$$ay^2 + by + c = dx^n,$$
$$a(b^2 - 4ac)d \neq 0, \qquad n \geqq 3,$$

has only a finite number of solutions as well as the equation

$$y^2 = ax^n + bx^{n-1} + \cdots + k$$

where, in the latter, the right-hand member has at least three different zeros.

After considering a number of special equations of the form

$$x^n + dy^n = \pm 1$$

where $n=3$ or 4, Skolem applies the theory of p-adic numbers to the equation

$$N(\alpha x + \beta y + \gamma z) = h$$

where α, β and γ are integers in an algebraic field K of degree n. He finds equations of this type which have only a finite number of solutions for $n=5$.

We now signalize a problem which seems fundamental in this subject. If we consider the irreducible equation

$$f(x_1, x_2, \cdots, x_k) = c$$

where f is of degree n with integral coefficients and with c integral and also f homogeneous we know from the theory of units in an algebraic field that for $k=n$ and $c=1$, there exist equations of this type with an infinity of integral solutions. On the other hand, if $k=2$, $n>2$, Thue's theorem states that there cannot be more than a finite number of solutions. The question is, how far must k be increased to obtain equations of this type with an infinity of solutions? If $n=3$, we have $k=3$.

The arithmetical theory of Hermitian forms is not considered, likewise Waring's theorem. It is not exactly surprising that the latter topic has been omitted, as it would merit a volume in itself.

Skolem has written a very interesting book. It is surprising how much arithmetical meat he has packed into the space he employs.

H. S. VANDIVER

Sur les Fonctions Orthogonales de Plusieurs Variables Complexes, avec les Applications à la Théorie des Fonctions Analytiques. By Stefan Bergman. New York, Interscience Publishers, 1941. 62 pp. $1.50.

This book was to appear as one of the Mémorial des Sciences Mathématiques series, but circumstances were such that the edition reached only the galley proof stage. The book is a photostatic edition made from the galley proofs. As a result, the characters are sometimes a little blurred and in some places the lines are uneven.

At the outset the author states that many important results in the theory of one complex variable cannot be extended to functions of several complex variables. To find a technique analogous to that of one variable it is necessary to introduce a new approach. One such approach is the method of orthogonal functions for n variables in a domain B. In this book, however, the author limits himself to the properties of functions of two complex variables. The methods used and the results obtained can be extended to functions of several complex variables. The power of the method of orthogonal functions lies in the following properties of the kernel function $\sum\phi^{(\nu)}(z_1, z_2)\cdot\overline{\phi^{(\nu)}(t_1, t_2)}=K_B(z_1, z_2, \bar{t}_1, \bar{t}_2)$, established by the author, of a system of functions closed for the class, $F(B)$, of functions analytic and of summable square in B. First, the function $K_B(z, \bar{t})$, $z\equiv\{z_1, z_2\}$, $\bar{t}\equiv\{\bar{t}_1, \bar{t}_2\}$ is regular in the variables $z_1, z_2, \bar{t}_1, \bar{t}_2$. Second, $K_B(z, \bar{t})$ is independent of the particular choice of the closed system $\{\phi^{(\nu)}\}$. Third, the minimum value of the integral $\int_B |h|^2 dw$, $dw=dx_1dy_1dx_2dy_2$, where h belongs to $F(B)$, and $h(t_1, t_2)=1$, that is, h is normalized at the point (t_1, t_2), is equal to $[K_B(t, \bar{t})]^{-1}=\lambda_B(t)$, and this minimum is attained for $h=f=K_B(z, \bar{t})/K_B(t, \bar{t})$. It is from these properties that he builds up a theory of orthogonal functions for two complex variables and uses this to deal with analytic functions of two complex variables. This method of the kernel function is very neatly applied with much success to the study of pseudo-conformal transformations. It is extremely interesting that these results can also be applied to the study of linear partial differential equations.

This theory of conformal transformations is essentially based on the Riemannian mapping theorem which states that every simply connected domain possessing at least two boundary points can be mapped conformally into a circle, and the Schwarz lemma. This lemma, as formulated by Pick, states that if a transformation takes the unit circle into its interior, then the hyperbolic length of a line segment included in the unit circle cannot be increased.

