74. S. B. Myers: *Complete Riemannian manifolds of positive mean curvature.*

The author has proved previously that if, on a complete n-dimensional Riemannian manifold M, the curvature at every point and with respect to every pair of directions is greater than a fixed positive constant, then M is closed (compact) and so is its universal covering manifold. In the present paper the same conclusions are drawn from the weaker hypothesis that the mean curvature of M at every point and with respect to every direction is greater than a fixed positive constant. In particular, a complete space of constant positive mean curvature is closed, and so is its universal covering manifold. Such spaces are important in the general theory of relativity. (Received November 25, 1940.)

Statistics and Probability

75. G. A. Baker: *Fundamental distributions of errors for agricultural field trials.*

Evidence from various sources is presented which shows that the fundamental error distribution for yield trials is represented by $[1/ab(t_1-t_0)]\int_0^a\int_0^b\int_{t_0}^{t_1}[1/\sigma(x, y, t)]\exp-\frac{1}{2}\{[\xi-f(x, y, t)]^2/\sigma^2(x, y, t)\}dtdydx$ where the integrals may be Stieltjes integrals. Under certain conditions the fundamental error distribution can be expressed as a Gram-Charlier series, but very rarely, if ever, as a normal distribution. For comparison with analysis of variance results based on the normal theory, the distribution of the ratio of independent estimates of the second moments of samples, if the fundamental distributions are Gram-Charlier series, are given. Similar considerations show that the distributions of the numbers attacked in field trials can rarely be represented by Poisson or binomial distributions as is usually assumed. (Received October 22, 1940.)

76. G. A. Baker: *Maximum likelihood estimation of the ratio of the components of nonhomogeneous populations.*

Let $f(x)=[1/(1+k)](f_1(x)+kf_2(x))$, $e\leqq x\leqq f$, $k>0$ and $k<\infty$, where $f_1(x)$ and $f_2(x)$ are probability functions. The problem is to find the maximum likelihood estimate of k, say $\bar{k}$. If $f_1(x)$ and $f_2(x)$ are rectangular with equal ranges that partially overlap, then the probability of a value of $\bar{k}=w/u$ (where u is the number of individuals drawn from the nonoverlapped interval of $f_1(x)$, w is the number of individuals drawn from the nonoverlapped interval of $f_2(x)$ and v is the number of individuals drawn from the interval overlapped by $f_1(x)$ and $f_2(x)$) is $(n!/u!v!w!)(p_1)^u(p_2)^v(p_3)^w$ where the p_i's are the probabilities of coming from the respective intervals. The cases for which $u=n$, $v=n$, $w=n$, $u=0$, $w=0$ are excluded because $\bar{k}$ is then indeterminate. Hence, the probability of a determinate value of $\bar{k}$ is $P=1+(p_2)^n-(p_2+p_3)^n-(p_1+p_2)^n$. The estimates of k are biased. (Received October 22, 1940.)

77. G. F. McEwen: *Statistical problems of the range divided by the mean in samples of size n.*

Certain quantitative climatological studies are based upon the "precipitation ratio" or ratio to the mean annual rainfall of the difference between the maximum and minimum annual rainfall corresponding to the standard number of years. Available observations correspond to various values of the number of years n. Accordingly it is necessary to compute the precipitation ratio I corresponding to a standard number of years from I_n corresponding to n years. Various assumptions regarding the frequency distribution of the precipitation lead to corresponding sets of theoretical factors for standardizing the precipitation ratio and certain of these agree well with observations. Results plotted on coordinate paper having a reciprocal scale for I_n and a logarithmic scale for n facilitate a graphical treatment of the observations since the graph approximates to a straight line. The standard error of I_n is also computed corresponding to the two most important frequency distributions of the precipitation. In order to reduce the accidental fluctuations of I_n it is sometimes desirable to use the difference between the averages of the rn highest and the rn lowest values, where rn equals some appropriate value, say 3. Theoretical factors derived for correcting such values to obtain I_n agree well with observations. (Received October 3, 1940.)

78. Jerzy Neyman: *A statistical problem in mass production and routine analyses.*

Let $x_{i1}, x_{i2}, \cdots, x_{in}$ denote the results of parallel analyses of some ith sample, $i=1, 2, \cdots, N$ (a sample of items of mass production). The x's are considered as random variables following the law $p(x_{ij})=(\sigma_i(2\pi)^{1/2})^{-1}\exp\{(x_{ij}-\xi_i)^2/2\sigma^2{}_i\}$. Denote by H the hypothesis that $\sigma_1=\sigma_2=\cdots=\sigma_N$, that is, that the precision of measuring ξ_i is maximum, and let $x_i=(\sum_j x_{ij})/n$, $S_i^2=\sum_j(x_{ij}-x_i)^2$, $V=\sum_i S_i^2$, $U=\sum_i S_i^4$. Also let E denote the observed point in the space W of the S_i^2's and $w(V)$ the part of the locus $V=$const. included in any region $w\subset W$. It is proved that (1) a necessary and sufficient condition for a test's probability of rejecting H when true to be a fixed value α is that its critical region $w=\sum_v w(V)$, with the $w(V)$ arbitrary except that $P\{E\in w(V) \mid E\in W(V)\}=\alpha$ for any $V>0$. (2) If H is not true and $h=\sigma^{-2}$ is a random variable with $p(h)=ch^{\alpha-1}e^{-\beta h}$, then the critical region w_0 determined by $U>U_\alpha V^2$, with $P\{U>U_\alpha V^2 \mid H, V\}=\alpha$, has the following useful properties: (a) if H be true, then the probability of w_0 rejecting H is equal to α, (b) the first derivative of the power function of w_0, taken at the point of H being true, is equal to zero, (c) the second derivative is maximum. ((b) is true for all regions satisfying (a).) (Received October 28, 1940.)

Theory of Numbers

79. E. T. Bell: *Selective equations.*

The symbols $(\ )'$, $[\ ]'$ denote the least, greatest of the integers occurring within the symbols. Each such integer may be replaced by a symbol $(\ )'$, $[\ ]'$ referring to a new set of integers, and so on, a finite number of times. The most general system of equations consisting of equalities between single power products formed from $(\ )'$, $[\ ]'$ is solved non-tentatively and finitely, by passing to a unique dual of the system, in which $(\ )$, the G.C.D., and $[\ ]$, the L.C.M., replace $(\ )'$, $[\ ]'$. The latter system is a simple multiplicative system, and hence is completely solvable non-tentatively and finitely. The problem solved arose in the detailed discussion of compound multiplicative systems. (Received October 26, 1940.)

80. Leonard Carlitz: *Finite differences and polynomials in a Galois field.*

This paper is concerned chiefly with the solution of equations such as $\sum f(M)M(t)=g(t)$ and $\sum f(M, u)M(t)=g(u, t)$, the summation extending over all polynomials $M=M(x)$ of degree less than m. (Received November 25, 1940.)