function $f(x)$ for which $(-1)^k f^{(2k)}(x) \geq 0$ on an arbitrary interval is necessarily entire. By use of this result a necessary and sufficient condition that a function can be expanded in an absolutely convergent Lidstone series is obtained. (Received November 25, 1940.)

55. J. R. Woolson: *A Theory of projections in complex Banach spaces.* Preliminary report.

A projection P is defined as a linear operator on a complex Banach space B to B, such that $P^2 = P$. Using an inter-space product $[F, f]$, $f \in B$, $F \in (B)$ the set of complex valued linear functionals defined on B, it is possible to prove the usual theorems concerning projections by certain analogies with the Hilbert space inner-product. (Cf. M. H. Stone, *Linear Transformations in Hilbert Space and Their Applications to Analysis*, American Mathematical Society Colloquium Publications, vol. 15.) The theory is used to show that $(1/n)\sum_{i=1}^{\infty} A^i$ converges weakly to a projection and to characterize a linear operator in terms of the linear manifolds which it leaves invariant. (Received October 28, 1940.)

APPLIED MATHEMATICS

56. Harry Bateman: *Aerodynamical effects of changes in the fundamental equations.*

Simple examples indicate that a change from the elliptic to the parabolic type may not be as drastic in its effects as a change from the elliptic to hyperbolic type. The effects of the various changes that have been made in the equations of viscous flow by Prandtl, Oseen, Kármán and others are reviewed by the author. Some remarks are made also on mixtures of fluids and changes of state. Some new equations are considered and reference is made to the work of Duhem, Schutz, Silberstein, Natanson, Bjerknes and Kozlowski. Some remarks are made on the equations of Burgers, Mattioli and others which differ from the classical equations and yet exhibit some of the phenomena of turbulence. (Received November 18, 1940.)

57. M. A. Biot: *Finite difference equations applied to aircraft engine vibrations.*

The vibration amplitudes of the crankshaft are shown to satisfy a finite difference equation of the second order with constant coefficients. The frequency equation derived from the end conditions is solved numerically by an artifice which shortens considerably the time required for the evaluation of the critical speeds. (Received November 18, 1940.)

58. J. Bjerknes: *Some uses of mathematics in meteorology.*

The atmosphere exhibits tide-like wave phenomena, but the lunar component of the atmospheric tide is only about one tenth of the solar component. If the atmospheric tide is gravitational, just as the ocean tides, the atmosphere must be able to act as a vibrating system with a proper period very close to 12 solar hours and thereby give the solar component of the tide a greater amplitude than the lunar component. That vibration problem is mathematically rather well defined but still unsolved in its general form. The atmospheric disturbances responsible for the day to day variations of weather are mainly aperiodic and disorderly but at times they are quasi-periodic and to some extent amenable to mathematical treatment. The near-

est approximation to wave-like conditions are found in the upper atmosphere. The general westerly current which prevails there takes a sine curve shape under disturbed conditions and the sinuosities move along with the current but with less speed than the air. That wave phenomenon is mathematically less tractable than the tidal waves, mainly because of the difficult boundary conditions, but whatever little progress can be made with it would be a valuable contribution to the science of weather forecasting. (Received November 18, 1940.)

59. B. E. Gatewood: *Thermal stresses in regions bounded by epitrochords.*

Muschelisvili's method of solving the biharmonic equation when the first derivatives are known on the boundary of the region is used in solving the thermal stress problem for a long cylindrical body whose cross section is bounded by an epitrochord (see abstract 45-9-314.) (Received November 14, 1940.)

60. F. B. Hildebrand: *The approximate solution of singular integral equations arising in engineering practice.*

In this paper a method of numerical solution of integral equations given by Crout (Journal of Mathematics and Physics, M.I.T., vol. 19 (1940), pp. 34–92) is extended to the solution of certain integral equations wherein the unknown function, as well as the kernel, may be singular. Considering an equation of the first kind, $\phi(x) = \int_a^b \sigma(\xi)G(x, \xi)d\xi$, the unknown function $\sigma(x)$ is approximated by a linear combination $s(x)$ of suitably chosen singular functions, $s(x) = \sum_{j=1}^r \chi_j s_j(x)$, where the χ's are undetermined constants, plus a polynomial $y(x)$ of order $2n$, in the Lagrangean form $y(x) \sum_{i=-n}^n K_i(x)y_i$, where the y's are the undetermined ordinates at $2n+1$ equally spaced points in (a, b). The conditions $\phi(x_k) \approx \int_a^b \{s(\xi)+y(\xi)\}G(x_k, \xi)d\xi$, $k = 1, 2, \cdots,$ m, where $m \geq 2n+r+1$, constitute m linear equations in the $2n+r+1$ parameters, the parameters then being determined by a method of least squares. Several specific problems arising in static field theory and elasticity are solved numerically, the results being in good agreement with exact results obtained by other methods. The computation involved is organized by the use of matrices and reduced principally to operations for which the modern computing machine is well adapted. (Received November 25, 1940.)

