meromorphic functions are treated. But here the work is sketchy, and in some places only references to papers are made. Furthermore, the author does not indicate very clearly what role the theory of orthogonal functions plays in the study of entire and meromorphic functions. This, of course, could not very well have been accomplished in the space devoted to this topic.

In Chapter IV some further properties of the invariant metric are given, and the differential geometric properties of the Hermitian metric are described.

A companion book to this one was to be written dealing with the invariant metric. We hope that this interesting book will soon appear in this country.

This book is essentially a collection of results by the author and other people working in this field. It is to be regretted that some of the value of this book, as a book, is diminished by the lack of elaboration. In many places results are merely stated without proof. However, a fairly complete bibliography is inserted at the end, and in all instances of the text, references are made to original papers. The material of this book is new and interesting, and it appears that this field is by no means exhausted. Anyone interested in this field would find it very stimulating to read this book.

ABE GELBART

An Introduction to Differential Geometry with Use of the Tensor Calculus. By Luther Pfahler Eisenhart. Princeton, Princeton University Press; London, Humphrey Milford and Oxford University Press. 1940. 10+304 pages. $3.50.

The author's *Differential Geometry of Curves and Surfaces*, which was published in 1909, has seen extensive use. Since that time, the tensor calculus has come to play an important role in Riemannian Geometry and in the Theory of Relativity and the author has considered it desirable to rewrite the differential geometry of curves and surfaces in terms of the tensor calculus. The differential geometry treated in the present book is about equivalent to that in the first half of the 1909 book, with the addition of the concept of parallelism in the sense of Levi-Civita.

There are four chapters. The first is concerned principally with the properties of curves in euclidean 3-space, but includes a definition of the parametric equations of a surface and the envelope of a one-parameter family of surfaces, in order to include consideration of the developables associated with a curve. The material of the chapter is for the most part included in the first two chapters of the 1909 book and except for changes in notation due to the use of the summation convention, the order and treatment have not been greatly changed. The principal addition is consideration of the theorem that the envelope of a one-parameter family of non-parallel planes is either a cone, a cylinder, or the tangent surface of a curve.

The second chapter is largely devoted to tensor algebra and calculus. Preparatory to the definition of a tensor, transformations of coordinates in euclidean 3-space and the effects on the fundamental quadratic form of such transformations are considered. Then contravariant and covariant vectors, and scalars, are defined and their geometrical significance is discussed. The definition of a tensor and the development of the algebra of tensors follow. The Christoffel symbols and Riemann tensor are introduced; covariant differentiation is defined; and the chapter closes with a classic existence theorem for a system of partial differential equations.

Chapter III deals with the intrinsic properties of a surface, that is, those properties of a surface which depend only on the coefficients of the first fundamental form. In addition to the fundamental concepts of arc length, angle and area, the concepts of Gaussian curvature, geodesic, geodesic curvature and parallelism in the sense of Levi-Civita are defined by intrinsic methods and studied in detail. Additional material includes the notions of isometric nets, isometric surfaces, differential parameters, conformal correspondence and geodesic correspondence.

Chapter IV deals with properties which depend on the imbedding of the surface in space, or upon the coefficients of the second fundamental quadratic form. The principal topics are: the equations of Gauss and Codazzi; principal radii of curvature and lines of curvature; conjugate nets and asymptotic lines; the spherical representation of a surface; tangential coordinates and lines of curvature. The chapter closes with a brief discussion of spherical and pseudo-spherical surfaces, and minimal surfaces.

The book is well organized, well written, and has excellent illustrations. There is an abundance of exercises of all varieties, some of which illustrate the material treated, others of which extend the theory. A student who has absorbed the contents of the book will know the fundamentals of differential geometry and tensor calculus and should find the passage to the study of n-dimensional Riemannian geometry easy.

The reviewer is not certain that the present text offers the most desirable introduction to the differential geometry of surfaces in three-dimensional euclidean space. The extensive use of tensors brings with it a needed emphasis on the notion of invariance. In the present case it also brings with it an order and an emphasis on symbolic manipulation which tend to obscure the geometry of the situation. For example, the Gaussian curvature is first met as a function depending on the Riemann tensor, which has been previously defined for no very obvious reason. The geometrical significance of the Gaussian curvature becomes clear only much later. As another example, principal directions for a tensor not the fundamental tensor are introduced abruptly and we must wait many pages before it becomes clear as to why anybody thought of them.

The author of a book on differential geometry faces vexing questions as regards analytical rigor. Since functions enter continually in the subject matter, a first question concerns the class (differentiability) of the functions involved. Many authors restrict their considerations to analytic functions, a method which assumes too much but which avoids pitfalls. In the present book we find the following statement on page 3: "Whenever throughout this book we consider any function, it is considered in a domain within which it is continuous in all its variables, together with such of its derivatives as are involved in the discussion." This process of asking the reader to determine the hypotheses of a theorem from the proof can be somewhat misleading. Since the proofs of theorems usually involve other theorems and since these in turn depend on still other theorems, the reader may find it difficult to determine just what his hypotheses are. Since functions are frequently expanded in series, there would appear to be the possibility of confusion between Class C^∞ and analytic. Since results obtained under the assumption of analyticity are used in the derivation of various theorems, these theorems are valid only under the assumption of analyticity, and yet no statement concerning analyticity appears in the proofs of these theorems. It seems to the reviewer that it would have been a happier situation if in each theorem the necessary class of the functions involved had been stated.

There are a number of other analytical details which appear objectionable to the reviewer, but in view of the fact that Fine's *Calculus* and the author's *Coordinate Geometry* are extensively used as references, it would appear that the attainments of the reader are not assumed to be extensive, and consequently the point of rigor should perhaps not be stressed.

Professor Eisenhart's new book is an interesting addition to the literature on differential geometry. It should prove to be highly useful.

GUSTAV A. HEDLUND