ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

397. Ben Dushnik and E. W. Miller: *Concerning similarity transformations of linearly ordered sets.*

This paper is concerned with the following questions: (1) Is every linearly ordered infinite set A similar to a proper subset of itself? (2) Let A be a linearly ordered set such that if f is any similarity transformation of A into a subset of itself, then $f(a) \geqq a$ for every a in A. Is it true that every such set A is well ordered? It is shown that the answer to each of these questions is in the affirmative if A is denumerable. An example is constructed to show that these conclusions need not hold if A is non-denumerable. (Received August 23, 1939.)

398. J. J. DeCicco: *The circular group in an infinite plane of the Kasner space.*

A horn-set (γ) consists of all the curves of the plane which pass through a given point in a common direction and have the same curvature γ. Let $x=2\gamma'$, $y=5^{1/2}\gamma''$, $z=2(\gamma'''-\gamma^2\gamma')$, where $(\gamma', \gamma'', \gamma''')$ are the first three derivatives of the curvature γ of any curve C of a horn-set (γ) at the common point. A horn-set (γ) may be regarded as a three-dimensional space, called the Kasner space K_3, where any point of K_3 is a curve $C(x, y, z)$ of the horn-set (γ). The group of conformal transformations induces a special affine five-parameter group G_5 between the Kasner spaces. The ∞^2 planes $4z=-b^2x+4by+4c$ are called the infinite planes of K_3. If $u=-x$, $v=bx/2-y$ denotes any point of an infinite plane, the group G_5 induces the metric $M_{12}=(u_2-u_1)^3/(v_2-v_1)^2$ between any two points of an infinite plane. The circles are the semicubical parabolas $(u-a)^3=R(v-b)^2$. The circular group is $U=au+c$, $V=bv+d$. A minimal characterization of this group is also obtained. (Received August 2, 1939.)

399. D. W. Hall (National Research Fellow) and G. T. Whyburn: *An analysis of arc-preserving transformations.*

Arc-preserving transformations have previously been defined and studied by one of the authors (G. T. Whyburn, American Journal of Mathematics, vol. 58 (1936), pp. 306–312) and the present paper results from a continuation of that investigation. Let K denote the set of all cut points and end points of a compact locally connected continuum A. A point of $A-K$ is called an internal point of the cyclic element of A containing it. Then (i) if $T(A)=B$ is continuous, in order that T be arc-preserving the following conditions are necessary and sufficient: (a) for each true cyclic element E_b of B there exists a true cyclic element E_a of A mapping onto E_b topologically under

T and such that T^{-1} is single-valued on the set $T(H)$, where H is the set of all internal points of E_a, and (b) the image of each cyclic chain in A is a cyclic chain in B. (ii) If $T(A)=B$ is arc-preserving, then the image of each true cyclic element E_a of A is a point, a free arc of B lying in no true cyclic element of B, or a true cyclic element E_b of B in which case $T(E_a)=E_b$ is topological. (Received September 2, 1939.)

400. Edward Kasner and J. J. DeCicco: *Characterization of the conformal group by horn angles of second order.*

This is a continuation of the paper by Kasner, *Characterization of the conformal and equilong groups by horn angles* (Duke Mathematical Journal, vol. 4 (1938), pp. 95–106). A horn angle is the configuration formed by two curves of a plane which pass through a common point (the vertex) in a given direction. It is said to be of order n if the curved sides have $n+1$ but not $n+2$ points in common at the vertex. Every horn angle of order n has a unique absolute conformal differential invariant of order $2n+1$. (See Kasner, *Conformal geometry*, Proceedings of the Fifth International Congress of Mathematicians, vol. 2 (1912), p. 81.) This is called the conformal measure of the horn angle. In this paper, it is proved that the conformal group is characterized by the preservation of the conformal measure of every horn angle of *second* order. In the Duke paper, the same thing was done for a horn angle of first order. Finally, it is found that the conformal group is characterized by the preservation of either of the two relative conformal invariants for a horn angle of second order, namely, one of the third and the other of the fifth order. (Received August 2, 1939.)

