DERRICK NORMAN LEHMER—IN MEMORIAM

After thirty-seven years of service in the department of mathematics at the University of California, Professor Derrick Norman Lehmer died on September 8, 1938. He came to the University of California in 1900, just after receiving the degree of Doctor of Philosophy at the University of Chicago. All of his academic career was therefore spent at California. He retired in 1937, apparently in robust health and with plans and ambitions for a life of continued activity with numerous interests, to all of which he had productively contributed.

Born in Indiana, July 27, 1867, he graduated in 1893 from the University of Nebraska, from which he also received the degree of Master of Arts in 1896, and the honorary degree of Doctor of Science in 1932. He married Clara Eunice Mitchell in 1900, and three daughters and two sons were born to them. One of the sons, D. H. Lehmer, followed in the footsteps of his father and is now an assistant professor of mathematics at Lehigh University.

Apart from mathematics he evidenced great interest and ability in both poetry and music. This was shown by his membership in numerous literary organizations accorded only to those who had indicated productive ability; by his editorship of the University of California Chronicle, devoted to the publication of articles of a literary character; and by numerous contributions of his own, mostly in the field of poetry.

His membership in honorary and learned societies included Phi Beta Kappa, Sigma Xi, the American Mathematical Society, the Mathematical Association of America, the Circolo Matematico di Palermo, and the American Association for the Advancement of Science, of which he was a fellow.

For a number of years he devoted much of his spare time to creative music, and was particularly interested in the music of western Indians, producing two operas based on Indian themes, "The Harvest" and "The Necklace of the Sun." In addition to these he wrote and produced a number of songs based on Indian legend and musical themes.

His earlier mathematical interests lay in the field of synthetic projective geometry and the theory of numbers; indeed, the latter subject continued to occupy Lehmer's attention throughout his entire career.

He found in rational numbers a fascination which lasted throughout his life. Primes, residues, quadratic forms, divisors of forms, continued fractions and their generalizations were seldom absent from his thoughts. Above all, the problem of factoring never failed to exert its appeal. The abstruse results of higher theory can be made to contribute to that tantalizing problem, though not yielding a complete solution. Such a situation was of the kind to especially attract him, and a large number was always a challenge. The possibility of experimental verification of a theorem or of a conjecture so often present in the theory of numbers led him to seek to increase the material available for such verification and so to extend and correct the factor table. This was the largest and most finished of his undertakings.

His table giving the least divisors of the first ten million numbers is a monument of accuracy and judgment. Others, notably Dase, Burckhardt, Glaisher, had labored successfully and had set a standard which he surpassed. This work occupied his available time for about nine years. Lehmer was not a computer, such as were Dase or Burckhardt, but like Glaisher, a mathematician, alive to the significance of theory and viewing the table as a contribution to it.

If an irrational number is developed in a continued fraction, the convergents have well known properties regardless of which irrational number is developed. But if the irrational is some distinguished number, the convergents may be expected to have special properties. To find these is a difficult problem toward the solution of which little has been done. An example may serve. Cotes found the continued fraction for e to be $2+1/1+1/2+1/1+1/1+1/4+1/1+\cdots$ where there are triads 1, $2k$, 1 in the quotients. Lehmer's result is this: the denominators of the convergents of orders $3n$, $3n-2$, $3n-6$ and the numerator of order $3n-3$ are divisible by n. Continued fractions whose quotients present a regularity of the type exemplified by e had been studied by Hurwitz, and Lehmer was able to extend his results to the point where particular theorems such as the one just given may be obtained.

Continued fractions may be generalized in different directions. Of these generalizations one due to Jacobi is interesting. Triads of numbers are determined by relations of recurrence just as pairs are determined in the case of the simple continued fraction. If there is periodicity in the recurrence relations, the triads may furnish convergents to the roots of a certain cubic equation, but they do not always do so. The distinction required elucidation which was given in several papers that cleared up the matter completely.

To analytical number theory an important contribution was made by Lehmer. A special result must suffice, but one typical of a wide class. Let $\theta(x)=1$ if all the prime divisors of x are of the form $4n+1$, but otherwise let $\theta(x)=0$. Let $V(x)$ denote the number of distinct prime divisors of x, then the quotient $(1/N)\sum_1^N 2^{V(x)}\theta(x)$ tends to a limit when N increases. The limit is $1/\pi$. Such determinations must be made if answers to many simple questions are required. Quotients similar to the above do not always have limits. Lehmer was able to obtain definitive results in important cases. His conjectures in the cases he was not able to treat fully were examined by Landau.

In the contrast to number theory which geometry offers, Lehmer often found relaxation. He taught projective geometry with enthusiasm and wrote a text introductory to the subject. His own contributions were directed to the application of synthetic methods to less usual topics such as the unicursal cubic, the covariant conic of a pencil, quadratic transformations. His insistence was on a solution in the fewest possible steps so that constructions should not only be theoretically possible but might really be effected. Variety of interest was added by studies on the tetrahedroid, planimetric devices, the quadratic complex, the spiral of Cornu.

He was a distinguished teacher with power to kindle enthusiasm not only in the classroom, but in his occasional lectures, when he presented the wonders of number theory, or exhibited curves in space traced by lights in a dark room by means of a mechanism devised by one of his students, Dr. Saul Pollock. He supervised personally, spending freely of time and thought, the production of more than twenty theses for the doctorate.

