systematic treatment of convergence factor theory in book form should have been written by him.

The author points out that all methods for the summation of a divergent series which have come into general use may be classified as mean-value methods or convergence-factor methods. In either case the object of the summability method is to determine a value or "generalized sum" for the series. Although convergence factors were first applied to summable series, it was soon seen that they could yield valuable information when applied to convergent series.

Moore classifies convergence factors into two types. In type I he places the factors which have only the property that they preserve convergence for a convergent series or produce convergence for a summable series. In type II he places the factors which not only maintain or produce convergence but have the additional property that they may be used to obtain the sum or generalized sum of the series. This book gives a generalized systematic treatment of the theory of convergence factors of both types, for simply infinite series and for multiple series, convergent and summable. For summability the author uses the method of Nörlund means instead of the Cesàro method, giving more general results. Many of the theorems and methods given in this work are original and the proofs have not been published elsewhere.

The book opens with an Introduction in which the early history of the idea of summability of series is discussed, and the emergence of the concept of convergence factors is traced historically. Chapter 1 deals with convergence factors in convergent series. For convergence factors of type I, necessary and sufficient conditions are derived in order that a convergent series may still converge after convergence factors are introduced into its terms. For factors of type II, additional necessary and sufficient conditions are obtained in order that the series with convergence factors may converge to the sum of the original series. Corresponding results are derived for multiple series whose partial sums are bounded. These convergence factor theorems serve to furnish criteria for the regularity of convergence-factor definitions and mean-value definitions of summability; they may also be used to obtain results concerning relations of equivalence and inclusion of summability methods.

In Chapter 2, a brief discussion is given of Nörlund summability of series, simple and multiple. Chapter 3 then takes up, for summable simple series, the corresponding questions to those of Chapter 1, for convergence factors of both types; Chapter 4 handles the case of summable double series, and Chapter 5 that of summable multiple series of higher order. Results for the Cesàro method of summability are obtained as special cases. The final chapter is concerned with convergence factors in restrictedly convergent multiple series. The book closes with a rather full bibliography of the subject.

Many of the convergence factor theorems hitherto published in the literature appear as special cases of the general theorems here derived. This book makes considerable progress in the attempt to present a definitive form to the subject of convergence factors.

L. L. SMAIL

Analyse Mathématique. Vol. 1. *Analyse des Courbes, Surfaces et Fonctions usuelles. Intégrales simples.* By Paul Appell. 5th edition, completely revised by Georges Valiron. Paris, Gauthier-Villars, 1937. 8+395 pp.

The first edition of this work appeared in 1898 and, according to the preface, it reproduced the course which for three years Paul Appell had been offering at l'École Central des Arts et Manufactures. It covered the essential elements of mathematical analysis from the standpoint of the applications to geometry, mechanics, and physics. In the early revisions of the text Appell made notable changes with a view to strengthening the preparation for the study of mechanics and physics. The fourth edition appeared in 1921 and, in addition to representing the course given at l'École Central, it included certain material which figured in the program of admission; the reader, however, was assumed to have a knowledge of the exponential and logarithmic functions, elementary differential calculus, complex numbers, and the elements of analytic geometry.

In presenting the first volume of the new edition, Georges Valiron has made numerous changes not only in the choice and order of the subject matter but also in the method of treatment. He reminds us that the book is not intended to be a mathematical encyclopedia, but rather a course of instruction which is progressively more difficult and introduces new notions only as they are needed. For example, complex numbers are not presented until very late in the course when the advantage of their use in certain problems is readily appreciated.

The first three chapters, which have been added by Valiron, serve as a geometric introduction and are based on lectures commonly given at the beginning of the course in mechanics at the Sorbonne. In these eighty pages the author introduces the vector notation and then develops the elements of plane and solid analytic geometry. Although Valiron's treatment is more extensive, it reminds one somewhat of Appell's *Éléments de la Théorie des Vecteurs et de la Géométrie Analytique.*

Beginning with the fourth chapter the new edition develops the present program of analysis and geometry for the certificate in general mathematics of the Faculty of Sciences of Paris. This program has been in use since 1931 and it preserves the spirit of Appell's work even though the general plan has been somewhat altered. The scope of this portion of the book is indicated by the chapter headings: 4. Fonctions d'une variable. Limites. Continuité. 5. Fonctions dérivables. 6. Fonctions primitives. Intégrales. Différentielles. 7. Fonctions exponentielle et logarithmique. 8. Méthodes d'intégration. 9. Intégrales définies dont une limite est infinie où portant sur une fonction non bornée. 10. Fonctions de plusieurs variables. 11. Courbes planes ou gauches. Courbure. Enveloppes. 12. Étude des courbes en coordonnées polaires. 13. Intégrales curvilignes. Calcul des aires et des volumes.

A comparison with the fourth edition shows that Valiron has rearranged the subject matter considerably. There are frequent additions; in particular, the chapter on exponential and logarithmic functions is new. Vector methods have simplified many of the discussions; this is especially noticeable in the study of plane and space curves. In this first volume very little use is made of infinite series. Numerous well-chosen examples from the fields of geometry and physics are solved in the text and much emphasis is placed on graphic representation and methods of approximation. The figures are numerous and very well drawn. The text is attractively printed and the number of typographical errors noted was small.

C. H. YEATON

The Theory of Linear Operators from the Standpoint of Differential Equations of Infinite Order. By H. T. Davis. Bloomington, Indiana, Principia 1936. 14+628 pp.

List of contents: 1. Linear operators. 2. Particular operators. 3. The theory of linear systems of equations. 4. Operational multiplication and inversion. 5. Grades defined by special operators. 6. Differential equations of infinite order with constant coefficients. 7. Linear systems of differential equations of infinite order with constant