401. E. R. Lorch: *Spectral analysis of weakly almost periodic transformations in reflexive vector spaces.*

It is proved that in a weakly compact space, a bicontinuous linear transformation V is weakly almost periodic if and only if it is uniformly bounded, $|V^n|\leqq K$, $n=0, \pm 1, \pm 2, \cdots$. If the underlying space $\mathfrak{B}$ is reflexive and if V is weakly almost periodic in $\mathfrak{B}$, then the following spectral analysis of V is possible: For every θ in the interval $-\pi<\theta\leqq\pi$, there exist two closed linear manifolds $\mathfrak{M}_\theta$ and $\mathfrak{N}_\theta$ such that $\mathfrak{M}_\theta$ and $\mathfrak{N}_\theta$ have only the element 0 in common and together span the space $\mathfrak{B}$. These manifolds reduce V. Conditions are given that $\mathfrak{M}_\theta$ and $\mathfrak{N}_\theta$ be disjoint thus generating a projection. The exact nature of the above analysis may best be indicated by applying it to a well known special case: If V is a unitary transformation in a Hilbert space, then $\mathfrak{M}_\theta$ and $\mathfrak{N}_\theta$ yield immediately the resolution of the identity of V. (Received August 29, 1939.)

402. Deane Montgomery and Leo Zippin: *Topological group foundations of "rigid" space geometries.*

The axioms of Hilbert on the foundations of plane geometry (*Grundlagen der Geometrie*, Mathematische Annalen, vol. 56 (1902); also, Appendix IV, 7th edition of *Grundlagen der Geometrie* (Teubner)) cannot be carried over, without some addition, to a characterization of 3-space geometries. The present paper deals with ordinary 3-space and a system G of homeomorphisms satisfying Hilbert's axioms I and III and the following II*: there exists at least one point p such that the group G_p, leaving p fixed, is a proper subgroup of G and furthermore such that the orbits $G_p(x)$, for at least one sequence of points converging to p, are two-dimensional. This asks less than Hilbert's II in that it is postulated of one point of space; it asks incomparably more at that point. It is shown that of any three systems satisfying these conditions at least two are abstractly identical. There are, of course, two cases, euclidean and hyperbolic. (Received August 2, 1939.)

403. Gabor Szegö: *On the gradient of solid harmonic polynomials.*

Let $u(x, y, z)$ be a real harmonic polynomial of the nth degree ($n \geqq 4$) which satisfies the inequality $|u(x, y, z)| \leqq 1$ in the unit sphere $x^2+y^2+z^2 \leqq 1$. Then $|\text{grad } u| \leqq 2n\{1/1-1/3+\cdots+(-1)^n/(2n-1)\}+[(-1)^n-1]/2$ holds for $x^2+y^2+z^2 \leqq 1$, and this is the precise upper bound. This is the analogue of a previous theorem of the author (Königsberger Gelehrte Gesellschaft, 1928, pp. 59–70) on the gradient of harmonic polynomials in two variables, which in turn is a more precise form of S. Bernstein's classical theorem on trigonometric polynomials. (Received August 15, 1939.)

404. V. W. Adkisson and Saunders MacLane: *On extending maps of plane Peano continua.*

Let the Peano continuum M be situated on a sphere S and let the homeomorphism T map M into a subset M' of a sphere S'. Then T can be extended to a homeomorphism which carries the whole sphere S into S' if and only if any two triods of M which have the same sense on S are mapped by T into triods of M' having the same sense on S'. This theorem is established by using Kline's analysis of sense on closed curves and by other arguments employed in Gehman's condition for the extendability of T (Transactions of this Society, vol. 28 (1926), pp. 252–265). In the condition for extendability, essentially only those triods with vertices at cut or split points of M need be considered. The proof of this fact requires the construction of a maximal triply connected subcontinuum of M containing any specified θ-graph in M, as well as certain existence theorems for triods. (Received September 19, 1939.)

405. R. P. Agnew: *On continuity and periodicity of measurable functions.*

Let $x(t)$ denote a complex-valued function measurable over $-\infty<t<\infty$. Then there is a decreasing sequence $H_1>H_2>\cdots$ converging to zero with the following property. If $h_1, h_2, \cdots$ is a real sequence with $|h_n|<H_n$ for each $n=1, 2, \cdots$, then for each t_0 with the exception of a null set (which may depend upon the particular sequence h_n), $x(t)$ is continuous at t_0 over the set $t_0, t_0+h_1, t_0+h_2, \cdots$. This result is used to show that if $x(t)$ and $y(t)$ are measurable over $-\infty<t<\infty$, then the measurable upper bound over $-\infty<t<\infty$ of $|x(t+\lambda)-y(t)|$ is a lower semicontinuous function $G(\lambda)$. In case $y(t)=x(t)$, zeros of $G(\lambda)$ are *essential periods* of $x(t)$. It is shown that if $x(t)$ is measurable, essentially periodic, and not essentially constant, then $x(t)$ has a least positive essential period h and each essential period of $x(t)$ is an integer multiple of h. This implies that if $x(t)$ is measurable, periodic, and not essentially constant, then $x(t)$ has a least positive period H and each period of $x(t)$ is an integer multiple of H. (Received September 22, 1939.)