The variety of his mathematical interests is well shown in the appended bibliography which is arranged chronologically.

PUBLICATIONS OF D. N. LEHMER

1896. *Proof of a theorem in continued fractions.* Annals of Mathematics, (1), vol. 11, p. 64.

1899. *Concerning the tractrix of a curve with planimetric applications.* Annals of Mathematics, (2), vol. 1, p. 14.

1900. *The asymptotic evaluation of certain totient sums.* American Journal of Mathematics, vol. 22, p. 293.

Rational triangles. Annals of Mathematics, (2), vol. 1, p. 97.

1901. *Multiply perfect numbers.* Annals of Mathematics, (2), vol. 2, p. 1.

A theorem in continued fractions. Annals of Mathematics, (2), vol. 2, p. 146.

1902. *Constructive theory of the unicursal cubic by synthetic methods.* Transactions of this Society, vol. 3, p. 372.

Errors in Legendre's tables of linear divisors. This Bulletin, vol. 8, p. 401.

A new short method of multiplication. Science, vol. 16, p. 71.

1903. *The parametric representation of the tetrahedroid surface.* American Journal of Mathematics, vol. 25, p. 1.

Note on negative digits. Science, vol. 17, p. 514.

1904. *Cornu's spiral as a transition curve.* California Journal of Technology, April.

1905. *On a cylinder the intersection of which with a sphere will develop into an ellipse.* American Mathematical Monthly, vol. 11, p. 186.

1906. *On the orderly listing of substitutions.* This Bulletin, vol. 13, p. 81.

1907. *On maximum and minimum values of the modulus of a polynomial.* Annals of Mathematics, (2), vol. 8, p. 175.

A theorem in the theory of numbers. This Bulletin, vol. 13, p. 501.

1908. *A discussion by synthetic methods of the covariant conic of two given conics.* American Mathematical Monthly, vol. 15, p. 29.

1909. *Factor Table for the First Ten Millions.* The Carnegie Institution, Washington.

1910. *On the theory of inversion as a special case of a reciprocal quadratic transformation.* American Mathematical Monthly, vol. 17, p. 135.

1911. *On the combination of involutions.* American Mathematical Monthly, vol. 18, p. 52.

1912. *On the arithmetical theory of pencils of binary quadratic forms.* American Journal of Mathematics, vol. 34, p. 21.

Note on prime numbers. American Mathematical Monthly, vol. 19, p. 50.

1913. *Certain theorems in the theory of quadratic residues.* American Mathematical Monthly, vol. 20, p. 151.

1914. *A theorem in number theory connected with the binomial formula.* American Mathematical Monthly, vol. 21, p. 15.

On a purely projective basis for the theory of involution. American Mathematical Monthly, vol. 21, p. 185.

List of Prime Numbers from 1 *to* 10,006,721. The Carnegie Institution, Washington.

On a special case of the tetrahedral complex. American Mathematical Monthly, vol. 21, p. 287.

1917. *Synthetic Projective Geometry.* Ginn and Co., Boston.

1918. *Arithmetical theory of certain Hurwitzian continued fractions.* Proceedings of the National Academy of Sciences, vol. 4, p. 214. American Journal of Mathematics, vol. 40, p. 375.

On Jacobi's extension of the continued fraction algorithm. Proceedings of the National Academy of Sciences, vol. 4, p. 360.

On the history of the problem of separating a number into its prime factors. Scientific Monthly, vol. 7, p. 227.

1919. *The general solution of the indeterminate equation* $Ax+By+Cz+\cdots=r$. Proceedings of the National Academy of Sciences, vol. 5, p. 111.

Note on the number of solutions of linear indeterminate equations. American Mathematical Monthly, vol. 26, p. 365.

1923. *On the multiplication of large numbers.* American Mathematical Monthly, vol. 30, p. 67.
1925. *A new method of factorization.* Proceedings of the National Academy of Sciences, vol. 11, p. 97.
Note on the construction of tables of linear forms. This Bulletin, vol. 31, p. 497.
1926. *On the construction of factor stencils.* This Bulletin, vol. 32, p. 149.
1928. *Waring Table of Fourth Powers.* The Carnegie Institution, Washington.
1929. *On Factor Stencils.* The Carnegie Institution, Washington.
On the congruences connected with certain magic squares. Transactions of this Society, vol. 31, p. 529.
1930. *On the factorization of large numbers.* University of California Chronicle, vol. 32, p. 326.
1931. *Inverse ternary continued fractions.* This Bulletin, vol. 37, p. 565.
1933. *Some new factorizations of* $2^n \pm 1$. This Bulletin, vol. 39, p. 105.
Hunting big game in the theory of numbers. Scripta Mathematica, vol. 1, p. 229.
On ternary continued fractions. Tôhoku Mathematical Journal, vol. 37, p. 436.
A complete census of 4×4 *magic squares.* This Bulletin, vol. 39, p. 764.
1934. *On the enumeration of magic cubes.* This Bulletin, vol. 40, p. 833.
1935. *Law of reciprocity for tables of linear forms.* Tôhoku Mathematical Journal, vol. 40, p. 405.

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