

249. J. A. Shohat: *On the best polynomial approximation for functions possessing derivatives.*

By virtue of the characteristic property of the polynomial of best approximation, of degree less than or equal to n , to $f(x)$ on a given interval, it may be also considered as a Lagrangean interpolation polynomial for $f(x)$. In this way are obtained very simply general theorems on best approximation in case $f^{(n+1)}(x)$ exists, also on the distribution of the points of deviation. (Received February 3, 1941.)

250. L. H. Swinford: *On Abel's equation.*

The substitution $y = \alpha + \beta z^{1/2} / \gamma + z^{1/2}$ allows one to replace Abel's equation $y' + g(x)y^2 + f(x)y^3 = 0$ by a system of two first order equations, by which means new cases of integrability are found. (Received March 7, 1941.)

251. Otto Szász: *On convergence and summability of trigonometric series.*

A new type of necessary or sufficient condition has been given recently for the convergence of a Fourier series at a point. This combines a continuity property of the function with an order condition on the coefficients, and is generalized in this paper. Moreover, instead of a Fourier series, more general trigonometric series are considered, associating for example the termwise integrated series with a function. The results are closely connected with two summability methods introduced by Riemann and Lebesgue. (Received March 5, 1941.)

252. Alexander Weinstein: *On the decomposition of a Hilbert space by its harmonic subspace.*

It is shown that a lemma playing a central part in a recent paper of H. Weyl on the method of orthogonal projection in potential theory (Duke Mathematical Journal, vol. 7 (1940), pp. 411-444, Lemma 2) is an almost immediate consequence of a problem of the unified theory of eigenvalues of plates and membranes considered some years ago in a joint paper of N. Aronszajn and the author (Comptes Rendus de l'Académie des Sciences, Paris, vol. 204 (1937), p. 96). The proof based on these results does not require any special construction or computation. (Received February 7, 1941.)

APPLIED MATHEMATICS

253. G. E. Hay: *The finite displacement of thin rods.*

The finite displacement of thin rods has been considered by G. Kirchhoff, who introduced approximation based on the thinness of the rod in a rather unsatisfactory manner. In the present paper the method of the tensor calculus is employed, and there is introduced a systematic method of approximation which involves the expansion of the fundamental equations as power series in a dimensionless parameter ϵ and permits a theoretical solution of the problem to any desired degree of accuracy. Finally, application of the theory is made to the problem of "straightening" certain thin rods by means of systems of forces applied to the ends. (Received March 13, 1941.)

254. A. E. Heins: *On the transformation theory of the solution of partial differential equations.* Preliminary report.

The regularity condition of the Fourier transform of a function defined over a

finite domain, and zero outside of this domain, plays an important role in pure mathematics. This condition also plays an important role in the transformation theory of the solution of linear partial differential equations of mathematical physics. By observing this regularity condition, boundary conditions which do not enter into a particular problem may be eliminated. With the aid of the finite Fourier transform the author obtains solutions of the two-dimensional wave equation and the equation of heat conduction under prescribed initial and boundary conditions and the two-dimensional Laplace equation under prescribed boundary conditions. The boundaries are taken to be either circles or rectangles. The treatment of these problems with other contours is now in progress. (Received March 10, 1941.)

255. A. S. Householder: *A theory of steady-state activity in nerve-fiber networks. II. The simple circuit.*

Let the neuron N_i have the origin s_i and terminus s_{i+1} , subscripts being reduced modulo n . These neurons constitute a simple circuit. A constant stimulus y_i applied to N_i at s_i is supposed to cause the production at s_{i+1} of a stimulus z_{i+1} according to the equation $z_{i+1} = \alpha_i(y_i - h_i)$, where $h_i > 0$, $\alpha_i = 0$ if $y_i \leq h_i$, $\alpha_i = a_i > 0$ or < 0 if $y_i > h_i$. If S_i is a constant stimulus applied from outside the circuit at s_i , then $y_i = S_i + z_i$. Then (1) the S_i do not necessarily suffice to determine the α_i , and hence, the y_i , unless $a_1 a_2 \cdots a_n < 1$. If $(\alpha_1, \cdots, \alpha_n)$ and $(\alpha'_1, \cdots, \alpha'_n)$ are distinct and both consistent with some (S_1, \cdots, S_n) , and if some $\alpha_i = 0$ and also some $\alpha'_i = 0$, then (2) $\alpha_i = 0$ implies $\alpha'_i \neq 0$; (3) $\alpha_i = \alpha'_i = 0$ implies $a_i a_{i+1} \cdots a_{j-1} < 0$; (4) $\alpha_i = \alpha_j = 0$ implies $a_i a_{i+1} \cdots a_{j-1} > 0$. (Received March 8, 1941.)