406. C. B. Allendoerfer: *The Euler number of an imbedded Riemann manifold.*

The space considered is a complete (closed) Riemann manifold R_n of n (even) dimensions imbedded in a euclidean space E_{n+q} where q may have any positive integral value. The total curvature of K of R_n is defined, and it is proved that the integral of K over R_n equals the Euler number of R_n times half the area of an n-sphere. The theorem is well known when $q=1$. (Received September 27, 1939.)

407. H. A. Arnold: *Postulates for the defective group.*

The postulates for the defective group D are (1) If x and y are distinct elements of D, there is a unique element $z=x\cdot y$ of D, corresponding to the ordered pair x, y. (2) If x, y, z are elements of D, and if $x\cdot y$, $y\cdot z$, $(x\cdot y)\cdot z$, $x\cdot(y\cdot z)$ all exist and are elements of D, then $(x\cdot y)\cdot z=x\cdot(y\cdot z)$. (3) If D has at least one element, then there exists at least one element e (identity) such that $e\cdot b$ exists and $eb=b$ for every element b of D. (4) If e is an element of D with the property in (3), and b is an element of D, there is at least one element x (the inverse of b) in D such that $x\cdot b$ exists and is an element of D and $x\cdot b=e$. The equality is taken to be logical identity. It is proved that $y\cdot e$ is an element of D for all y, that $y\cdot e=y$; that e is unique; that the inverse of an element is unique. A theory of homomorphism and its connection with Galois theory is formulated. The usual strengthening of Postulate (2) results in a Brandt Gruppoid or a Loewy Mischgruppe. A Brandt Gruppoid is homomorphic to a defective group. The strengthened defective groups of lower orders that are not groups are enumerated. (Received October 2, 1939.)

408. L. A. Aroian: *New continued fractions for the incomplete beta function.*

After a brief review of previous methods for the evaluation of the incomplete beta function $B_x(p, q)=\int_0^x x^{p-1}(1-x)^{q-1}dx$, the author shows that J. H. Muller's continued fraction in conjunction with a new continued fraction of the corresponding type furnishes a general method covering the range of the incomplete beta function for all values of x, p, and q. Convergence properties of Muller's continued fraction and the new continued fraction are established. The equivalent and associated types of continued fractions for the new continued fraction with their general properties are also given. While an upper bound is given for the error term in all cases, a remainder to the new continued fraction has not been found. Numerical examples illustrate these methods. Comparisons with previous methods show the superiority of the two continued fractions both from the point of view of theory and practice. (Received September 29, 1939.)

409. L. M. Blumenthal and C. V. Robinson: *A new metric characterization of the straight line.*

In this paper the following new metric characterization of the straight line is obtained: A necessary and sufficient condition that a complete convex externally convex metric space M (containing at least two points) be congruent with the straight line is that M does not possess three points a, b, c such that dist (a, b) $=$dist $(b, c)=$dist $(a, c)>0$. An earlier characterization of the straight line, given by Menger (Ergebnisse eines mathematischen Kolloquiums, Vienna, vol. 4 (1932), pp. 41–43) in terms of the notion of *quasi congruence order* is an immediate consequence of the above characterization, the proof of which is much less difficult than that offered for the earlier theorem. It follows from the above result that the simple property "absence of equilateral triples" is logically equivalent to the property "quasi congruence order 3" for complete convex externally convex metric spaces. (Received October 2, 1939.)

410. Salomon Bochner: *Harmonic surfaces in Riemann metric.*

A two-dimensional manifold in a given Riemannian space is a harmonic surface if it is a solution of the system of equations $\partial^2x^i/\partial u^2+\partial^2x^i/\partial v^2+\Gamma^i_{pq}(x)(\partial x^p\partial x^q/\partial u\partial u$

$+\partial x^p \partial x^q/\partial v\partial v)=0$. The paper presents some general properties of such surfaces and a compactness theorem for families of surfaces. (Received September 27, 1939.)

