})(1/s^2)$ sin $\cdot$ sin $(2s\pi r/R)\int_0^1 x\phi(Rx)$ sin $s\pi x\,dx$, for the contribution to the earth's temperature arising from radioactivity of the rocks. This series is very slowly convergent. However, if we write $T_1(r)=(2R^3/\pi^2Kr)\sum_{s=1}^{\infty}(1/s^2)$ sin $(s\pi r/R)\int_0^1 x\phi(Rx)$ sin $s\pi x\,dx$, and $T_2(r, t)=(2R^3/\pi^2Kr)\sum_{s=1}^{\infty}e^{-s^2c}(1/s^2)$ sin $(s\pi r/R)\int_0^1 x\phi(Rx)$ sin $s\pi x\,dx$, then $T(r, t) =T_1(r)-T_2(r, t)$. Here $T_1(r)$ represents the distribution of temperature corresponding to the steady state and is therefore a solution of the differential equation of heat conduction, in which the term $\partial/\partial t$ has been dropped; $T_1(r)$ is thus obtained in closed form suitable for computations. The convergence of $T_2(r, t)$ is considerably faster. Upper bounds for the remainder, when the series has been truncated after n terms, are obtained (by replacing the infinite series representing the remainder by suitable definite integrals) for the case $\phi=Ae^{-\alpha(R-r)}$ in the two alternative forms $|E(n, r)|<2C(d/r)[1-\mathrm{Erf}(nc^{1/2})]$ and $E(n, r)<C(R/r)[-Ei(-en^2)]$, wherein C is a certain dimensional constant. On the basis of the last two expressions for the remainder, it is shown that $T(r, t)$ may be computed to a depth of 1800 miles by keeping not more than 25 terms in the infinite series representation of $T(r, t)$. (Received August 26, 1938.)

396. A. N. Lowan: *On wave motion for subinfinite domains.*

In a previous paper (*On wave motion with subinfinite domains*, to appear in the

Philosophical Magazine), the author investigated the integration of the system S: (I) $\nabla^2 u(P, t)=2b\partial u(P, t)/\partial t+(1/a^2)\partial^2 u(P, t)/\partial t^2+\Phi(P, t)$; (II) $\lim_{t\to 0} u(P)=f(P)$; (III) $\lim_{t\to 0} \partial u(P, t)/\partial t=g(P)$, for infinite domains of one, two, and three dimensions. In the present paper, (I) is integrated for the following domains and boundary conditions: (1) $0<x<\infty$, $u(0, t)=\phi(t)$; (2a) $0<x<\infty$, $-\infty<y<\infty$, $u(0, y, t)=\phi(y, t)$; (2b) $0<x<\infty$, $0<y<\infty$, $u(0, y, t)=\phi_1(y, t)$, $u_2(x, 0, t)=\phi_2(x, t)$; (3a) $0<x<\infty$, $-\infty<y<\infty$, $-\infty<z<\infty$, $u(0, y, z, t)=\phi(y, z, t)$; (3b) $0<x<\infty$, $0<y<\infty$, $-\infty<z<\infty$, $u(0, y, z, t)=\phi_1(y, z, t)$, $u(x, 0, y, t)=\phi_2(x, y, t)$; (3c) $0<x<\infty$, $0<y<\infty$, $0<z<\infty$, $u(0, y, z, t)=\phi_1(y, z, t)$, $u(x, 0, z, t)=\phi_2(x, z, t)$, $u(x, y, 0, t)=\phi_3(y, z, t)$. In all the above cases $U(P, t)=u(P, t)+v(P, t)$, $u(P, t)$ satisfies (I) and vanishes at the given boundaries, $v(P, t)$ satisfies the system S' obtained from S by putting $f=g=\phi=0$ and assumes the prescribed boundary conditions; $u(P, t)$ may be obtained from the corresponding solutions given in the previous paper by suitably extending the definitions of f, g, and ϕ. The solutions $v(P, t)$ may be obtained by deriving the Laplace transform $v^*(P, p)$ and then reverting to $v(P, t)$. (Received August 23, 1938.)

397. L. A. MacColl: *Differential-geometric aspects of relativistic dynamics.*

Kasner has determined a set of five geometrical properties which characterizes the families of trajectories of a particle moving in a plane, according to the laws of Newtonian dynamics, under forces which are functions of position only (Transactions of this Society, vol. 7 (1906), pp. 401–424; also *Differential-geometric aspects of dynamics*, American Mathematical Society Colloquium Publications, vol. 3_2, 1913, pp. 9–17). In the present paper, the modification of Kasner's problem which is obtained by assuming that the particle moves according to the laws of special relativistic dynamics is discussed. It is found that the families of trajectories can be characterized by a set of seven geometrical properties. Five of these resemble, in various degrees, the properties obtained by Kasner, while the remaining two properties have no classical analogs. (Received August 31, 1938.)