256. William Mersman: *Heat conduction in an infinite composite solid.*

The problem of one-dimensional heat conduction in a doubly infinite composite solid, with given initial temperature distribution, is solved uniquely. The classical solution for a semi-infinite solid with surface temperature known is used to obtain an integral equation, which is solved formally by means of the Laplace transformation. The solution is then established and proved unique by classical methods. (Received March 8, 1941.)

257. I. S. Sokolnikoff: *Deflection of non-isotropic elastic plates. Preliminary report.*

The general solution of the equation for the deflection w of a non-isotropic plate with one plane of elastic symmetry, $\alpha_{11} \partial^4 w / \partial x^4 + 2\alpha_{12} \partial^4 w / \partial x^3 \partial y + 2\alpha_{13} \partial^4 w / \partial x^2 \partial y^2 + 2\alpha_{31} \partial^4 w / \partial y^3 \partial x + \alpha_{22} \partial^4 w / \partial y^4 = p(x, y)$, can be written in terms of two analytic functions $F_1(z_1)$ and $F_2(z_2)$ of the independent complex variables $z_1 = x + k_1 y$ and $z_2 = x + k_2 y$, where the complex numbers k_i depend on the elastic constants α_{ij} . The determination of the functions F_i can be reduced to the search for certain analytic functions of a complex variable ζ related to z_1 and z_2 via a pair of mapping functions $z_i = \omega_i(\zeta)$. The development of the theory is analogous to the treatment of certain two-dimensional problems in isotropic theory initiated by N. Muskhelishvili (Mathematische Annalen, vol. 107 (1932), pp. 282–312). Some applications of the theory to a clamped elliptic plate are made. The formulas specialize to the known results of the theory of isotropic and orthotropic plates. (Received March 8, 1941.)

258. R. M. Sutton: *An instrument for drawing confocal conics.*

A simple device is demonstrated for drawing families of orthogonal ellipses and hyperbolae. On a spring roller is wound a fishline in two strands so that as the line is pulled out both strands elongate at the same rate and are held under tension by the roller. Confocal hyperbolae are described by holding chalk against the cord without slippage; confocal ellipses are drawn in the usual manner by slipping the chalk within a loop of fixed length. The eccentricity of either kind of curve is changed with ease, and the distance between focal points can be altered quickly. The instrument offers convenience, flexibility, and rapidity of operation. (Received March 28, 1941.)

259. C. J. Thorne and J. V. Atanasoff: *The application of a general functional approximation method to thin plate problems.*

The approximate solutions to two problems of thin plate theory are obtained by a general functional method. This method includes, indeed unifies into a connected whole, many of the standard approximation methods and many more. In particular it includes the methods of Trefftz, Boussinesq or "least square," and Rayleigh-Ritz. The first problem represents that of a centrally loaded, square, clamped plate. The solution to this problem illustrates convergence to boundary values as well as the differential equation. The solution agrees accurately with the results obtained by others. The second problem is that of an infinite, corner loaded plate supported by an elastic foundation. A solution containing seventeen constants is obtained. (Received March 31, 1941.)

260. Alexander Weinstein: *On the vibrations of a clamped plate under tension.*

This problem, of importance for acoustic reception, has been solved in terms of Bessel's functions in the elementary case of a circular plate by W. G. Bickley, *Philosophical Magazine*, vol. 15 (1933), pp. 776-797. It will be shown here, for a plate of any shape, that a convergent sequence of lower bounds for the frequencies can be computed by using the solutions of the membrane problem for the same domain. In the case of a rectangular plate the resulting equations involve only trigonometric and hyperbolic functions and can be more easily solved numerically than the equations for the circular plate. Upper bounds for the frequencies can be computed by the Rayleigh-Ritz method. However, lower bounds are more important in the present case. (Received April 1, 1941.)

GEOMETRY

261. Reinhold Baer: *Homogeneity of projective planes.*

It is the main object of this note to show that the theorems of Desargues and Pappus are valid in a projective plane if and only if there exist two one-parameter families of dualities of this plane which meet certain requirements (like existence of axis and center). (Received March 12, 1941.)

262. Y. K. Bal: *Study on graphs.* Preliminary report.

This is an abstract study of generalized polygons and their properties invariant under the independent rigid motion of their vertices. A relation, called "next," is introduced, in general by: I. If $A \leftrightarrow B$ then $B \leftrightarrow A$, II. $A \leftrightarrow A$ cannot hold; in particular by: II'. If $A \leftrightarrow B \leftrightarrow C$ then not $A \leftrightarrow C$. I and II' give II. (a) S_n is not null. (b) If