411. Alfred Brauer: *On the non-existence of the euclidean algorithm in certain quadratic number fields.*

The problem of determining all real euclidean quadratic fields $P(m^{1/2})$ has not been solved completely (cf. Hardy and Wright, *An Introduction to the Theory of Numbers*, 1938, pp. 212–217). Fifteen such fields are known. Except for these 15 fields the euclidean algorithm can only exist in the following cases (p and q denote primes): (1) $m=p\equiv 13 \pmod{24}$; (2) $m=p\equiv 1 \pmod{8}$; (3) $m=pq\equiv 1 \pmod{24}$. Moreover, the algorithm does not exist if m is sufficiently large. This was proved by analytical methods by Erdös and Ko for cases 1 and 2 (Journal of the London Mathematical Society, vol. 13 (1938), pp. 3–8), and by Heilbronn for case (3) (Proceedings of the Cambridge Philosophical Society, vol. 34 (1938), pp. 521–526). These results, however, give no bound m_0 such that the fields $P(m^{1/2})$ are not euclidean for $m>m_0$. In this paper it is shown by elementary methods that the algorithm does not exist in case (1) for $p>109$. In the proof, an inequality for the least odd quadratic non-residue, obtained by the author some years ago (Mathematische Zeitschrift, vol. 33 (1930), pp. 161–176), is improved. (Received September 29, 1939.)

412. R. P. Dilworth: *Lattices with unique irreducible decompositions.*

The main result of this paper is the following theorem: Let $\mathfrak{S}$ be a lattice with unit element in which every quotient lattice has finite dimensions. Then each element of $\mathfrak{S}$ is uniquely expressible as a reduced crosscut of irreducibles if and only if $\mathfrak{S}$ is a Birkhoff lattice in which every modular sublattice is distributive. (Received September 26, 1939.)

413. R. P. Dilworth: *The arithmetical theory of Birkhoff lattices.*

Let $\mathfrak{S}$ be a Birkhoff lattice with unit element in which every quotient lattice has finite dimensions. Let u_a denote the union of the elements covering a, and $\mathfrak{S}_a$ denote the quotient lattice of elements x such that $u_a \supseteq x \supseteq a$. An element b of $\mathfrak{S}_a$ is said to be *characteristic* if there exists an irreducible of $\mathfrak{S}$ which divides exactly the same points of $\mathfrak{S}_a$ as b. It is shown that a has a reduced irreducible decomposition $a=[q_1, \cdots, q_k]$ if and only if a has the reduced decomposition $a=[b_1, \cdots, b_k]$ where b_i is a characteristic element of $\mathfrak{S}_a$ divisible by q_i. Characteristic elements are determined in terms of the structure of the lattice as follows: an element b of $\mathfrak{S}_a$ is characteristic if and only if there exists a divisor x of b such that (x, b') covers x and $[x, b']=a$ for every complement b' of b in $\mathfrak{S}_a$. It is then shown that the number of components in the reduced irreducible decompositions of the elements of $\mathfrak{S}$ is unique if and only if $\mathfrak{S}_a$ is a modular sublattice of $\mathfrak{S}$ for each element a of $\mathfrak{S}$. A number of related arithmetical questions are also treated. (Received September 26, 1939.)

414. J. L. Doob and Warren Ambrose: *On continuous stochastic processes.*

The theory of continuous stochastic processes is the theory of measure on a space of functions of a real variable. One way of establishing such a measure has been given by Paley and Wiener (*Fourier Transforms in the Complex Domain*, American Mathematical Society Colloquium Publications, vol. 19, 1934, pp. 140–178; also see earlier papers by Wiener which are cited in this reference) and another has been given by Doob (Transactions of this Society, vol. 42 (1937), pp. 107–140). This paper establishes an equivalence between these two theories of measure. (Received October 2, 1939.)

415. Aaron Fialkow: *The conformal theory of curves.*

For a curve C in any Riemann space V_n, $(n>2)$, define an integral invariant S and $(n-1)$ differential invariants $J_1, J_2, \cdots, J_{n-1}$ which remain unchanged by any conformal transformation of V_n. The J's and S are the "conformal curvatures" and "conformal length" respectively of C. The J's (except J_{n-1}) appear in conformal analogues of the ordinary Frenet formulas. If $V_n \longleftrightarrow \overline{V}_n$, $C \longleftrightarrow \overline{C}$ by a conformal map, then the J's are the same functions of S for C and $\overline{C}$. Conversely, if V_n and $\overline{V}_n$ are conform-euclidean spaces and the J's for C and $\overline{C}$ are the same functions of S, then a conformal mapping exists for which $V_n \longleftrightarrow \overline{V}_n$, $C \longleftrightarrow \overline{C}$. If V_n is conform-euclidean, then $J_q \equiv 0$, $(1 \leqq q < n-1)$, if and only if C is conformally equivalent to a curve in an euclidean n-space whose $(q+1)$st metric curvature vanishes. For $n=2$, this theory applies if the conformal mappings are restricted to inversions of spaces of constant curvature. Throughout this paper, systematic use is made of a new kind of tensor differentiation which has conformal meaning. (Received September 28, 1939.)

