258. R. M. Sutton: *An instrument for drawing confocal conics.*

A simple device is demonstrated for drawing families of orthogonal ellipses and hyperbolae. On a spring roller is wound a fishline in two strands so that as the line is pulled out both strands elongate at the same rate and are held under tension by the roller. Confocal hyperbolae are described by holding chalk against the cord without slippage; confocal ellipses are drawn in the usual manner by slipping the chalk within a loop of fixed length. The eccentricity of either kind of curve is changed with ease, and the distance between focal points can be altered quickly. The instrument offers convenience, flexibility, and rapidity of operation. (Received March 28, 1941.)

259. C. J. Thorne and J. V. Atanasoff: *The application of a general functional approximation method to thin plate problems.*

The approximate solutions to two problems of thin plate theory are obtained by a general functional method. This method includes, indeed unifies into a connected whole, many of the standard approximation methods and many more. In particular it includes the methods of Trefftz, Boussinesq or "least square," and Rayleigh-Ritz. The first problem represents that of a centrally loaded, square, clamped plate. The solution to this problem illustrates convergence to boundary values as well as the differential equation. The solution agrees accurately with the results obtained by others. The second problem is that of an infinite, corner loaded plate supported by an elastic foundation. A solution containing seventeen constants is obtained. (Received March 31, 1941.)

260. Alexander Weinstein: *On the vibrations of a clamped plate under tension.*

This problem, of importance for acoustic reception, has been solved in terms of Bessel's functions in the elementary case of a circular plate by W. G. Bickley, Philosophical Magazine, vol. 15 (1933), pp. 776–797. It will be shown here, *for a plate of any shape*, that a convergent sequence of *lower bounds* for the frequencies can be computed by using the solutions of the membrane problem for the same domain. In the case of a rectangular plate the resulting equations involve only trigonometric and hyperbolic functions and can be more easily solved numerically than the equations for the circular plate. *Upper bounds* for the frequencies can be computed by the Rayleigh-Ritz method. However, lower bounds are more important in the present case. (Received April 1, 1941.)

Geometry

261. Reinhold Baer: *Homogeneity of projective planes.*

It is the main object of this note to show that the theorems of Desargues and Pappus are valid in a projective plane if and only if there exist two one-parameter families of dualities of this plane which meet certain requirements (like existence of axis and center). (Received March 12, 1941.)

262. Y. K. Bal: *Study on graphs.* Preliminary report.

This is an abstract study of generalized polygons and their properties invariant under the independent rigid motion of their vertices. A relation, called "next," is introduced, in general by: I. If $A\leftrightarrow B$ then $B\leftrightarrow A$, II. $A\leftrightarrow A$ cannot hold; in particular by: II′. If $A\leftrightarrow B\leftrightarrow C$ then not $A\leftrightarrow C$. I and II′ give II. (a) S_n is not null. (b) If

$A \in S$ there are n and only n elements next to it. (c) If A, $B \in S_n$ there are $(A_1, \cdots, A_n) \in S_n$ for which $A_1 \leftrightarrow A_2 \leftrightarrow \cdots \leftrightarrow A_r \leftrightarrow B$. n is finite and r is finite or zero. $|S_n|$ is the number of elements in S_n. Definition of $S_{n,p}$: For distinct elements $A_1 \leftrightarrow A_2 \leftrightarrow \cdots \leftrightarrow A_q$ and not $A_i \leftrightarrow A_j$ when $1 < |i-j| < p-1$. If $A_i \leftrightarrow A_p$ whenever $|i-j| = p-1$, $S_{n,p}$ is shown by $S'_{n,p}$. It is proved: $|S'_{n,4}| = 2n$ and $S'_{n,5}$ cannot exist. For distinct elements $A_1 \leftrightarrow A_2 \leftrightarrow \cdots \leftrightarrow A_r \leftrightarrow A_1$ is a cycle C_n. A cycle is either S_1 or S_2. If $|C_n| = |S_n|$ it is a boundary cycle. The order of S_n is the number of distinct b-cycles not differing by circular permutation. The order of $S'_{n,4}$ is shown equal to $(n!)^2/n$. For A_1 and B_1, $A_1 \leftrightarrow \cdots \leftrightarrow A_{m_1} \leftrightarrow B_1$, then the smallest m_i is the distance (A_1, B_1) with $(A, A) = 0$. It is shown that $|S_n|$ is finite and by further study S_n's are classified. (Received March 31, 1941.)