Transformations of the domains in four-dimensional space by a pair of analytic functions are called pseudo-conformal transformations. As was first pointed out by Poincaré, two arbitrary simply-connected four-dimensional domains cannot be transformed one into the other by a pseudo-conformal transformation. Also Hartogs has shown that every analytic function regular in a simply connected four-dimensional region assumes on the boundary of this region all the values it assumes in the interior, that is, a function cannot be constructed that vanishes at some point in the interior of the domain B and has an absolute value of one on the boundary. An immediate generalization of the Schwarz-Pick lemma therefore cannot be obtained for functions of two complex variables. Therefore, analogous methods to those described above cannot be applied in the case of functions of two complex variables.

To overcome these difficulties the author employs the kernel function $K_B(t, \bar{t})$. In the case of conformal transformations $ds^2 = K_B(z, \bar{z})|dz|^2$ defines a metric which is invariant with respect to conformal transformations. If the domain is simply connected, the metric becomes a hyperbolic metric. Analogously, using the kernel function for functions of two complex variables, the author defines a metric which is invariant with respect to pseudo-conformal transformations. It is clear that if a domain $G \subset B$ then $\lambda_G(t) \leqq \lambda_B(t)$, and since λ is the inverse of the kernel K, $K_G(z, \bar{z}) \geqq K_B(z, \bar{z})$ and hence the metric $K_G(z, \bar{z})|dz|^2 \geqq K_B(z, \bar{z})|dz|^2$. This is a form of the Schwarz-Pick lemma.

The book consists of an introduction and four chapters. In the introduction the author describes his approach to the problem of conformal mapping using orthogonal functions.

Chapter I deals with the characteristics of the space of two complex variables. The author describes domains in the z_1, z_2-space which are of particular importance in the study of analytic functions. In particular, he deals with analytic surfaces, hypersurfaces, and domains with distinguished boundary surfaces. These geometrical studies are not employed in the book, but without an understanding of them one might not fully appreciate the value of the results obtained. These geometrical properties, however, are of fundamental importance in quite another aspect of the theory of functions of two complex variables that the author has developed in other papers. There are several points in Chapter I that would be of particular interest to differential geometers.

Chapter II, which forms the heart of the book, is devoted to the properties of orthogonal functions. The differences between real and analytic orthogonal functions are stressed. The essential and interesting theorem is that the kernel function of complex analytic functions converges at every interior point of the domain. The connection between the theory of orthogonal functions and conformal mapping is described. Special systems of orthogonal functions are also studied.

In Chapter III the author concerns himself with minima problems of the type described above. The problem of interpolation and the application of orthogonal functions to the theory of entire and meromorphic functions are treated. But here the work is sketchy, and in some places only references to papers are made. Furthermore, the author does not indicate very clearly what role the theory of orthogonal functions plays in the study of entire and meromorphic functions. This, of course, could not very well have been accomplished in the space devoted to this topic.

In Chapter IV some further properties of the invariant metric are given, and the differential geometric properties of the Hermitian metric are described.

A companion book to this one was to be written dealing with the invariant metric. We hope that this interesting book will soon appear in this country.

This book is essentially a collection of results by the author and other people working in this field. It is to be regretted that some of the value of this book, as a book, is diminished by the lack of elaboration. In many places results are merely stated without proof. However, a fairly complete bibliography is inserted at the end, and in all instances of the text, references are made to original papers. The material of this book is new and interesting, and it appears that this field is by no means exhausted. Anyone interested in this field would find it very stimulating to read this book.

ABE GELBART

An Introduction to Differential Geometry with Use of the Tensor Calculus. By Luther Pfahler Eisenhart. Princeton, Princeton University Press; London, Humphrey Milford and Oxford University Press. 1940. 10+304 pages. $3.50.

The author's *Differential Geometry of Curves and Surfaces*, which was published in 1909, has seen extensive use. Since that time, the tensor calculus has come to play an important role in Riemannian Geometry and in the Theory of Relativity and the author has considered it desirable to rewrite the differential geometry of curves and surfaces in terms of the tensor calculus. The differential geometry treated in the present book is about equivalent to that in the first half of the 1909 book, with the addition of the concept of parallelism in the sense of Levi-Civita.

There are four chapters. The first is concerned principally with the properties of curves in euclidean 3-space, but includes a definition of the parametric equations of a surface and the envelope of a one-parameter family of surfaces, in order to include consideration of the developables associated with a curve. The material of the chapter is for the most part included in the first two chapters of the 1909 book