416. M. C. Foster: *Note on autopolar curves.*

This paper considers autopolar curves as special solutions of differential equations invariant under the dual substitutions, for which the conic of reference is a parabola. (Received September 25, 1939.)

417. H. L. Garabedian: *Theorems associated with the Riesz and the Dirichlet's series methods of summation.*

The following theorems suggested by a theorem due to G. H. Hardy are established. (1) Suppose that $\lim_{n\to\infty} s_n e^{-\nu_n s}=0$ for all values of s for which $\Re(s)>0$, that the series $\sum u_n$ is summable $(\lambda, 1)$ to the sum U, and that ν_n is a logarithmico-exponential function of λ_n such that $\nu_n = O(\lambda_n^c)$, where c is any positive constant. Then the series $\sum u_n e^{-\nu_n s}$ converges for $\Re(s)>0$ and $\lim_{s\to 0} \sum u_n e^{-\nu_n s} = U$. (2) The Dirichlet's series definitions of summability include $(\lambda, 1)$ summability provided that ν_n is a logarithmico-exponential function of λ_n which tends to infinity with n but not as slowly as $\log n$ nor faster than λ_n^c, where c is any positive constant however large. The special case of the second theorem which pertains to logarithmic summability $(\log n, 1)$ is of particular interest. (Received August 15, 1939.)

418. Abe Gelbart: *On functions of two complex variables given by power series expansion.*

Let $f(z_1, z_2)=\sum a_{mn} z_1^m z_2^n$ be an entire function and let $W^2(r)$ be the smallest convex enveloping domain containing the set of points that $f(z_1, z_2)$ assumes on $E[z_2 = Ar^s e^{i\phi},\ z_1 = h(z_2, re^{i\lambda});\ 0 \leqq \phi,\ \lambda \leqq 2\pi;\ r_0 \leqq r < \infty]$. Consider the coefficients of $\exp[f(z_1, z_2)e^{i\alpha}]$. By using a certain generalized Newton polygon, determine a quantity $M(\alpha, r)$ for every α and r. With r constant, construct a line making an angle α with the x axis and having a distance $M(\alpha, r)$ from the origin. $R^2(r)$ is the domain enveloped by these lines, $0 \leqq \alpha \leqq 2\pi$. Then, using the theory of distinguished surfaces, especially the integral formula given by Bergmann (Mathematische Zeitschrift, vol. 39 (1935), pp. 76–94), it is shown that $W^2(r) \supset R^2(r)$. (Received September 28, 1939.)

419. T. S. George and A. J. Maria: *Equilibrium point of Green's function for an n-dimensional spherical shell.*

Let the pole and the corresponding equilibrium point of the Green's function be at distances r_0 and r respectively from the center of the concentric spheres bounding the shell. The authors show that dr/dr_0 is not zero for any position of the pole in the shell and that as the pole moves toward either boundary the equilibrium point remains away from the boundary. (Received September 28, 1939.)

420. P. W. Gilbert: *Two-to-one transformations on acyclic continuous curves.*

It is shown that if M is an acyclic continuous curve which is the sum of a finite number of arcs, there does not exist a continuous 2–1 transformation defined on M. An acyclic curve K is found having the following property: If M is any acyclic continuous curve on which it is possible to define a continuous 2–1 transformation, then M contains K topologically. (Received September 30, 1939.)

421. Nathan Jacobson: *A note on hermitian forms.*

Let Φ_0 be any field of characteristic not equal to 2 and $\Phi=\Phi_0(i)$ or $\Phi_0(i, j)$, respectvely, a quadratic extension or a quaternion division algebra over Φ_0 with $\alpha\rightarrow\bar{\alpha}$, the usual conjugation in these division algebras. To each hermitian matrix A $(\bar{A}'=A)$ with elements in Φ associate a symmetric matrix A_1 with elements in Φ_0 and prove that a necessary and sufficient condition that two such matrices be cogredient $(B=\bar{M}'AM,\ M$ nonsingular) is that the corresponding symmetric matrices be cogredient $(B_1=N'A_1N,\ N$ nonsingular). It is thus possible to extend the theory of symmetric matrices over certain special fields (for example, algebraic number fields, p-adic fields) to these two types of hermitian matrices. (Received October 3, 1939.)