398. Dorothy Manning: *On simply transitive groups with transitive abelian subgroups of the same degree.* III.

A simply transitive group of degree $p_1^{a_1}p_2^{a_2}\cdots p_k^{a_k}$ ($p_1, p_2, \cdots, p_k$ primes, not necessarily distinct, $k\geqq 2$) which contains a transitive abelian subgroup of the same degree generated by k permutations of orders $p_1^{a_1}, p_2^{a_2}, \cdots, p_k^{a_k}$, respectively, is imprimitive and compound when $p_1^{a_1}>p_2^{a_2}>\cdots>p_k^{a_k}$. It may be primitive when $p_1^{a_1}=p_2^{a_2}=\cdots=p_k^{a_k}$. (Received August 25, 1938.)

399. Kyrille Popoff: *Application des méthodes d'intégration de Poincaré à la balistique extérieure.*

Les méthodes d'intégration de Poincaré, établiés dans la *Mécanique Céleste*, s'appliquent aussi bien à l'étude du mouvement du centre de gravité du projectile sur la trajectoire qu'à l'étude du mouvement du projectile autour de son centre de gravité. La considération des intégrales des équations différentielles comme fonctions des paramètres figurant dans ces équations nous a permis de développer les intégrales du mouvement du centre de gravité du projectile en séries procédant suivant les puissances de $\sin^2 \phi/2$, où on a $\phi=\pi/2+\alpha$ et où α est l'angle de projection. Nous étudions ensuite le rayon de convergence de ces séries et par une transformation conforme nous arrivons à des séries convergentes dans un intervalle $(0, T)$ de temps, où T est un nombre fini, choisi d'avance d'une manière arbitraire. Pour arriver à cela nous étudions d'abord les points singuliers des équations différentielles du mouvement et établissons l'existence des solutions asymptotiques à ces points et le caractère analytique de ces solutions. L'étude du mouvement de précession du projectile autour de son centre de gravité nous conduit à un système d'équations différentielles linéaires, que nous intégrons en formant soit les équations aux variations, soit en les réduisant à un système d'équations intégrales de Volterra. Nous établissons ainsi l'existance de solutions asymptotiques aux points singuliers de nos équations différentielles du mouvement de précession, et l'existence de solutions périodiques et de solutions asymptotiques à ces solutions périodiques. L'étude du mouvement de nutation se fait après suivant les mêmes principes. Le calcul des coefficients des séries que nous établissons se ramène a des quadratures. (Received August 29, 1938.)

400. Hillel Poritsky: *On algebraic equations whose roots lie in the negative half-plane.*

The tests of Routh, Hurwitz, and Schur are reviewed and deduced from Cauchy's theorem, and an extension of the Routh criterion to equations with complex coefficients is obtained. (Received September 6, 1938.)

401. J. H. Roberts: *A problem on acyclic continuous curves.*

The following theorem answers a question of O. H. Hamilton (Transactions of this Society, vol. 44 (1938), p. 24): If M is any acyclic continuous curve, there exists a topological transformation T, other than the identity, which maps M into a subset (not necessarily proper) of M. (Received August 15, 1938.)

402. J. H. Roberts: *Concerning homeomorphisms of the plane into itself.*

The following result has been obtained: The space of homeomorphisms of a plane S into itself consists of exactly two arc-wise connected components. More precisely, if π is any homeomorphism of S into itself, there exists a homeomorphism π_1 of S into itself which is either the identity or a reflection in the y-axis and a single-valued continuous function $f(p, t)$ defined over the range $p\varepsilon S$, $(0 \leqq t \leqq 1)$, with values in S, such that (1) for fixed t, $f(p, t)$ is a homeomorphism of S into S, and (2) $f(p, 0) = \pi(p)$ and $f(p, 1) = \pi_1(p)$. (Received August 15, 1938.)

403. Louis Weisner: *Condition that a finite group be multiply isomorphic with each of its irreducible representations.*

The author proves the following theorem: A necessary and sufficient condition that a finite group G be multiply isomorphic with each of its irreducible representations as a linear group is that G contain a subgroup P whose order is a power of a prime p, such that (1) P is generated by the minimal invariant subgroups of G whose orders are powers of p; and (2) every subgroup of index p of P contains an invariant subgroup of G different from the identity group. (Received September 1, 1938.)

404. Louis Weisner: *The subgroup of order n of a transitive group of degree n and class $n-1$.*

It is well known that a transitive permutation group of degree n and class $n-1$ contains an invariant subgroup of order n, composed of the identity and the permutations of degree n. The author proves that this subgroup of order n is an abelian group. (Received August 23, 1938.)

405. R. M. Robinson: *A generalization of Picard's and related theorems.*

According to Picard's theorem, a meromorphic function which leaves out more than two values is a constant. We shall say that a is an exceptional value of order m for $f(z)$ if the equation $f(z)=a$ has no root of multiplicity less than m (where m is a positive integer or ∞). To such an exceptional value we assign the weight $1-1/m$. A value which is never assumed by the function is an exceptional value of order ∞ and hence of weight 1. The generalization of Picard's theorem may then be stated: *A meromorphic function which admits exceptional values of total weight more than 2 is a constant.* Corresponding generalizations are made of the theorems of Landau, Schottky, Montel (criterion for normal family), and of the second Picard theorem. Of these, only the last is new; but the proofs given are simpler than those already known. The previous proofs depended on Nevanlinna's theory of meromorphic functions. The present proof is based on Ahlfors's recent generalization of Schwarz's lemma, which appeared in the May issue of the Transactions of this Society. (Received August 26, 1938.)