61. Theodore von Kármán and W. R. Sears: *Solution of the problem of the airplane wing of finite span by expansion in eigen-functions.* Preliminary report.

The problem of the calculation of the lift distribution along the span of a wing according to the lifting-line theory of L. Prandtl can be reduced to the determination of a two-dimensional potential function which satisfies on the boundary of a circle a linear relation with variable coefficients between the values of the function and its normal derivative. In engineering practice this problem is usually solved by means of a development of the boundary values of the potential function in a trigonometric series. In the present paper the classical method of expansion in a series of the eigen-functions of the problem is carried out. It is shown that this method has several practical advantages; moreover, the eigen-functions and eigen-values required for the calculation are independent of the twist of the wing and each set of them is applicable for a certain family of planforms. They can be estimated by various approximate methods. (Received November 18, 1940.)

62. W. D. Rannie: *Tensor methods in the theory of turbulence*. Preliminary report.

Applications of tensor analysis to the statistical theory of isotropic turbulence as developed by T. von Kármán and L. Howarth, and later treated by H. P. Robertson, are reviewed. (Received November 18, 1940.)

63. Eric Reissner: *A new derivation of the equations for the deformation of elastic shells*.

The equations of the theory of small deformations of shells, first given by A. E. H. Love, are rederived in a simpler manner. The simplifications are accomplished by using (1) vector stress resultants and equilibrium conditions in vector form and (2) the three-dimensional system of orthogonal coordinates which goes with the lines of curvature on the middle surface of the shell and the strain components with respect to this system. The assumption that the normal to the undeformed middle surface is deformed into the normal to the deformed middle surface, satisfied by determining appropriate displacement components, is introduced into these strain components. (Received November 25, 1940.)

64. H. J. Stewart: *Steady state oscillations in an atmosphere on a rotation sphere*. Preliminary report.

If one plots the mean surface atmospheric pressure, averaged over a period of at least a week, one finds that in addition to the mean westerly flow of air, there exist large scale closed isobaric systems which change very slowly with time. Attempts to develop long range weather forecasting techniques have shown the positions of these systems to be of primary importance and a knowledge of the factors which control these systems is very useful as a guide in formulating forecasting methods. In the present paper certain steady state oscillations of the stratosphere are investigated and are shown to vary with the mean velocity in the same manner as the observed oscillations. (Received November 18, 1940.)

GEOMETRY

65. P. O. Bell: *On differential geometry intrinsically connected with a surface element of projective arc length*.

In this paper a surface element of projective arc length is interpreted geometrically and used to obtain a new geometric interpretation for each of the following: a generalization of Bompiani's projective curvature, a generalization of Fubini's asymptotic curvature, a projective torsion introduced in this paper, conjugate tangents, the tangents of Darboux, and the tangents of Segre. The associate conjugate net of an arbitrary net $N_{\lambda_1 \lambda_2}$ of a surface S (introduced in this paper) is defined as the conjugate net whose tangents at a point P of S separate harmonically the tangents at P of the net $N_{\lambda_1 \lambda_2}$. The following characteristic property of this net is a typical result: Let arcs PP_1, PP_2 of equal projective length s be measured, with respect to the form $ds = (2Rv')^{1/2} du$, from the point P along the curves C_{λ_1}, C_{λ_2}, respectively, of the net $N_{\lambda_1 \lambda_2}$. The tangent plane to S at P intersects the line joining $P_1 P_2$ in a point P_3 which tends to a limit point P_0, distinct from P, as s tends to zero. The tangent line joining PP_0 and its conjugate tangent envelop the conjugate associate of the net $N_{\lambda_1 \lambda_2}$, as P varies over S. (Received November 20, 1940.)