422. S. A. Jennings: *The structure of the group ring of a p-group over a modular field.*

A basis for the radical N of the group ring of a p-group over a modular field whose characteristic divides the order of the group consists of all elements of the form $A-E$, where A is any element, and E is the unit element, of the group. By considering the characteristic subgroups K_i of all group elements which may be written in the form $E+n_{i+1}$, where n_{i+1} belongs to N^{i+1}, we get a series $K_0\geqq K_1\geqq\cdots\geqq K_i\geqq\cdots$, terminating with the unit element. We determine a basis for, and the rank of, N^w modulo N^{w+1} in terms of the elements and orders of these subgroups, and identify them abstractly. (Received September 19, 1939.)

423. E. S. Kennedy: *Exponential analogues of the Lambert series.*

The general class of series considered is obtained by replacing z^n in the classical Lambert series with $e^{-\lambda_n z}$, $(0<\lambda_n<\lambda_{n+1}\rightarrow\infty)$. However, the major portion of the paper deals with the "ordinary" series, the special case resulting when $\lambda_n=\log n$. Convergence theorems previously stated by Iyer are proved in detail. It is shown that any series of the general class can be developed as a unique Dirichlet series, and vice versa. In the ordinary case the abelian theorem of Iyer is generalized to include "complex," or Stolz-path, approach. Natural boundary theorems are proved with the aid of a theorem analogous to that of Franel in the Lambert series theory. This theorem is shown to hold for Stolz-path, as well as for "real," approach. Based on the general series, a method of summability is defined which generalizes the "Lambert summability" of Ananda-Rau. (Received September 18, 1939.)

424. Alfred Korzybski: *General semantics: extensionalization in mathematics, mathematical physics, and general education.* III. *Over/ under defined terms.*

This paper, the third of its series, shows that even in mathematics, physics, and life we deal only with over/under defined terms, resulting in extreme difficulties of thinking. Thinking$_1$ of apes, insane, and so on, differs from thinking$_2$ of normal humans. Formalism$_1$ of insane, and so on, contrasts with normal formalism$_2$, including mathematics. Calculus of propositions requires formalism$_2$, needed for sanity. The author shows mathematics as a language similar in structure to imperical facts, and the structure of the nervous system, allowing predictability in science and life. Undefined terms and postulates solve the dilemma of knowing everything before we know anything. The multiordinality of knowledge is explained. Plain human ignorance is often impossible, only false knowledge exists, inducing insanity. Ignorance is static in character; false knowledge is dynamic. Physico-mathematical methods uniquely translate static into dynamic, and vice versa. Evaluation by intensional methods, or mere verbal definition, induces dementia praecox. It is explained how physico-mathematical, extensional methods are essential for sanity. The two methods prove irreconcilable. Analysis shows the inherent dualism of over/under defined terms, depending on whether intensional or extensional methods are used, eliminating conflicting dualism in science and semantic (evaluational) reactions. (Received September 14, 1939.)

425. W. T. Martin: *On a minimal problem in the theory of analytic functions of several variables.*

In 1932 Wirtinger proposed and solved, in explicit form, the following problem. Given a region G in the complex z-plane and a function $\phi(z, \bar{z})$ once continuously differentiable in G, to find an analytic function $f(z)$ for which $\int_G |\phi - f|^2 \cdot d\omega = \min.$, where $d\omega$ is the element of area (Monatshefte für Mathematik und Physik, vol. 39 (1932), pp. 377–384). This year he stated the analogous question for several variables and obtained a unique and explicit solution in the case in which the region under consideration is a hypersphere, and ϕ is merely integrable (Monatshefte für Mathematik und Physik, vol. 47 (1939), pp. 426–431). He conjectured the existence of a solution for general regions. Using the theory of the kernel of a region for functions of several complex variables (Bergmann, Mathematische Zeitschrift, vol. 29 (1929), pp. 640–677), the author proves Wirtinger's conjecture for a wide class of regions which includes all bounded regions and for $\phi \in L^2$ over the region. Certain extensions of the problem are also considered, for example, where f is required, in addition, to take on given values at given points. The solution is again unique and explicit. (Received September 25, 1939.)

426. Deane Montgomery and Leo Zippin: *Note on rotation-group of the two-sphere.*

Let S denote the unit two-sphere (in ordinary space) and G the familiar rotation-group of S. This note shows by elementary topological considerations that G has no subgroup (itself excepted) which is transitive on S. The subgroups considered are of entirely arbitrary nature. This is of interest in connection with the problem of "rigid geometries" in 3-space (see abstract 45-11-402 by these authors). (Received September 28, 1939.)