263. Nathaniel Coburn: *Frenet formulas for curves in unitary space.*

The object of this paper is to develop a set of Frenet formulas for any curve X_1 whose equations are functions of a real parameter (t) and which lies imbedded in a unitary space of n dimensions K_n. The derivation proceeds analogously to that of the Frenet formulas for a curve V_1 which lies imbedded in a Riemannian space of n dimensions, V_n. However, in the Riemannian case, the curve possesses $(n-1)$ independent curvatures; in the unitary space, the curve possesses $(2n-1)$ independent curvatures. Furthermore, in Riemannian space, the curvature matrix of V_1 in V_n is skew-symmetric. Similarly, the curvature matrix of X_1 in K_n possesses Hermitian symmetry. Finally, corresponding to the similar theorem in Riemannian space, it is shown that "a complex curve X_1 in a unitary space K_n is uniquely determined when: (1) an initial point; (2) an initial orientation of the n-uple of normals; and (3) the $(2n-1)$ independent curvatures are given." The paper concludes with a study of geodesic X_1 in a unitary K_n with semi-symmetric connection. It is shown that these geodesics are characterized by the first Frenet curvatures and the orientation of the n-uple of normals. (Received February 11, 1941.)

264. V. G. Grove: *The transformation T of congruences.*

In a projective space of three dimensions two congruences are said to be related by a transformation T if their lines are in one-to-one correspondence, their developable surfaces correspond, and each congruence possesses transversal surfaces whose tangent planes at the points of intersection with a line of that congruence pass through the corresponding line of the other congruence. The transformation T is of two types. In the first, called the asymptotic type, the curves on the transversal surfaces of the congruences corresponding to developables of these congruences are asymptotic curves. If one of the congruences of the pair is a W-congruence, the other is also. In the second or conjugate type the curves on the transversal surfaces corresponding to the developables form conjugate nets in relation F. If these nets are in the relation of a transformation K of Koenigs, they are in the relation of Eisenhart. Associated with the conjugate case there exists a one-parameter family, or pencil, of congruences such that each focal point of a line of a variable member of the pencil lies on a line. If one congruence of this pencil is a W-congruence, all congruences of the pencil are W. (Received March 8, 1941.)

265. Edward Kasner and John De Cicco: *Geometry of dual-velocity systems.*

A dual-velocity system consists of the ∞^2 curves such that the osculating circles of the ∞^1 curves tangent to any line l, constructed at the elements of l, are tangent to another line L. Any such set may be considered to be analogous to the velocity systems as developed by Kasner. Any line transformation, not preserving all parallel pencils of lines, converts exactly one dual-velocity system into a dual-velocity system. The group preserving all dual-velocity systems is $X=\phi(x)$, $Y=y\psi(x)+\chi(x)$. This is the contact group leaving invariant the set of all dual-isothermal families. Examples of dual-velocity systems are equitangential, dual-natural, Δ, and dual-Γ families. Characterizations of these are obtained by the correspondence between the lines l and L mentioned above. Any dual-velocity system contains exactly ∞^2, ∞^1, one, or zero dual-isothermal families. Finally, the invariant theory of dual-velocity systems under both the dual-isothermal and equilong groups is developed. A dual-analogue of natural family has been discussed in an earlier paper. (Received February 11, 1941.)

266. L. J. Savage: *Distance spaces.*

By a distance space is meant a set M of elements $p, q, \cdots$ over which is defined a real valued function $D(p, q)$ such that $D(p, q)=D(q, p)$, and $D(p, p)=0$. $D(p, q)$ may be thought of as the square of the distance from p to q. This concept is some generalization of metric space. A vector space V over which a scalar product $x \cdot y$ is defined can be considered as a distance space by setting $D(x, y)=(x-y)\cdot(x-y)$. For every distance space M there is a "smallest" scalar product space $V(M)$ in which M is imbeddable. An interesting class of distance spaces is that of differentiable manifolds over which a $D(p, q)$ is so defined as to be suitably differentiable when considered as a function of the coordinates. Because of the remark about scalar product spaces these differentiable distance manifolds can be handled much like differentiable submanifolds of euclidean space. In particular the concepts of regularity, tangent-flat, and first and second fundamental form, can be extended to them. Finally there are theorems connecting the possibility of imbedding such manifolds into euclidean and pseudo-euclidean spaces with certain restrictions on the second fundamental form. (Received March 13, 1941.)

267. R. K. Wakerling: *On the rational loci of ∞^1 $(\rho-1)$-spaces in r-space.*

The representation upon a ρ-space of the hypersurface W_ρ^n in S_r, which is the rational locus of ∞^1 $(\rho-1)$-spaces, is investigated in this paper. The hyperplane sections of W_ρ^n are represented in S_ρ by a system of hypersurfaces $V_{\rho-1}^\nu$ passing through a given $(\rho-2)$-space $\nu-1$ times, and having in common σ simple points. Some of the properties of W_ρ^n are discussed together with those of its projection upon a $(\rho+1)$-space. A special case of the transformation between two r-spaces is given, and the paper is concluded with a brief note on rational ruled surfaces of order $r-1$ in S_r. (Received March 8, 1941.)

STATISTICS AND PROBABILITY

268. A. H. Copeland: *If.*

If one attempts to apply Boolean algebra to the theory of probability, he discovers that it is inadequate for the treatment of conditional probabilities, selections, and observations—all three of which are of prime importance in the modern theories of probability and statistics. In a number of recent formalizations of the theory of probability an additional operator (or logical constant) "if" has been introduced in order to handle conditional probabilities. Selections and observations were not introduced