427. D. C. Murdoch: *Further theory of quasi-groups.*

An "abelian" quasi-group has been previously defined by the author as one which satisfies the law $(ab)(cd) = (ac)(bd)$. The structure of these quasi-groups is now studied from the point of view of extension theory. There are two types of extensions: those which preserve the right-unit quasi-group and those which enlarge it. Every abelian quasi-group can be obtained from a "self-unit" quasi-group (one which is identical with its own right-unit quasi-group), by a series of extensions by quotient quasi-groups which satisfy certain restrictions. Some properties of self-unit quasi-groups are discussed and the Shreier conditions for the extensions are obtained. Certain non-abelian quasi-groups are also treated, namely those satisfying the associative law $a(bc) = (a^t b)c^s$, where s and t are automorphisms and $st = ts$. (Received September 27, 1939.)

428. Rufus Oldenburger: *Exponent trajectories in symbolic dynamics.*

In treating symbolic trajectories, Marston Morse has used exponents on elements. In the present paper, it is proved that the exponents on the elements in a recurrent trajectory T with two or more generating symbols form a unique recurrent trajectory T_e. The trajectory T_e is termed the exponent trajectory of T. If T is periodic, then T_e is periodic, and if T contains exactly two generating symbols, T is periodic if and only if T_e is periodic. If T is periodic, $T \neq T_e$. There exist nonperiodic trajectories T such that $T = T_e$, and if T contains the generating symbols 1, 2 only, there is one and only one such trajectory T. The introduction of the notion of exponent trajectory gives a new method of constructing recurrent trajectories. Certain methods of constructing recurrent trajectories from a given recurrent trajectory have been found by Morse and Hedlund. In a manner analogous to the treatment of exponent trajectories, it is also proved in the present paper that the exponents on the elements in a transitive ray form a unique transitive ray. (Received September 28, 1939.)

429. Walter Prenowitz: *Projective geometry as a group-like system.*

Garrett Birkhoff (Annals of Mathematics, (2), vol. 36 (1935), pp. 743–748) gives a lattice theoretic treatment of projective geometry taking as undefined the set S of all linear subspaces of a projective space and the operations meet and join in S. In this paper a less abstract treatment is given. The primitive terms are point and the operation join as applied to a pair of points. The postulates characterize a projective space as a type of "group" (closely related to the so-called multi-group) in which join, the operation of composition, is many-valued. This leads to a group theoretic development of projective geometry in which linear subspaces appear in the guise of subgroups. The properties of meet and join of linear subspaces in n-dimensional projective geometry are studied by algebraic methods, the austauschsatz of modern algebra playing an important role in the treatment. (Received September 20, 1939.)

430. J. F. Randolph: *Metric separability.*

The results of this paper were announced in this Bulletin, abstract 43-1-50, but the methods of proof are considerably improved. (Received October 2, 1939.)

431. C. J. Rees: *Differential equation of elliptic orthogonal polynomials.*

Following the method of Shohat (Duke Mathematical Journal, vol. 5 (1939)), a differential equation is derived for elliptic orthogonal polynomials. As is known from the general theory, it is a linear homogeneous differential equation of the second order. Its coefficients are given in explicit form and involve one unknown parameter only, for which a recurrence relation is given. An application of this differential equation is made to the study of roots of the elliptic orthogonal polynomial. (Received September 14, 1939.)

432. H. J. Riblet: *Symmetric differential expressions and applications.*

This paper introduces a theory for symmetric differential expressions comparable to that for ordinary symmetric functions. It is shown that every integral rational differential function of n quantities $y_1, \cdots, y_n$ which is of first order and which has coefficients in a commutative field R_1 can be expressed as a rational integral function of the elementary symmetric functions of the n quantities and their first derivatives with coefficients in R_1. The identity $S_m^*+a_1S_{m-1}^*+\cdots+a_{m-1}S_1^*+a_m'=0$ is established, wherein $S_j^*=\sum x_i^{j-1}x_i'$ and the primes denote differentiation. Applications are made to Waring's formula for the power sums and to solutions of total differential equations. (Received October 5, 1939.)

433. J. H. Roberts: *A theorem on two-to-one transformations.*

O. G. Harrold has shown (this Bulletin, abstract 45-1-20) that there does not exist a continuous 2–1 transformation of an arc into a circle. A transformation is said to be 2–1 if every image point has exactly 2 inverse images. In the present paper it is shown that it is not possible to define a continuous 2–1 transformation on a closed 2-cell. The author has not been able to generalize this result to the closed n-cell. (Received September 30, 1939.)

434. Benjamin Rosenbaum: *Ideals of a quadratic field in canonical form.*

The canonical forms of the product, quotient (when it exists), greatest common divisor, and least common multiple of two ideals of a quadratic field are obtained from the canonical forms of the two ideals. (A definition of the canonical form of an ideal in a quadratic field can be found in one of the books on algebraic numbers by Hancock, Reid or Sommer.) Also, new results are obtained in the determination of necessary and sufficient conditions that an ideal of a quadratic field be a principal ideal. (Received September 11, 1939.)

435. W. T. Scott and H. S. Wall: *A convergence theorem for continued fractions.* III.

If the inequalities in the convergence theorem announced in abstracts 45-7-277 and 45-7-285, this Bulletin, are satisfied by $r_1, r_2, r_3, \cdots$ which are real and positive and such that the products $r_1r_3r_5\cdots r_{2n-1}$ and $r_2r_4r_6\cdots r_{2n}$ are bounded, with actual inequality for at least one odd value of n and for at least one even value of n, then the continued fraction converges if (and only if) the series $\sum|b_n|$ diverges, where $a_1=1/b_1=1$, $a_n=1/b_nb_{n-1}$, $(n=2,3,4,\cdots)$. In particular, if the a_n lie within or upon the boundary of the parabola $|z|-R(z)=1/2$, then the continued fraction converges if and only if the series $\sum|b_n|$ diverges. With the aid of this convergence theorem, it is shown that the secondary requirement of the divergence of the series $\sum(p_1-1)(p_2-1)\cdots(p_n-1)$ in Pringsheim's uniform convergence theorem (Perron, p. 262, Theorem 30) is not essential. (Received September 12, 1939.)

436. D. T. Sigley: *k-set groups.*

A k-set group is defined as a finite abstract group which contains precisely k complete sets of non-invariant conjugate subgroups. The order of a k-set group is divisible by not more than $k+1$ distinct prime factors. There exists at least one k-set group for every integral value of k. This paper includes the determination of all 1-set and 2-set groups. (Received August 25, 1939.)

437. A. W. Tucker: *The algebraic structure of complexes.* Preliminary report.

Let $x_1, x_2, \cdots, x_n$ denote the cells (of all dimensions) of a complex X. Let E and F be square matrices of order n which are such that $\sum x_j E_i^j = (-1)^p x_i$, where $p = \dim x_i$, and $\sum x_j F_i^j$ is the boundary chain of x_i. Then (1) $EE=1$, (2) $FF=0$, and (3) $EF+FE=0$. Conversely, any two matrices E and F which satisfy these three conditions give rise to an algebraic system which behaves in many respects like a complex. Included in the part of the theory of complexes which is characterized in this simple algebraic fashion is the theory of chain-mappings, their coincidences and graphs, as well as a general theory of products. (Received October 2, 1939.)

438. W. J. Trjitzinsky: *Some problems in the theory of singular integral equations.*

An extensive and systematic treatment of integral equations of any finite or transfinite rank has been given recently by Trjitzinsky (Transactions of this Society, vol. 41 (1939), pp. 202–279). In this connection, Carleman's kernels, which constitute a very important extension of the better known classical kernels, were designated as of rank one. The kernels of rank $\alpha > 1$ are limits, in a suitable sense, of kernels of lower ranks. Thus there arose a transfinite hierarchy of theories of ascending generality. In Carleman's and in Trjitzinsky's work emphasis is on results for non-real values of the parameter. Now, both from the point of view of mathematical interest and from the point of view of important applications to mathematical physics, developments for real values of the parameter are essential. Such developments, including a treatment of equations of the first kind, are extensively given in the present paper, which will appear in the Annals of Mathematics. (Received September 30, 1939.)

439. A. H. Wheeler: *A one-sided polyhedron having one hundred twenty faces.*

This paper deals with some of the properties of a one-sided polyhedron which has 120 faces, 90 vertices and 210 edges satisfying the relation $e-k+f=0$. Thirty of its edges can be placed in coincidence with the 30 edges of a regular icosahedron, and when so placed it surrounds and lies entirely outside of the icosahedron. It is a member of a family of one-sided polyhedra having $6n$ faces, where $n \geqq 2$. (Received September 13, 1939.)